

Lecture 27m1: Noise & MDS

Equivalent Input-Referred Voltage and Current Noise Sources

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LecM 17

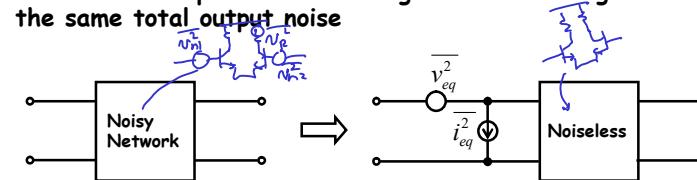
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Equivalent Input v , i Noise Generators

- Take a noisy 2-port network and represent it by a noiseless network with input v and i noise generators that generate the same total output noise



• Remarks:

- Works for linear time-invariant networks
- v_{eq} and i_{eq} are generally correlated (since they are derived from the same sources)
- In many practical circuits, one of v_{eq} and i_{eq} dominates, which removes the need to address correlation
- If correlation is important → easier to return to original network with internal noise sources

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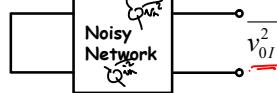
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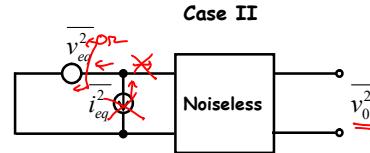
Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$

- a) To get $\overline{v_{eq}^2}$ for a two-port:

Case I



Case II



- Short input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$)
- For eq. network, short input, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$)
- Set $\overline{v_{0I}^2} = \overline{v_{0II}^2} \rightarrow$ solve for $\overline{v_{eq}^2}$ (or $\overline{i_{eq}^2} = \overline{i_{0I}^2} = \overline{i_{0II}^2}$)

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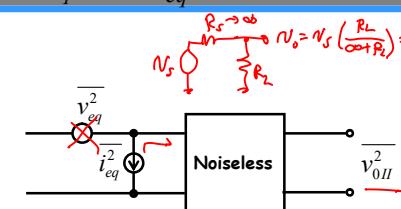
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Calculation of $\overline{v_{eq}^2}$ and $\overline{i_{eq}^2}$ (cont)

- b) To get $\overline{i_{eq}^2}$ for a 2-port:



- Open input, find $\overline{v_{0I}^2}$ (or $\overline{i_{0I}^2}$)
- Open input for eq. circuit, find $\overline{v_{0II}^2}$ (or $\overline{i_{0II}^2}$)
- Set $\overline{v_{0I}^2} = \overline{v_{0II}^2} \overline{i_{eq}^2} \rightarrow$ solve for $\overline{i_{eq}^2}$ (or $\overline{i_{0I}^2} = \overline{i_{0II}^2} \overline{i_{eq}^2}$)

- Once the equivalent input-referred noise generators are found, noise calculations become straightforward as long as the noise generators can be treated as uncorrelated

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Cases Where Correlation Is Not Important

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- There are two common cases where correlation can be ignored:
 - Source resistance R_s is **small** compared to input resistance $R_i \rightarrow$ i.e., voltage source input
 - Source resistance R_s is **large** compared to input resistance $R_i \rightarrow$ i.e., current source input

1) $R_s = \text{small}$ (ideally = 0 for an ideal voltage source):

... For $R_s = \text{small}$, $\overline{i_{eq}^2}$ can be neglected \rightarrow only $\overline{v_{eq}^2}$ is important!
(Thus, we need not deal with correlation)

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Cases Where Correlation Is Not Important

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2) $R_s = \text{large}$ (Ideally = ∞ for an ideal current source)

Voltage $\overline{v_{eq}^2}$ effectively "opened" out!

$$\overline{v_i} = \frac{R_{in}}{\infty + R_{in}} \overline{v_{eq}^2} = 0!$$

... For $R_s = \text{large}$, $\overline{v_{eq}^2}$ can be neglected!
 \rightarrow only $\overline{i_{eq}^2}$ is important!
(... and again, we need not deal with correlation)

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Example: TransR Amplifier Noise

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Input-referred current noise:
Open inputs; equate output voltage noise.

Case I:
 $N_{0I1} = \overline{i_{ia}^2} R_f$
 $N_{0I2} = \overline{i_f^2} R_f$
 $N_{0I3} = \overline{i_{ia}^2} \xrightarrow{R_i} \overline{i_{ia}^2}$
 More N_{ia} through R_{in} to
This is unity gain!

$$\therefore N_{0I}^2 = \overline{i_{ia}^2} R_f^2 + \overline{i_f^2} R_f^2 + \overline{i_{ia}^2}$$

Case II:
 $\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2}$
 $\therefore N_{0II}^2 = \overline{i_{eq}^2} R_f^2$
 $\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2} + \frac{\overline{i_{ia}^2}}{R_f^2}$

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Example: TransR Amplifier Noise (cont)

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Input-referred voltage noise:
Short inputs; equate output voltage noise

Case I:
 $\overline{N_{0I}^2} = \overline{N_{ia}^2} a^2$
 (Both $\overline{i_{ia}^2}$ & $\overline{i_f^2}$ are shorted out.)

