



EE C247B - ME C218
Introduction to MEMS Design
Spring 2016

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Lecture Module 14: Sensing Circuits

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Lecture Outline

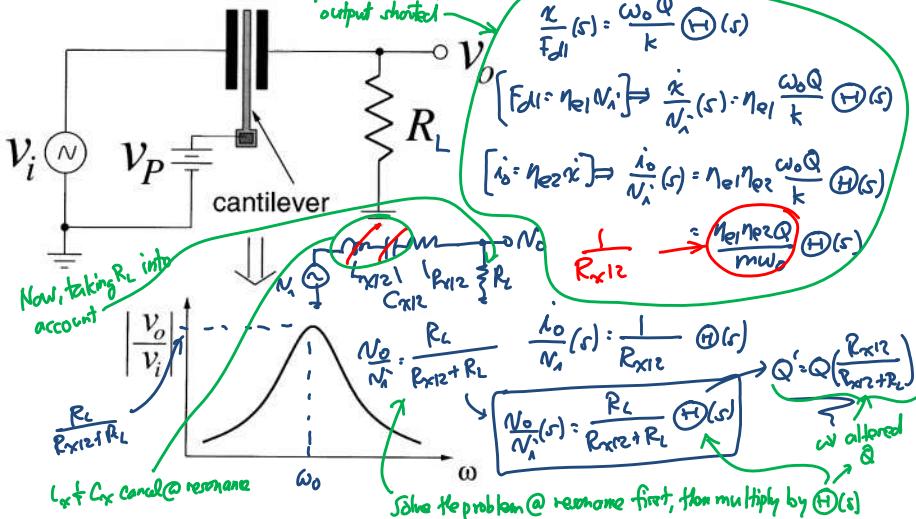
- **Reading:** Senturia, Chpt. 14
- **Lecture Topics:**
 - ↳ **Detection Circuits**
 - **Velocity Sensing**
 - **Position Sensing**

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Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load

Consider the mechanical device by itself first, w



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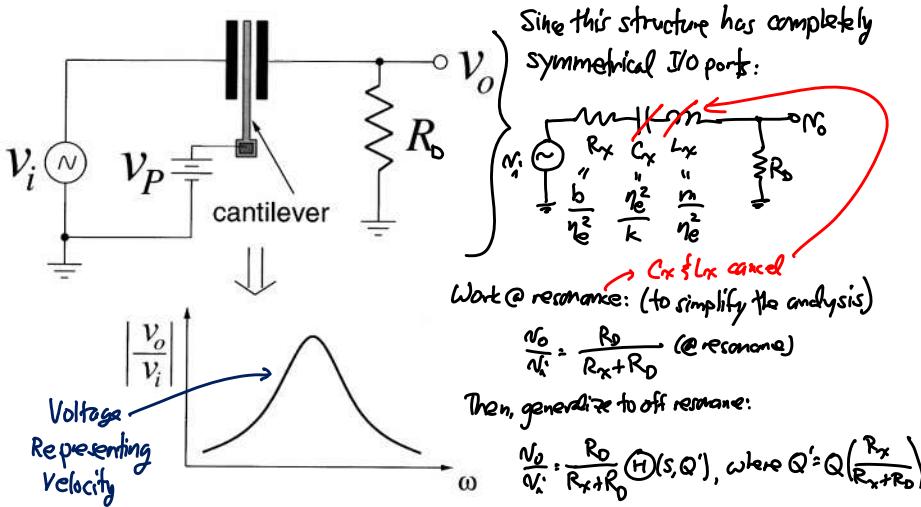
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Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load



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Velocity-to-Voltage Conversion

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- To convert velocity to a voltage, use a resistive load

Since this structure has completely symmetrical I/O ports:

$$Q = \frac{\omega_0 L_x}{R_x} \rightarrow Q' = \frac{\omega_0 L_x}{R_x + R_D} \rightarrow \frac{R_x + R_D}{L_x} = \frac{\omega_0}{Q'}$$

Brute force approach:

$$\frac{N_0}{V_i}(s) = \frac{R_D}{R_x + \frac{1}{sC_x} + sL_x + R_D} = \frac{sR_D C_x}{sR_D C_x + 1 + s^2 L_x C_x + sR_x C_x} = \frac{s \frac{R_D}{L_x}}{s^2 + s \frac{R_x + R_D}{L_x} + \frac{1}{L_x C_x}}$$

$$= \frac{R_D}{R_x + R_D} \frac{s \left(\frac{R_x + R_D}{L_x} \right)}{s^2 + s \left(\frac{R_x + R_D}{L_x} \right) + \frac{1}{L_x C_x}}$$

$$\Rightarrow \frac{N_0}{V_i}(s) = \frac{R_D}{R_x + R_D} \frac{s (\omega_0 / Q')}{s^2 + s (\omega_0 / Q') + \omega_0^2} = \frac{R_D}{R_x + R_D} H(s, Q')$$

