EE 247B / ME 218 Discussion 12

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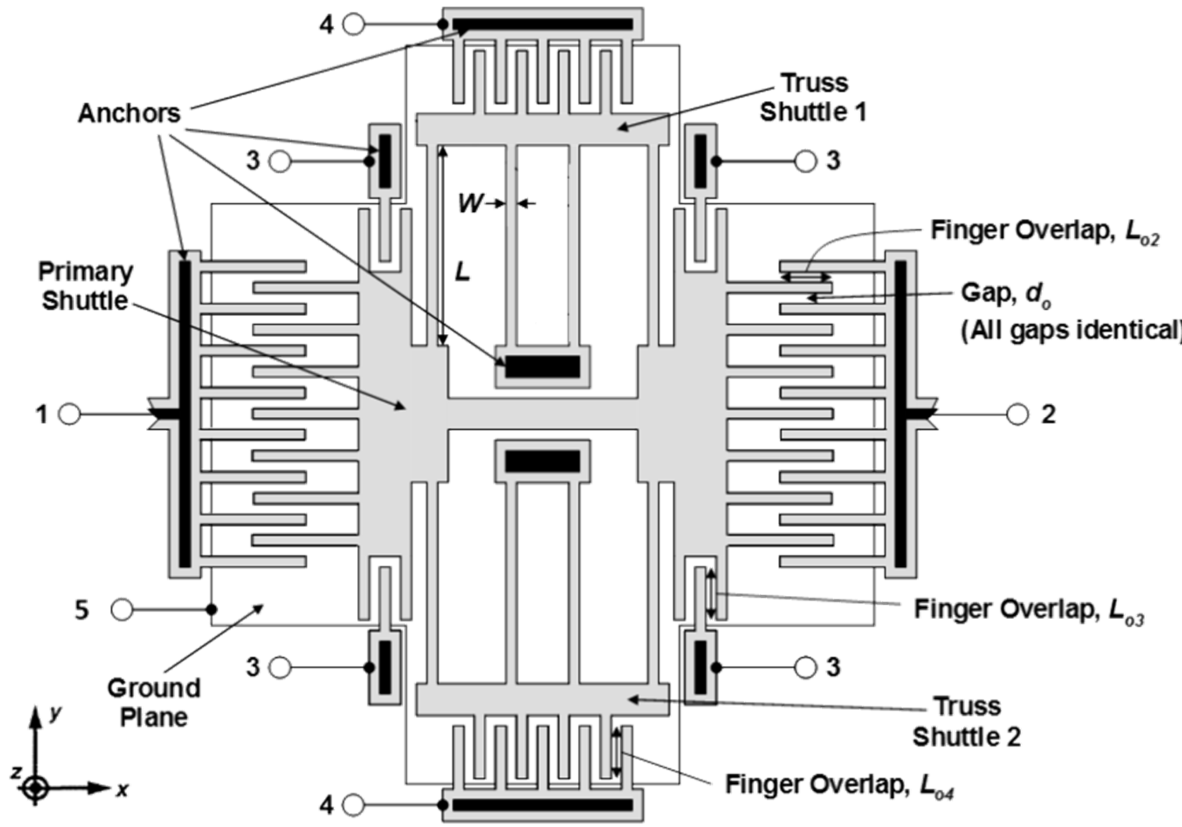
Equivalent Circuits & Sensing

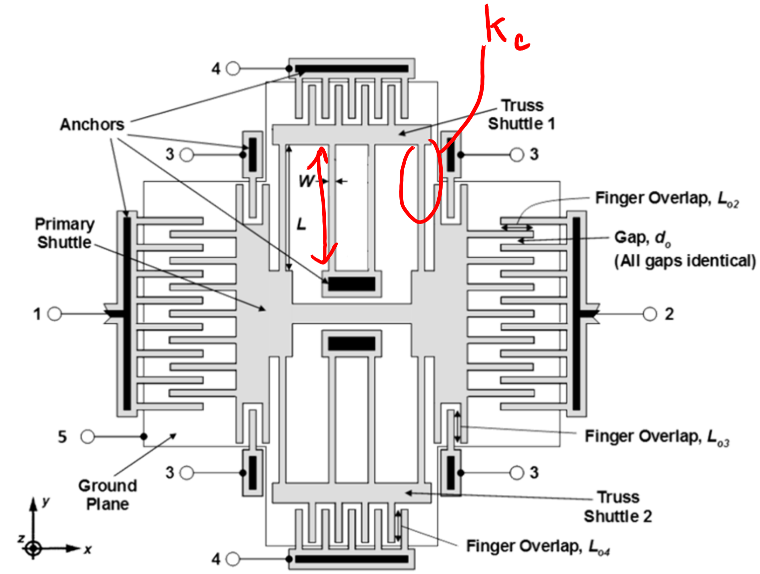
Let's do some analysis on the resonance behavior of this comb-drive structure