Case II:
 $\overline{N_{0II}^2} = \overline{N_{eq}^2} a^2$
 $\therefore \overline{N_{eq}^2} = \overline{N_{ia}^2}$

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Example: TransR Amplifier Noise (cont)

$$\overline{i_{eq}^2} = \overline{i_{ia}^2} + \overline{i_f^2} + \frac{\overline{v_{ia}^2}}{R_f^2}$$

$$\overline{v_{eq}^2} = \overline{v_{ia}^2}$$

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Back to Gyro Noise & MDS

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Example: Gyro MDS Calculation

- The gyro sense presents a large effective source impedance
- Currents are the important variable; voltages are "opened" out
- Must compare i_o with the total current noise i_{eqTOT} going into the amplifier circuit

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Example: Gyro MDS Calculation (cont)

- First, find the rotation to i_o transfer function:

$$\dot{x}_s = \frac{\omega_s Q}{k_s} \Theta_s(j\omega_d) F_s = \frac{\omega_s Q}{k_s} \cdot 2\omega_d r_d S_m \cdot \Theta(j\omega_d)$$

$$[F_s = F_c = 2\omega_d r_d S_m]$$

$$\dot{x}_s = 2 \frac{\omega_d}{\omega_s} Q r_d \Theta(j\omega_d) \cdot S_m$$

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Example: Gyro MDS Calculation (cont)

$i_o = \eta_e \dot{x}_s = 2 \frac{w_d}{w_s} Q \chi_d \eta_e \Theta(j\omega_d) \cdot \dot{x}_s \rightarrow i_o = A \dot{x}_s$
Where $A = 2 \frac{w_d}{w_s} Q \chi_d \eta_e \Theta(j\omega_d)$

When $\dot{x}_s = \dot{x}_{s\min} \triangleq \text{MDS}$, $i_o = i_{eqTOT}$ input-referred noise current entering the sense amplifier \rightarrow in $\text{pA}/\sqrt{\text{Hz}}$

$\therefore i_{eqTOT} = A \dot{x}_{s\min} \rightarrow \dot{x}_{s\min} = \frac{i_{eqTOT}}{A} \left(\frac{3600\pi}{\text{hr}} \right) \left(\frac{180^\circ}{\pi} \right) \left[(\%)/\sqrt{\text{Hz}} \right]$

Angle Random Walk: $\text{ARW} = \frac{1}{60} \dot{x}_{s\min} [\%/\text{hr}]$

Earlier to determine directional error as a function of elapsed time.

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Example: Gyro MDS Calculation (cont)

$\vec{F}_c = m \vec{a}_c = m \cdot (2 \vec{x}_d \times \vec{\Omega})$

$F_c = l_x c_x \vec{f}_{r_x}^2 r_x \eta_e : 1$

$i_o = \eta_e \dot{x}_s$

$i_{eqTOT} = \sqrt{\frac{i_s^2 + i_{ia}^2 + i_f^2 + \frac{N^2}{R_f}}{R_s}}$

$R_s: \text{large} \therefore N^2 \text{"opened" out}$

Now, find the i_{eqTOT} entering the amplifier input:

$i_{eqTOT} = i_s + i_{ia} \rightarrow i_{eqTOT} = \sqrt{i_s^2 + i_{ia}^2 + i_f^2 + \frac{N^2}{R_f^2}}$

$\vec{f}_{r_x}^2 = 4kT r_x$

Brownian motion noise of the sense element \rightarrow determined entirely by the noise in $r_x \rightarrow \vec{f}_{r_x}^2$

easiest to convert to an all electrical equiv. ckt.

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Example: Gyro MDS Calculation (cont)

$N \vec{R}_x = 4kT R_x$

where $L_x = \frac{R_x}{\eta_e}$, $C_x = \eta_e^2 C_x$, $R_x = \frac{r_x}{\eta_e}$

$\therefore i_s = N R_x \left(\frac{1}{R_x} \right) \Theta(j\omega_d) \rightarrow \frac{i_s^2}{\Delta f} = 4kT R_x \left(\frac{1}{R_x^2} \right) |\Theta(j\omega_d)|^2$

$\Rightarrow \frac{i_s^2}{\Delta f} = \frac{4kT}{R_x^2} |\Theta(j\omega_d)|^2$

Thus:

$\frac{i_{eqTOT}^2}{\Delta f} = \frac{4kT}{R_x} |\Theta(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{i_{ia}^2}{\Delta f} + \frac{N^2}{R_f^2} \left(\frac{1}{R_f^2} \right)$

Learn to get these from EE240.
or just get them from a data sheet ...