$Q' = Q \left(\frac{R_x}{R_x + R_D} \right)$

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Position-to-Voltage Conversion

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- To sense position (i.e., displacement), use a capacitive load

Again, here port-to-port I/O symmetry:

Brute force approach:

$$\frac{N_0}{V_i}(s) = \frac{1}{sC_D}$$

$$\frac{N_0}{V_i}(s) = \frac{\frac{C_x}{C_D}}{sR_x C_x + 1 + s^2 L_x C_x + \frac{1}{sC_D}}$$

$$= \frac{C_x / C_D}{1 + C_x / C_D} \frac{\frac{1}{s^2 L_x C_x}}{1 + \frac{sR_x C_x}{1 + C_x / C_D} + s^2 \frac{1}{1 + C_x / C_D}}$$

$$= \frac{C_x / C_D}{1 + C_x / C_D} \frac{\frac{1}{s^2 L_x C_x}}{1 + \frac{sR_x C_x}{L_x C_x} + s^2 \left(\frac{R_x}{L_x} + \frac{1 + C_x / C_D}{L_x C_x} \right)}$$

$$\left[\omega_0^2 \frac{1}{L_x C_x} \rightarrow (\omega_0')^2 = \omega_0^2 (1 + C_x / C_D) \right]$$

$$\left[Q' = \frac{\omega_0' L_x}{R_x} = \frac{R_x}{L_x} \frac{\omega_0'}{Q'}, Q' = Q \sqrt{1 + C_x / C_D} \right] \text{ over}$$

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Position-to-Voltage Conversion

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- To sense position (i.e., displacement), use a capacitive load

$$\frac{V_o}{V_i}(s) = \frac{C_x/C_D}{1 + C_x/C_D} \cdot \frac{(w_0')^2}{s^2 + (w_0'/Q')s + (w_0')^2}$$

DC Gain Term Low-Pass Biquad
 To maximize gain $\rightarrow 1$, need $C_x \ll C_D$.
 (must minimize C_D)

Note: Can we similar short-cut to the R case.

- Get DC response $\rightarrow C$'s dominate.
- Then:

$$\frac{V_o}{V_i}(s) = (\text{DC Gain}) \cdot \frac{1}{s} \cdot \Theta(s, w_0', Q') \cdot w_0' Q'$$

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Velocity Sensing Circuits

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Velocity-to-Voltage Conversion

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- To convert velocity to a voltage, use a resistive load

Since this structure has completely symmetrical I/O ports:

$$\frac{V_o}{V_i} = \frac{R_D}{R_x + R_D}$$

Work @ resonance: (to simplify the analysis)

$$\frac{V_o}{V_i} = \frac{R_D}{R_x + R_D} \quad (@\text{resonance})$$

Then, generalize to off resonance:

$$\frac{V_o}{V_i} = \frac{R_D}{R_x + R_D} H(s, Q'), \text{ where } Q' = Q \left(\frac{R_x}{R_x + R_D} \right)$$

Cx & Lx cancel

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Problems With Purely Resistive Sensing

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Now, we get: (approximately)

$$\frac{V_o}{V_i}(s) \sim \frac{R_D}{R_x + R_D} \cdot \frac{1}{1 + \frac{s}{\omega_p}} \cdot H(s, \omega_b, Q')$$

