



**EE C247B - ME C218**  
**Introduction to MEMS Design**  
**Spring 2016**

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**Lecture Module 14: Sensing Circuits**

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**Lecture Outline**

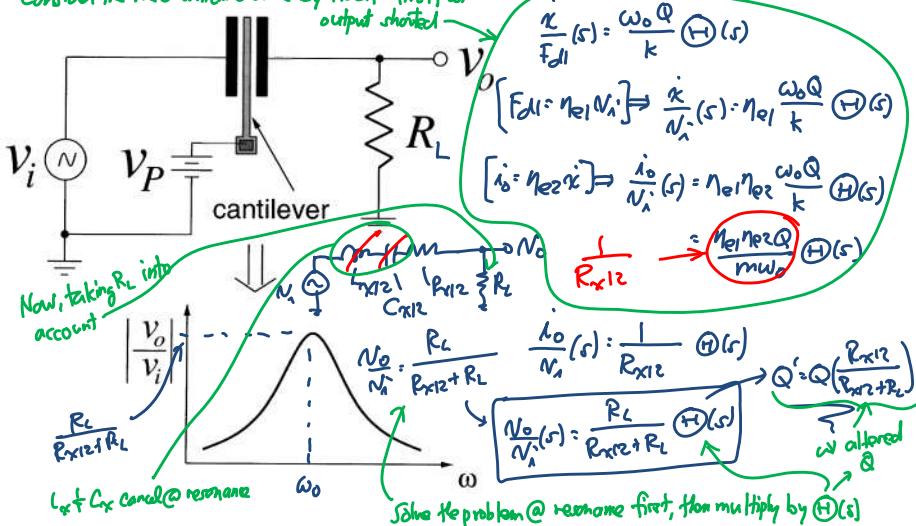
- **Reading:** Senturia, Chpt. 14
- **Lecture Topics:**
  - ↳ **Detection Circuits**
    - **Velocity Sensing**
    - **Position Sensing**

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# Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load

Consider the mechanical device by itself first, we



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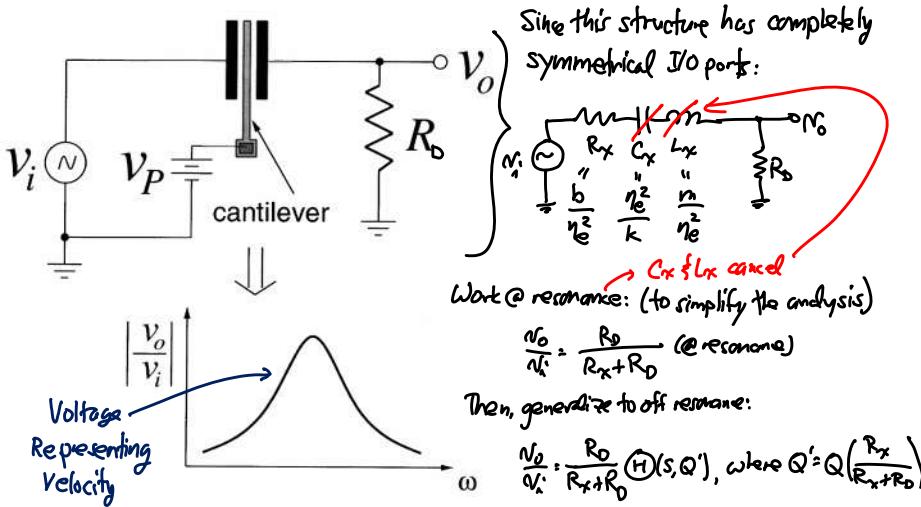
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# Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load



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## Velocity-to-Voltage Conversion

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- To convert velocity to a voltage, use a resistive load

Since this structure has completely symmetrical I/O ports:

$$Q = \frac{\omega_0 L_x}{R_x} \rightarrow Q' = \frac{\omega_0 L_x}{R_x + R_D} \rightarrow \frac{R_x + R_D}{L_x} = \frac{\omega_0}{Q'}$$

Brute force approach:

$$\frac{N_0(s)}{V_i} = \frac{R_D}{R_x + \frac{1}{sC_x} + sL_x + R_D} = \frac{sR_D C_x}{sR_D C_x + 1 + s^2 L_x C_x + sR_x C_x} = \frac{s \frac{R_D}{L_x}}{s^2 + s \frac{R_x + R_D}{L_x} + \frac{1}{L_x C_x}}$$

$$= \frac{R_D}{R_x + R_D} \frac{s \left( \frac{R_x + R_D}{L_x} \right)}{s^2 + s \left( \frac{R_x + R_D}{L_x} \right) + \frac{1}{L_x C_x}}$$

$$\Rightarrow \frac{N_0(s)}{V_i} = \frac{R_D}{R_x + R_D} \frac{s (\omega_0 / Q')}{s^2 + s (\omega_0 / Q') + \omega_0^2} = \frac{R_D}{R_x + R_D} H(s, Q')$$