Structural Material Properties:
Young's Modulus, E = 150 GPa; Density, ρ = 2,300 kg/m^3
Poisson ratio, ν = 0.226; Q = 100,000
Geometric Dimensions:
L = 50μm; W = 2μm; Thickness, h = 2μm; All Finger Gaps, d_o = 1 μm
All Finger Overlaps, L_o= 10 μm, Truss Shuttle 1 Area = 300 μm^2
Truss Shuttle 2 Area = 300 μm^2, Primary Shuttle Area = 4,000 μm^2

x-Direction Resonance Frequency

First, let's find f_o when all ports are grounded



$$\omega_o = \sqrt{\frac{k_s}{m_s}}$$

$$k_s = k_c = \frac{2Ew^3h}{L^3} = 2(150G)(2\mu)\left(\frac{2\mu}{50\mu}\right) = 38.4\ \mathrm{N/m}$$

$$m_s = M_s + \frac{1}{4}M_t + \frac{12}{35}M_b = 20.35\ \mathrm{ng}$$

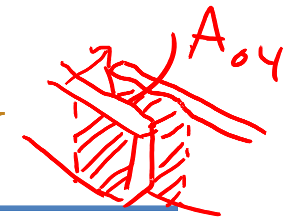
$$M_s = \rho h A_s = 18.4\ \mathrm{ng}$$

$$M_t = 2\rho h A_t = 2.76\ \mathrm{ng}$$

$$M_b = 8\rho h L W = 3.68\ \mathrm{ng}$$

$$\omega_o = \sqrt{\frac{38.4}{20.3n}} = 1.37\ \frac{\mathrm{Mrad}}{s} \Rightarrow f_o = 218.618\ \mathrm{kHz}$$

x-Direction Resonance Frequency

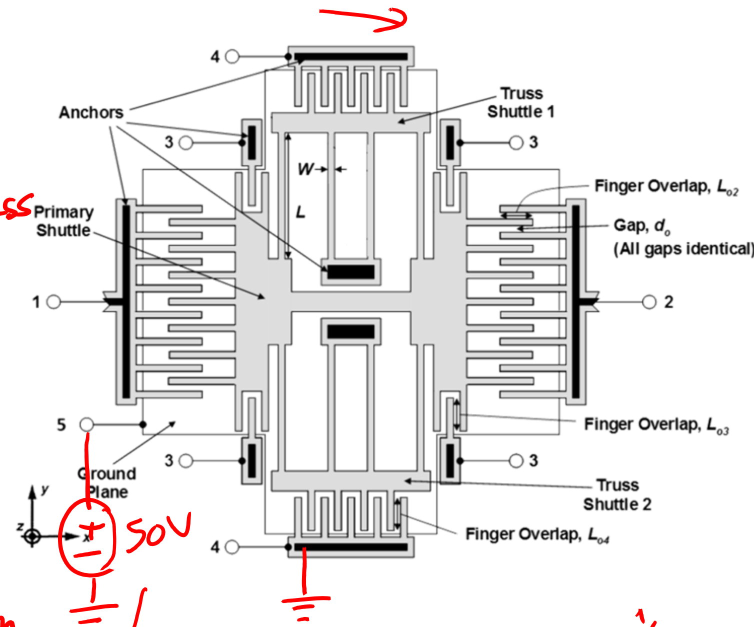


Now, let's find f_o when ports 1, 2 & 4 are grounded, and all other ports are biased to 50 V