$\omega_p = \frac{1}{(R_x || R_D) C_p}$

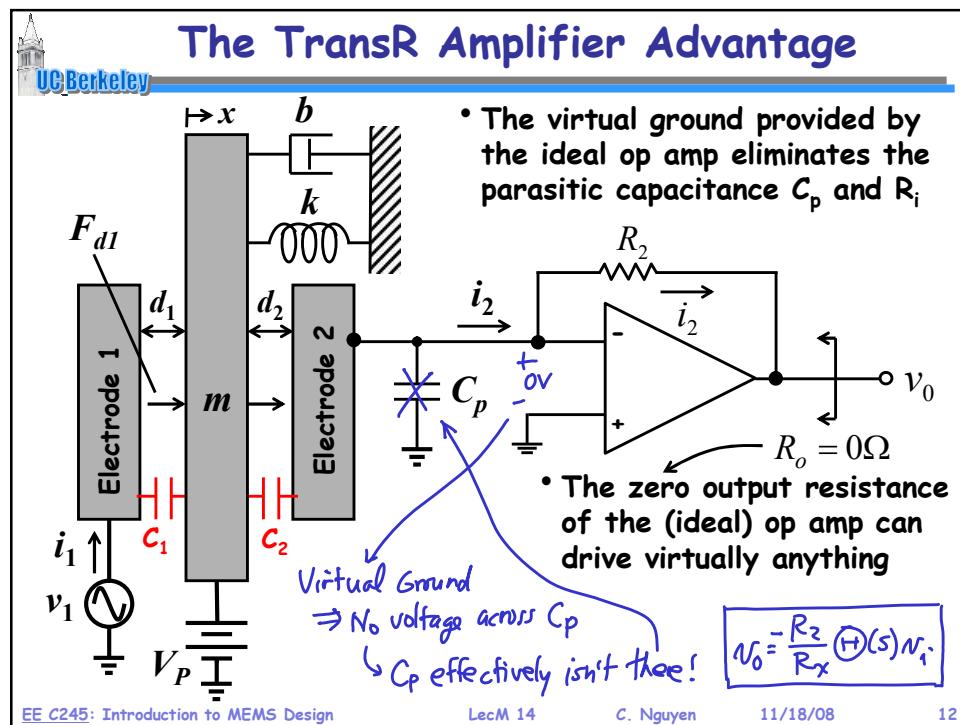
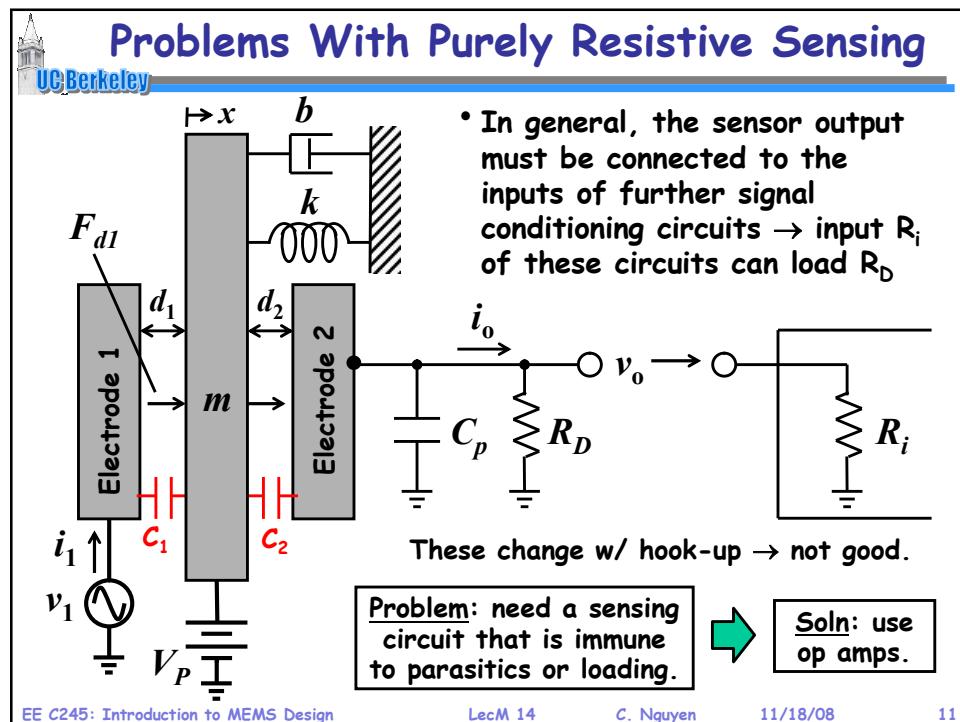
Depend on both R_D & C_p .

Impact depends on where ω_p is relative to ω_0 .

Not Good *Okay*

ω_c ω_b ω_D

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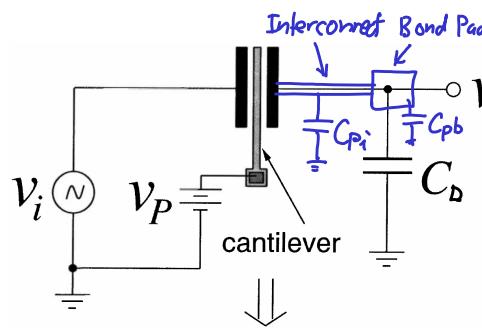
Position Sensing Circuits

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Problems With Pure-C Position Sensing

- To sense position (i.e., displacement), use a capacitive load



$$\frac{V_O}{V_i} = \frac{C_x/C_D}{1 + C_x/C_D} \cdot \frac{1}{s} \cdot H(s, \omega_0, Q') \cdot \omega_0 \alpha$$

Integration yields displacement.