$Q' = Q \left( \frac{R_x}{R_x + R_D} \right)$

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## Position-to-Voltage Conversion

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- To sense position (i.e., displacement), use a capacitive load

Again, here port-to-port I/O symmetry:

Brute force approach:

$$\frac{N_0(s)}{V_i} = \frac{1}{sC_D}$$

$$\frac{N_0(s)}{V_i} = \frac{\frac{C_x}{C_D}}{sR_x C_x + 1 + s^2 L_x C_x + \frac{1}{sC_D}}$$

$$= \frac{C_x / C_D}{1 + C_x / C_D} \frac{\frac{1}{L_x C_x}}{s^2 + s \left( \frac{R_x}{L_x} \right) + \frac{1 + C_x / C_D}{L_x C_x}}$$

$$\left[ \omega_0^2 \frac{1}{L_x C_x} \rightarrow (\omega_0')^2 = \omega_0^2 (1 + C_x / C_D) \right]$$

$$\left[ Q' = \frac{\omega_0' L_x}{R_x} = \frac{R_x}{L_x} \frac{\omega_0'}{Q'}, Q' = Q \sqrt{1 + C_x / C_D} \right] \text{ over } \rightarrow$$

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## Position-to-Voltage Conversion

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- To sense position (i.e., displacement), use a capacitive load

$$\frac{V_o}{V_i}(s) = \frac{C_x/C_D}{1 + C_x/C_D} \cdot \frac{(w_0')^2}{s^2 + (w_0'/Q')s + (w_0')^2}$$

DC Gain Term      Low-Pass Biquad  
 To maximize gain  $\rightarrow 1$ , need  $C_x \ll C_D$ .  
 (must minimize  $C_D$ )

**Note:** Can we similar short-cut to the R case.

- Get DC response  $\rightarrow C$ 's dominate.
- Then:

$$\frac{V_o}{V_i}(s) = (\text{DC Gain}) \cdot \frac{1}{s} \cdot \Theta(s, w_0', Q') \cdot w_0' Q'$$

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## Velocity Sensing Circuits

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## Velocity-to-Voltage Conversion

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- To convert velocity to a voltage, use a resistive load

Since this structure has completely symmetrical I/O ports:

$$\frac{V_o}{V_i} = \frac{R_D}{R_x + R_D} \quad (@\text{resonance})$$

Then, generalize to off resonance:

$$\frac{V_o}{V_i} = \frac{R_D}{R_x + R_D} H(s, Q'), \text{ where } Q' = Q \left( \frac{R_x}{R_x + R_D} \right)$$

*Cx & Lx cancel*

Voltage Representing Velocity

$\omega$

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## Problems With Purely Resistive Sensing

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Now, we get: (approximately)

$$\frac{V_o}{V_i}(s) \sim \frac{R_D}{R_x + R_D} \cdot \frac{1}{1 + \frac{s}{\omega_p}} \cdot H(s, \omega_b, Q')$$

$\omega_p = \frac{1}{(R_x || R_D) C_p}$

Depend on both  $R_D$  &  $C_p$ .