$$f_o' = f_o\left(1-\frac{k_{e4}}{k_t}\right)^{\frac{1}{2}}$$

electrical stiffness from port 4

stiffness @ truss



$$k_{e4} = V_P^2\frac{C_o}{d_o^2} = V_P^2\frac{\varepsilon_o A_{o4}}{d_o^3} = \frac{V_P^2\varepsilon_o(2N_{f4}hL_o)}{d_o^3}$$

$$= \frac{(50)^2(8.85\times10^{-12})(2)(8)(2\mu)(10\mu)}{(1\mu)^3} = 7.08\ \mathrm{N/m}$$

$$\cancel{\tfrac{1}{2}}m_t v_t^2 = \cancel{\tfrac{1}{2}}m_s v_s^2$$

$$m_t = \left(\frac{v_s^2}{v_t^2}\right)m_s = 4m_s \Rightarrow k_t = 4k_s = 4(38.4\ \mathrm{N/m})$$

$$k_t = 153.6\ \mathrm{N/m}$$

$$v_t = \tfrac{1}{2}v_s \qquad \omega_{ot} = \omega_{os}$$

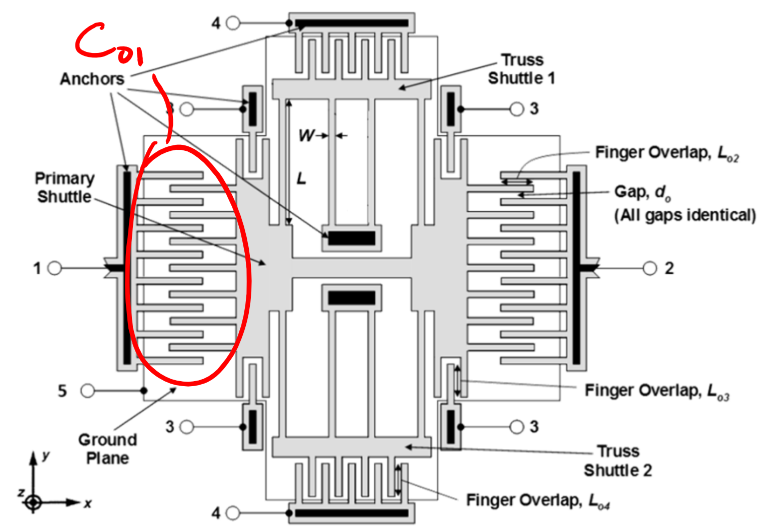
$$f_o' = (218k)\left(1-\frac{7.08}{153.6}\right)^{1/2}$$

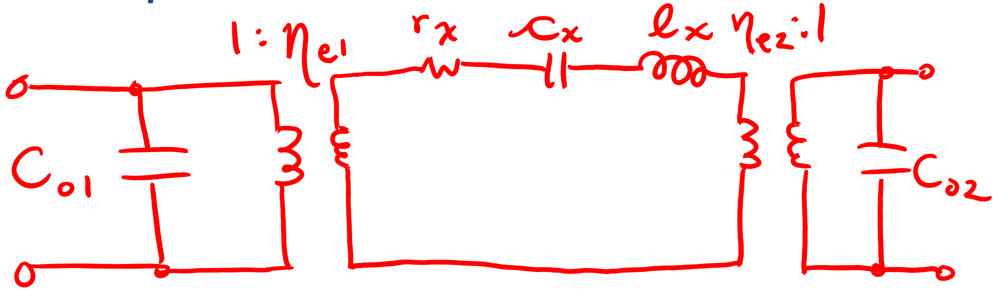
$$f_o = 0.9767 f_o$$

$$= 213.517\ \mathrm{kHz}$$

Equivalent Circuit

Now draw the transformer-based equivalent circuit (between ports 1 & 2) when ports 1, 2 & 4 are grounded and all other ports are biased to 50 V





$$l_x = m_s = 2.035 \times 10^{-11}\ \text{kg (or H)}$$

$$c_x = \frac{1}{k_s} = 0.026\ \frac{\text{m}}{\text{N}}\ \text{(or F)}$$

$$r_x = b_x = \frac{\sqrt{k_s m_s}}{Q} = 0.2795\ \text{n}\frac{\text{kg}}{\text{s}}\ (\text{or n}\Omega)$$

$$\eta_{e1} = \eta_{e2} = V_P \frac{\partial C}{\partial x} = V_P \frac{2N_{fl}\varepsilon_o(2\mu)}{1\mu}$$