To maximize gain, minimize C_D .
 ⇒ Problem: parasitic capacitance
 $C_D \rightarrow C_D + C_{Pi} + C_{Pb}$

⇒ DC Gain: $\frac{C_x/(C_D + C_{Pi} + C_{Pb})}{1 + C_x/(C_D + C_{Pi} + C_{Pb})}$

Output will get smaller!

Remedy: Suppress C_D via use of op amps.

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The Op Amp Integrator Advantage

The diagram shows a MEMS mass-spring system. A mass m is attached to a spring with stiffness k , which is fixed to a wall. Two electrodes, "Electrode 1" and "Electrode 2", are positioned facing the mass. The distance between Electrode 1 and the mass is d_1 , and the distance between the mass and Electrode 2 is d_2 . A force F_{d1} acts on Electrode 1. The center of mass is at position x . The system is subject to parasitic capacitance C_p between the electrodes and ground.

The output current i_o is measured between Electrode 2 and ground. The input voltage v_1 is applied to Electrode 1 through a capacitor C_1 . The output voltage v_o is sensed across a capacitor C_2 .

The op-amp integrator circuit consists of a non-inverting input connected to ground, a resistor R_2 , a capacitor C_2 , and a feedback resistor $R_f = 0\Omega$. The output v_o is fed back through R_f to the inverting input of the op-amp.

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p .

Handwritten notes and equations:

$$N_o = -i_o \left(R_2 \frac{1}{sC_2} \right)$$

$$\approx - \frac{V_i}{R_x} \frac{1}{sC_2} \xrightarrow{\text{well defined}} \text{good!}$$

$$\frac{N_o}{N_s}(s) = - \frac{1}{R_x C_2} \frac{H(s)}{s}$$

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Differential Position Sensing

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Differential Position Sensing

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- Example: ADXL-50

Tethers with fixed ends

Proof Mass

Sense Finger

Applied Acceleration

Fixed Electrodes

V_p

C_1

C_2

V_o

C_p

$V_o = -V_p + \frac{(2V_p)}{C_1+C_2}$

$$= -\frac{V_p C_1 - V_p C_2 + 2V_p C_1}{C_1+C_2} = \frac{V_p (C_1 \cdot C_2)}{C_1+C_2} = V_o$$

Issue: Parasitic Capacitance

$$V_o = \frac{C_1 \cdot C_2}{C_1+C_2+C_p} V_p$$

As before, C_p reduces gain → Soln: Use op amp!

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Buffer-Bootstrapped Position Sensing

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$+V_p$

$-V_p$

V_0

C_p

C_{gd}

Unity Gain Buffer

$C_{gd} = \text{gate-to-drain capacitance of the input MOS transistor}$

- Bootstrap the ground lines around the interconnect and bond pads
 - ↳ No voltage across C_p
 - ↳ It's effectively not there!

Interconnect

Ground Plane

1x

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Effect of Finite Op Amp Gain

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+ V_P Total ADXL-50 Sense $C \sim 100\text{fF}$

- V_P $N_o = A_o(N_{+} - N_{-}) = A_o(V_i - V_o) \rightarrow N_o(1+A_o) = A_o N_i \rightarrow \frac{N_o}{N_i} = \frac{A_o}{1+A_o}$

Get $Z_i = \frac{V_i}{i_i}$: $i_i = (N_i - N_o) sC_p = N_i(1 - \frac{A_o}{1+A_o}) sC_p = N_i \frac{1}{1+A_o} sC_p$

$\therefore \frac{N_i}{i_i} : Z_i = \frac{1}{s[\frac{C_p}{1+A_o}]} \rightarrow C_{eff} = \frac{C_p}{1+A_o}$

Ex: $A_o = 100, C_p = 2\text{pF} \Rightarrow C_{eff} = \frac{2\text{pF}}{101} = 20\text{fF}$

No longer zero!

Not negligible compared w/ ADXL-50 $C_{tot} \sim 100\text{fF}!$

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Integrator-Based Diff. Position Sensing

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+ V_P

- V_P $i_o = i_1 + i_2 = N_p(sC_1) - N_p(sC_2) = V_p s(C_1 - C_2)$

$\therefore N_o = -i_o (\frac{1}{sC_F}) = -N_p (\frac{C_1 - C_2}{C_F})$

$\frac{N_o}{V_P} = -\frac{C_1 - C_2}{C_F}$

$R_2 >> \frac{1}{sC_2}$ (for biasing)

Can drive next stage's R_i w/o interference to transfer function!

A seemingly perfect differential Sensor/amplifier output! ...but only when the op amp is ideal...

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