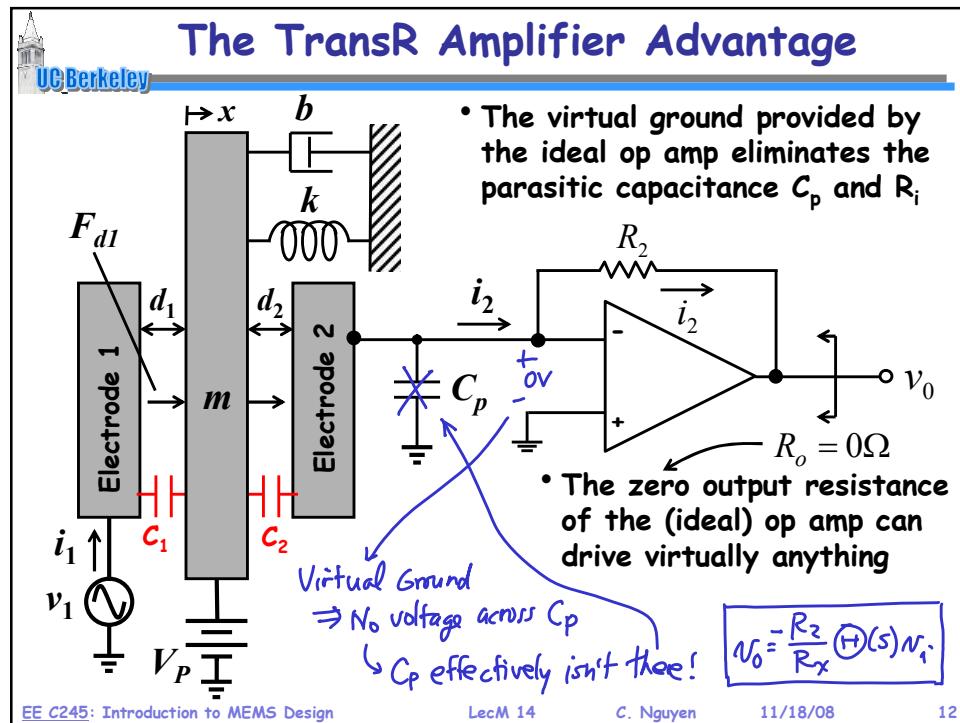
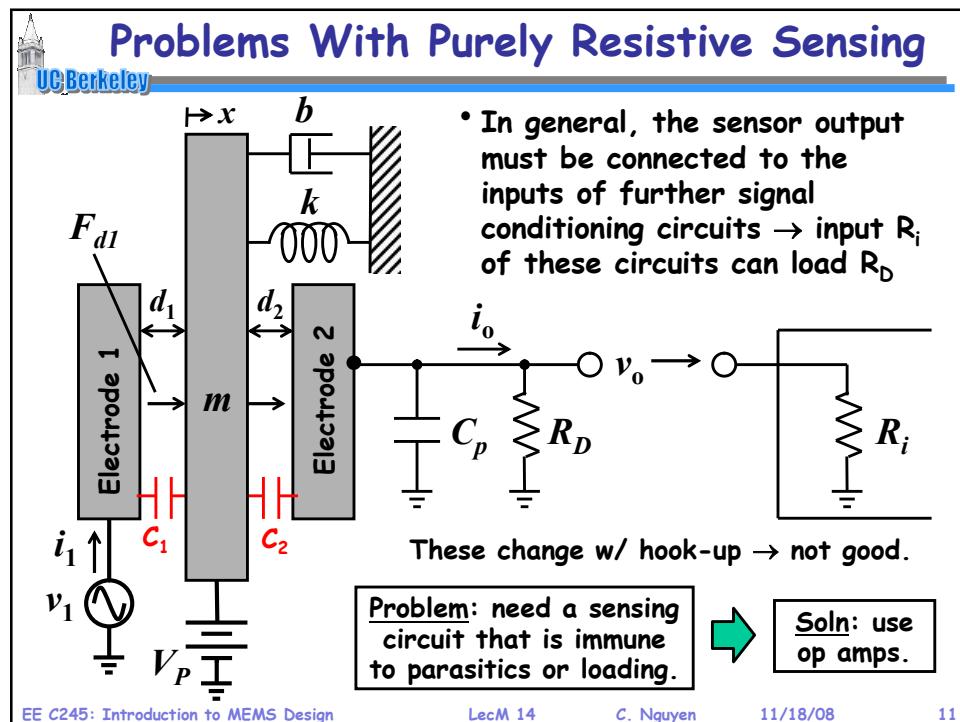
Impact depends on where  $\omega_p$  is relative to  $\omega_0$ .

*Not Good*    *Okay*

$\omega_c$      $\omega_b$      $\omega_D$

$\omega$

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## Position Sensing Circuits

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### Problems With Pure-C Position Sensing

- To sense position (i.e., displacement), use a capacitive load

$$\frac{V_o}{V_i} = \frac{C_p/C_b}{1+C_p/C_b} \cdot \frac{1}{s} \cdot H(s, w_0, Q') \cdot w_0 \alpha$$

Integration yields displacement.

To maximize gain, minimize  $C_D$ .  
 ⇒ Problem: parasitic capacitance  
 $C_D \rightarrow C_D + C_{Pi} + C_{Pb}$

⇒ DC Gain:  $\frac{C_x/(C_D + C_{Pi} + C_{Pb})}{1 + C_x/(C_D + C_{Pi} + C_{Pb})}$

Output will get smaller!  
 Remedy: Suppress  $C_D$  via use of op amps.

