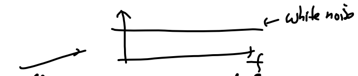
Noise Sources

Thermal Noise

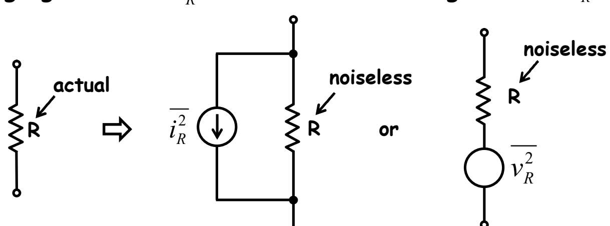
- Thermal Noise in Electronics: (Johnson noise, Nyquist noise)
  - Produced as a result of the thermally excited random motion of free e⁻'s in a conducting medium
  - Path of e⁻'s randomly oriented due to collisions
- Thermal Noise in Mechanics: (Brownian motion noise)
  - Thermal noise is associated with all dissipative processes that couple to the thermal domain
  - Any damping generates thermal noise, including gas damping, internal losses, etc.



- Properties:
  - Thermal noise is white (i.e., constant w/ frequency)
  - Proportional to temperature
  - Not associated with current
  - Present in any real physical resistor

Circuit Representation of Thermal Noise

- Thermal Noise can be shown to be represented by a series voltage generator $\overline{v_R^2}$ or a shunt current generator $\overline{i_R^2}$



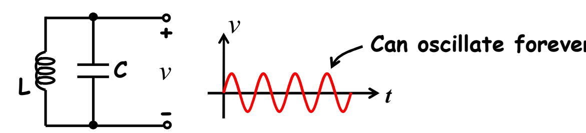
$$\frac{\overline{i_R^2}}{\Delta f} = \frac{4kT}{R} \qquad \frac{\overline{v_R^2}}{\Delta f} = 4kTR$$

where $4kT = 1.66x10^{-20} V \cdot C$
and where these are spectral densities.

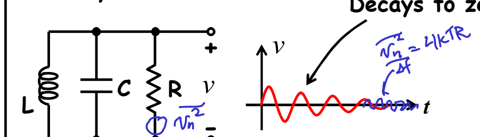
Note: These are one-sided mean-square spectral densities! To make them 2-sided, must divide by 2.

Noise in Capacitors and Inductors?

- Resistors generate thermal noise
- Capacitors and inductors are noiseless → why?



- Now, add a resistor:

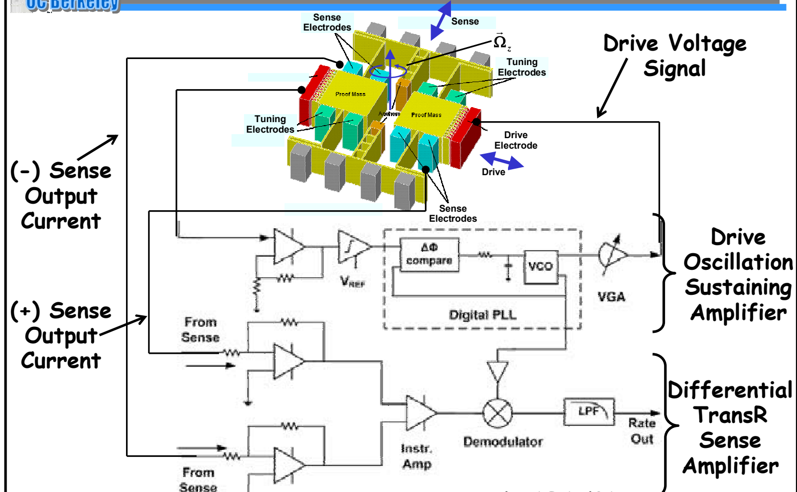


But this violates the laws of thermodynamics, which require that things be in constant motion at finite temperature

Need to add a forcing function, like a noise voltage $\overline{v_R^2}$ to keep the motion going → and this noise source is associated with R

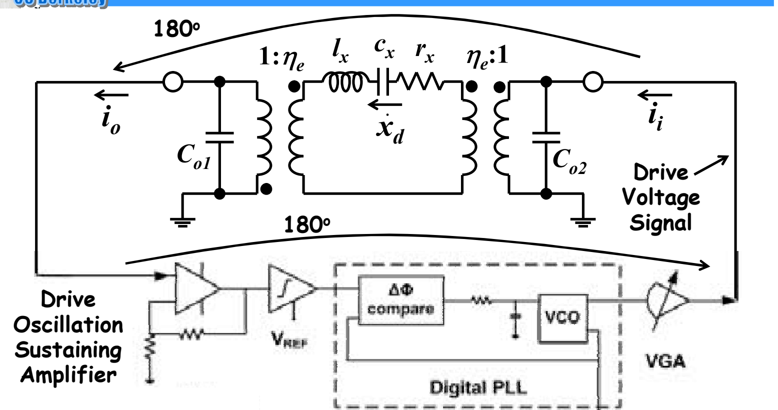
Back to Determining Sensor Resolution

MEMS-Based Tuning Fork Gyroscope



[Zaman, Ayazi, et al, MEMS'06]