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LF356 Op Amp Data Sheet

LF155/LF156/LF256/LF257/LF355/LF356/LF357 JFET Input Operational Amplifiers

General Description

These are the first monolithic JFET input operational amplifiers to incorporate well matched, high voltage JFETs on the same chip with standard bipolar transistors (Bi-FET™ Technology). These amplifiers feature low input bias and offset currents/low offset voltage and offset voltage drift, coupled with offset adjust which does not degrade drift or common-mode rejection. The devices are also designed for high slew rate, wide bandwidth, extremely fast settling time, low voltage and current noise and a low 1/f noise corner.

Features

Advantages

- Replace expensive hybrid and module FET op amps.
- Rugged JFETs allow blow-out free handling compared with MOSFET input devices.
- Excellent for low noise applications using either high or low source impedance—very low 1/f corner.
- Offset adjust does not degrade drift or common-mode rejection as in most monolithic amplifiers.
- New output stage allows use of large capacitive loads (5,000 pF) without stability problems.
- Internal compensation and large differential input voltage capability.

Common Features

- Low input bias current: 30pA
- Low Input Offset Current: 3pA
- High Input Impedance: $10^{12}\Omega$
- Low Input Noise Current: 0.01 pA/ $\sqrt{\text{Hz}}$
- High Common-Mode Rejection Ratio: 100 dB
- Large DC Voltage Gain: 106 dB

Uncommon Features

	LF155/ LF156/	LF257/	Units
	LF355	LF256/	LF357
	(A _v =5)		
Extremely fast settling time to 0.01%	4	1.5	1.5 μs
Fast slew rate	5	12	50 V/ μs
Wide gain bandwidth	2.5	5	20 MHz
Low Input noise voltage	20	12	$\text{nV}/\sqrt{\text{Hz}}$

$\frac{i_{ia}^2}{\Delta f} = 0.01 \text{ pA}/\sqrt{\text{Hz}}$

$\frac{i_{ia}^2}{\Delta f} = 12 \text{ nV}/\sqrt{\text{Hz}}$

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Example ARW Calculation

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- Example Design:**
 - Sensor Element:
 $m = (100\mu\text{m})(100\mu\text{m})(20\mu\text{m})(2300\text{kg/m}^3) = 4.6 \times 10^{-10}\text{kg}$
 $\omega_s = 2\pi(15\text{kHz})$
 $\omega_d = 2\pi(10\text{kHz})$
 $k_s = \omega_s^2 m = 4.09 \text{ N/m}$
 $x_d = 20 \mu\text{m}$
 $Q_s = 50,000$
 $V_p = 5\text{V}$
 $h = 20 \mu\text{m}$
 $d = 1 \mu\text{m}$
 - Sensing Circuitry:
 $R_f = 100\text{k}\Omega$
 $i_{ia} = 0.01 \text{ pA}/\sqrt{\text{Hz}}$
 $v_{ia} = 12 \text{ nV}/\sqrt{\text{Hz}}$

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Example ARW Calculation (cont)

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Get rotation rate to output current scale factor:

$$A = 2 \frac{w_d}{\omega_s} Q_s \chi_d \eta_e |\Theta(j\omega_d)| = 2 \left(\frac{10\text{K}}{15\text{K}} \right) (50\mu\text{m}) (5) (2000\epsilon_0) (0.00024) = 2.83 \times 10^{-12} \text{ C}$$

$$\Theta(j\omega_d) = \frac{(j\omega_d)(\omega_s/\omega_s)}{-\omega_d^2 + j\omega_d\omega_s + \omega_s^2} = \frac{j(10\text{K})(15\text{K})/(50\text{K})}{(15\text{K})^2 - (10\text{K})^2 + j(10\text{K})/(5\text{K})} = \frac{j(3\text{K})}{1.25 \times 10^8 + j(3\text{K})}$$

$$\Rightarrow |\Theta(j\omega_d)| = \frac{3\text{K}}{\sqrt{(1.25 \times 10^8)^2 + (3\text{K})^2}} = 0.000024 \quad 8.854 \times 10^{-8} \text{ F/m}$$

$$\left[\frac{\partial C}{\partial x} = \frac{C_0}{d} = \frac{\epsilon_0 h W_p}{d} = \frac{\epsilon_0 (20\mu\text{m})(100\mu\text{m})}{(1\mu\text{m})^2} = 2000\epsilon_0 \rightarrow \eta_e = V_p \frac{\partial C}{\partial x} = 5(2000\epsilon_0) \quad \begin{matrix} \uparrow \\ \text{Assume electrode covers} \\ \text{the whole sidewall.} \end{matrix} \quad 8.854 \times 10^{-12} \text{ F/m} \right]$$