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## The Op Amp Integrator Advantage

The diagram shows a MEMS mass-spring system. A mass  $m$  is attached to a spring with stiffness  $k$ , which is fixed to a wall. The mass is positioned between two electrodes, Electrode 1 and Electrode 2. The distance from the wall to the center of the mass is  $b$ . The distance from the wall to Electrode 1 is  $d_1$ , and the distance from Electrode 1 to Electrode 2 is  $d_2$ . A force  $F_{d1}$  acts on Electrode 1. The total capacitance between the electrodes and the wall is  $C_p$ . The input voltage is  $v_1$ , and the output current is  $i_1$ . The output voltage is  $v_o$ .

The op-amp integrator circuit consists of a non-inverting input terminal connected to ground through a capacitor  $C_2$  and a resistor  $R_2$ . The inverting input terminal is connected to ground through a capacitor  $C_p$  and a resistor  $R_x$ . The output of the op-amp is  $v_o$ . The feedback path from the output to the inverting input is through a resistor  $R_o = 0.5R_2$ .

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance  $C_p$ .

Handwritten notes:

$$N_o = -i_o \left( R_2 / (sC_2) \right)$$

$$\approx - \frac{V_i}{R_x} \frac{1}{sC_2} \xrightarrow{\text{well defined}} \text{good!}$$

$$\frac{N_o}{N_s}(s) = - \frac{1}{R_x C_2} \frac{H(s)}{s}$$

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## Differential Position Sensing

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## Differential Position Sensing

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- Example: ADXL-50

Tethers with fixed ends

Proof Mass

Sense Finger

Applied Acceleration

Fixed Electrodes

$V_p$

$C_1$

$C_2$

$V_o$

$C_p$

$V_o = -V_p + \frac{(2V_p)}{C_1+C_2}$

$$= -\frac{V_p C_1 - V_p C_2 + 2V_p C_1}{C_1+C_2} = \frac{V_p (C_1 - C_2)}{C_1+C_2} = V_o$$

Issue: Parasitic Capacitance

$$V_o = \frac{C_1 \cdot C_2}{C_1 + C_2 + C_p} V_p$$

As before,  $C_p$  reduces gain → Soln: Use op amp!

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## Buffer-Bootstrapped Position Sensing

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$+V_p$

$-V_p$

$V_0$

$C_p$

$C_{gd}$

Unity Gain Buffer

$C_{gd} = \text{gate-to-drain capacitance of the input MOS transistor}$

- Bootstrap the ground lines around the interconnect and bond pads
  - ↳ No voltage across  $C_p$
  - ↳ It's effectively not there!

Interconnect

Ground Plane

1x

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### Effect of Finite Op Amp Gain

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**Total ADXL-50 Sense  $C \sim 100\text{fF}$**

$$+V_P \quad -V_P \quad N_o = A_o(N_i - N_o) = A_o(N_i - V_o) \rightarrow N_o(1 + A_o) = A_o N_i \rightarrow \frac{N_o}{N_i} = \frac{A_o}{1 + A_o}$$

$$\text{Get } Z_i = \frac{V_i}{i_i}: i_i = (N_i - N_o) s C_p = N_i \left(1 - \frac{A_o}{1 + A_o}\right) s C_p = N_i \frac{1}{1 + A_o} s C_p$$

$$\therefore \frac{N_i}{i_i} = Z_i = \frac{1}{s \left[ \frac{C_p}{1 + A_o} \right]} \rightarrow C_{eff} = \frac{C_p}{1 + A_o}$$

No longer zero!

Ex:  $A_o = 100, C_p = 2\text{pF}$   
 $\Rightarrow C_{eff} = \frac{2\text{pF}}{101} = 20\text{fF}$

Not negligible compared w/ ADXL-50  $C_{tot} \sim 100\text{fF}$ !

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### Integrator-Based Diff. Position Sensing

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$i_1$

$i_2$

$$+V_P \quad -V_P \quad i_o = i_1 + i_2 = N_p(sC_1) - N_p(sC_2) = V_p s(C_1 - C_2)$$

$$\therefore N_o = -i_o \frac{1}{sC_F} = -N_p \frac{(C_1 - C_2)}{C_F}$$

$$\frac{N_o}{V_p} = -\frac{C_1 - C_2}{C_F} \rightarrow \text{A seemingly perfect differential Sensor/amplifier output! ... but only when the op amp is ideal...}$$

(for biasing)

$R_2$

$R_o = 0\Omega$

Can drive next stage's  $R_i$  w/o interference to transfer function!

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