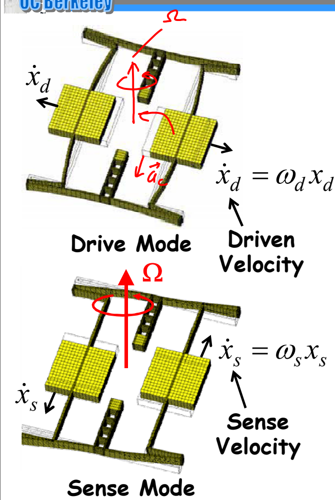
Drive Axis Equivalent Circuit



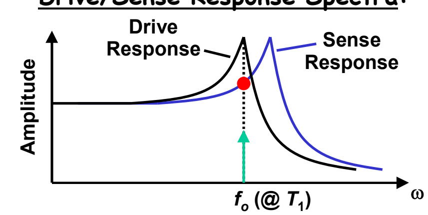
- Generates drive displacement velocity $\dot{x}_d$ to which the Coriolis force is proportional

To Sense Amplifier (for synchronization)

Drive-to-Sense Transfer Function



Drive/Sense Response Spectra:



Rotation-Induced Coriolis Force:

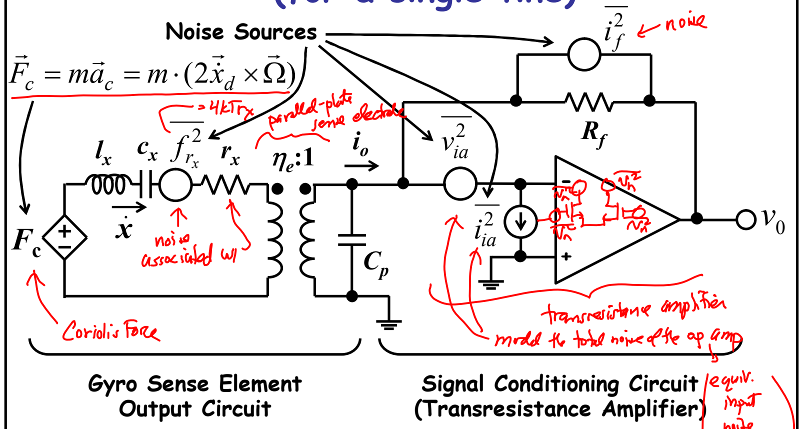
$\vec{a}_c = 2\dot{\vec{x}}_d \times \vec{\Omega} = 2\omega_d \vec{x}_d \times \vec{\Omega}$

⇒ Acts in the sense mode direction

$a_s = 2\omega_d x_d \Omega \sin 90°$

$\boxed{a_s = 2\omega_d x_d \Omega}$

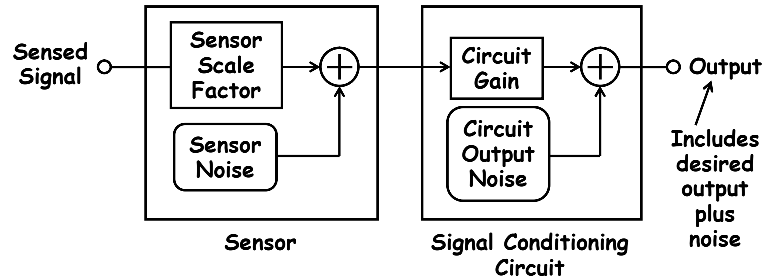
Gyro Readout Equivalent Circuit (for a single tine)



- Easiest to analyze if all noise sources are summed at a common node

Minimum Detectable Signal (MDS)

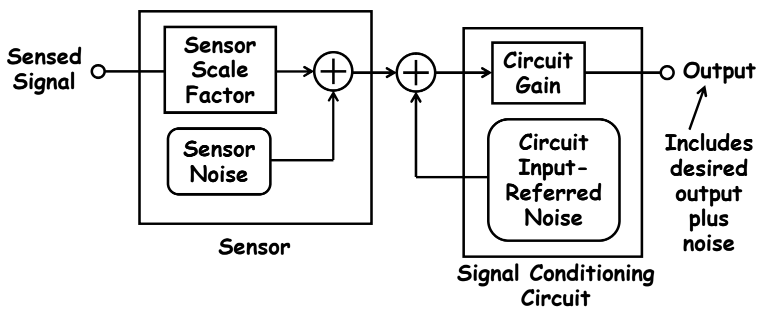
- Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity



- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

Move Noise Sources to a Common Point

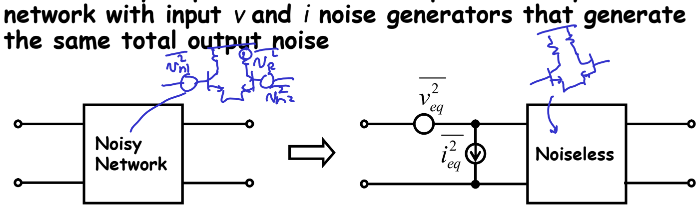
- Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)
- Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS



Equivalent Input-Referred Voltage and Current Noise Sources

Equivalent Input v, i Noise Generators

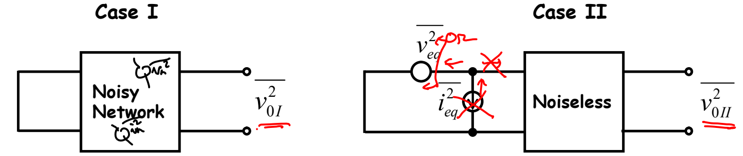
- Take a noisy 2-port network and represent it by a noiseless network with input *v* and *i* noise generators that generate the same total output noise



- Remarks:
  1. Works for linear time-invariant networks
  2. v_{eq} and i_{eq} are generally correlated (since they are derived from the same sources)
  3. In many practical circuits, one of v_{eq} and i_{eq} dominates, which removes the need to address correlation
  4. If correlation is important → easier to return to original network with internal noise sources

Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$

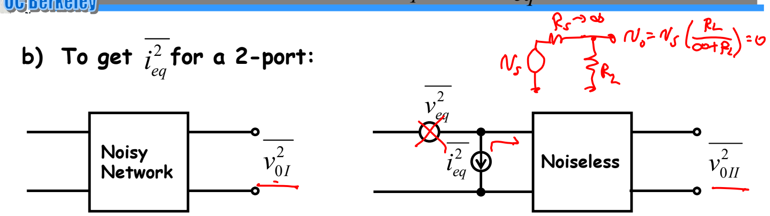
a) To get $\overline{v_{eq}^2}$ for a two-port:



1) Short input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$)
2) For eq. network, short input, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$)
$$\overline{v_{0II}^2} = f\left(\overline{v_{eq}^2}\right) \qquad \overline{i_{0II}^2} = f\left(\overline{v_{eq}^2}\right)$$
3) Set $\overline{v_{0I}^2} = \overline{v_{0II}^2}$ → solve for $\overline{v_{eq}^2}$ (or $\overline{i_{0I}^2} = \overline{i_{0II}^2}$)

Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$ (cont)

b) To get $\overline{i_{eq}^2}$ for a 2-port:



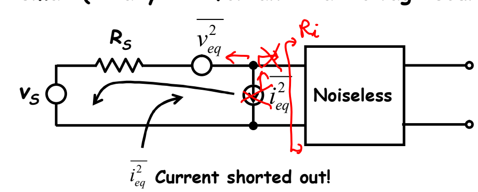
1) Open input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$)
2) Open input for eq. circuit, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$)
3) Set $\overline{v_{0I}^2} = \overline{v_{0II}^2}\left(\overline{i_{eq}^2}\right)$ → solve for $\overline{i_{eq}^2}$ (or $\overline{i_{0I}^2} = \overline{i_{0II}^2}\left(\overline{i_{eq}^2}\right)$)

- Once the equivalent input-referred noise generators are found, noise calculations become straightforward as long as the noise generators can be treated as uncorrelated

Cases Where Correlation Is Not Important

- There are two common cases where correlation can be ignored:
  1. Source resistance R_s is **small** compared to input resistance R_i → i.e., voltage source input
  2. Source resistance R_s is **large** compared to input resistance R_i → i.e., current source input

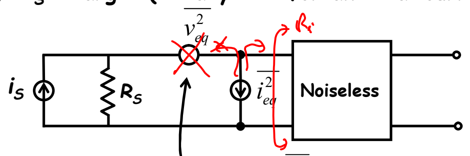
1) R_S = small (ideally = 0 for an ideal voltage source):



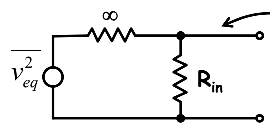
∴ For R_S= small, $\overline{i_{eq}^2}$ can be neglected → only $\overline{v_{eq}^2}$ is important!
(Thus, we need not deal with correlation)

Cases Where Correlation Is Not Important

2) R_S = large (Ideally = ∞ for an ideal current source)



Voltage $\overline{v_{eq}^2}$ effectively "opened" out!



$$v_i = \frac{R_{in}}{\infty + R_{in}} v_{eq} = 0!$$

∴ For R_S= large, $\overline{v_{eq}^2}$ can be neglected!

→ only $\overline{i_{eq}^2}$ is important!

(... and again, we need not deal with correlation)