Then, get noise:

$$\frac{i_{eq,TOT}^2}{\Delta f} = \frac{4kT}{R_f} |\Theta(j\omega_d)|^2 + \frac{4kT}{R_f} + \frac{i_{ia}^2}{R_f} + \frac{V_{ia}^2}{R_f^2} \left(\frac{1}{\Delta f} \right)$$

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Example ARW Calculation (cont)

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$$R_{eq} = \frac{w_s m}{Q_s \eta_e^2} = \frac{2\pi(15\text{K})(4.6 \times 10^{-10})}{(50\text{K})(8.854 \times 10^{-8})^2} = 110.6 \text{ k}\Omega$$

$$\frac{i_{eq,TOT}^2}{\Delta f} = \frac{(1.66 \times 10^{-29})}{(110.6\text{K})} (0.000024)^2 + \frac{(1.66 \times 10^{-29})}{1\text{M}} + (0.01\text{p})^2 + \frac{(12\text{n})^2}{(1\text{M})^2}$$

$$\xrightarrow{8.64 \times 10^{-25} \text{ A}^2/\text{Hz}} \xrightarrow{1.66 \times 10^{-26} \text{ A}^2/\text{Hz}} \xrightarrow{1 \times 10^{-28} \text{ A}^2/\text{Hz}} \xrightarrow{1.44 \times 10^{-28} \text{ A}^2/\text{Hz}}$$

sensor element noise
Insignificant

Noise from R_f dominates!

$$\therefore \frac{i_{eq,TOT}^2}{\Delta f} = 1.68 \times 10^{-26} \text{ A}^2/\text{Hz} \rightarrow i_{eq,TOT} = \sqrt{\frac{i_{eq,TOT}^2}{\Delta f}} = 1.30 \times 10^{-13} \text{ A}/\sqrt{\text{Hz}}$$

$$\therefore \Sigma_{min} = \frac{i_{eq,TOT}}{A} \left(\frac{3600\pi}{\text{hr}} \right) \left(\frac{180^\circ}{\pi} \right) = \frac{1.30 \times 10^{-13}}{2.83 \times 10^{-12}} (3600) \left(\frac{180}{\pi} \right) = 9448 (\%/\text{hr})/\sqrt{\text{Hz}}$$

And finally:
 $ARW = \frac{1}{60} \Sigma_{min} = \frac{1}{60} (9448) = 157 \%/\text{hr} = ARW$ \rightarrow Almost turned around in 1 hour!

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What if $\omega_d = \omega_s$?

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If $\omega_d = \omega_s = 15\text{kHz}$, then $|\Theta(j\omega_d)| = 1$ and

$$A = 2 \frac{w_d}{\omega_s} Q_s \chi_d \eta_e = 2 Q_s \chi_d \eta_e = 2(50\mu\text{m}) (20\mu\text{m}) (5) (2000\epsilon_0) = 1.77 \times 10^{-7} \text{ C}$$

$$\frac{i_{eq,TOT}^2}{\Delta f} = \frac{(1.66 \times 10^{-29})}{(110.6\text{K})} (1)^2 + \frac{(1.66 \times 10^{-29})}{1\text{M}} + (0.01\text{p})^2 + \frac{(12\text{n})^2}{(1\text{M})^2}$$

$$\xrightarrow{1.51 \times 10^{-25} \text{ A}^2/\text{Hz}} \xrightarrow{1.66 \times 10^{-26} \text{ A}^2/\text{Hz}} \xrightarrow{1 \times 10^{-28} \text{ A}^2/\text{Hz}} \xrightarrow{1.44 \times 10^{-28} \text{ A}^2/\text{Hz}}$$

Now, the sensor element dominates!

$$\therefore \frac{i_{eq,TOT}^2}{\Delta f} = 1.67 \times 10^{-25} \text{ A}^2/\text{Hz} \rightarrow i_{eq,TOT} = \sqrt{\frac{i_{eq,TOT}^2}{\Delta f}} = 4.08 \times 10^{-13} \text{ A}/\sqrt{\text{Hz}}$$

$$\therefore \Sigma_{min} = \frac{i_{eq,TOT}}{A} \left(\frac{3600\pi}{\text{hr}} \right) \left(\frac{180^\circ}{\pi} \right) = \frac{4.08 \times 10^{-13}}{1.77 \times 10^{-7}} (3600) \left(\frac{180}{\pi} \right) = 0.476 (\%/\text{hr})/\sqrt{\text{Hz}}$$

And finally:
 $ARW = \frac{1}{60} \Sigma_{min} = \frac{1}{60} (0.476) = 0.0079 \%/\text{hr} = ARW$ \rightarrow Navigation grade!

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