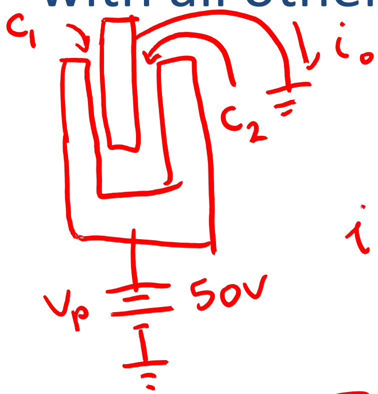
$$\eta_{e1} = 1.24 \times 10^{-8}\ \text{C/m}$$

$$C_{o1} = C_{o2} = \frac{\varepsilon_o h L_o (2N_{fl})}{d_o}$$

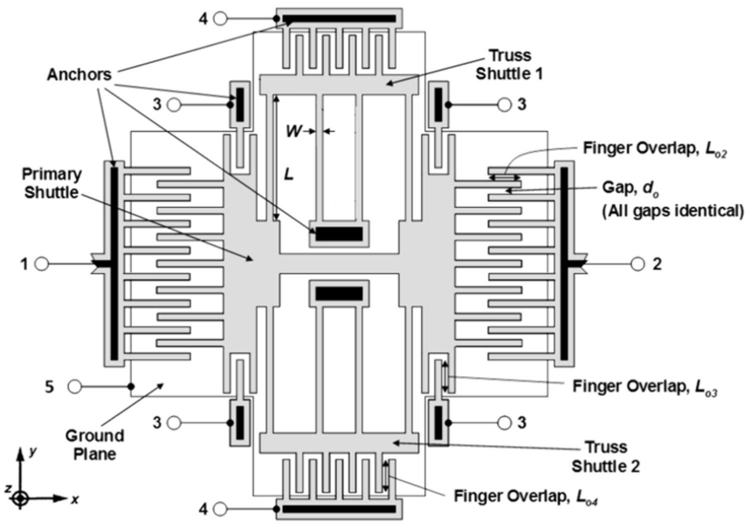
$$C_{o1} = 2.48\ \text{fF} = C_{o2}$$

Equivalent Circuit

Next, draw the transformer-based equivalent circuit between ports 1 & 3 with all other ports are biased to 50 V



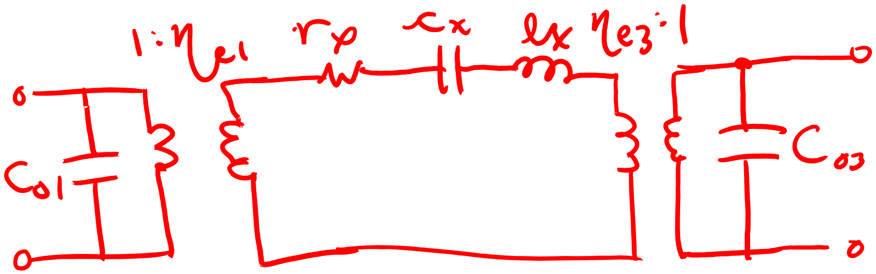
$$\frac{\partial C_1}{\partial x} \approx \frac{C_o}{d_o} \qquad \frac{\partial C_2}{\partial x} \approx -\frac{C_o}{d_o}$$



$$i_o = V_P\left(\frac{\partial C_1}{\partial x} + \frac{\partial C_2}{\partial x}\right)\frac{dx}{dt} = 0$$

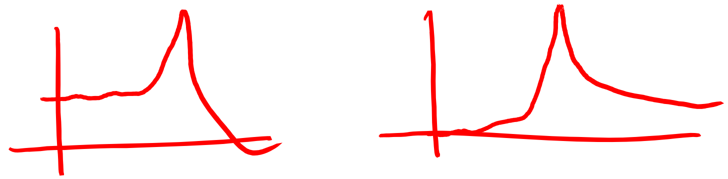
$$F_d = V_P\left(\frac{\partial C_1}{\partial x} + \frac{\partial C_2}{\partial x}\right)v_i = 0$$

$$\eta_{e3} = V_P\left(\frac{\partial C_1}{\partial x} + \frac{\partial C_2}{\partial x}\right) = 0$$

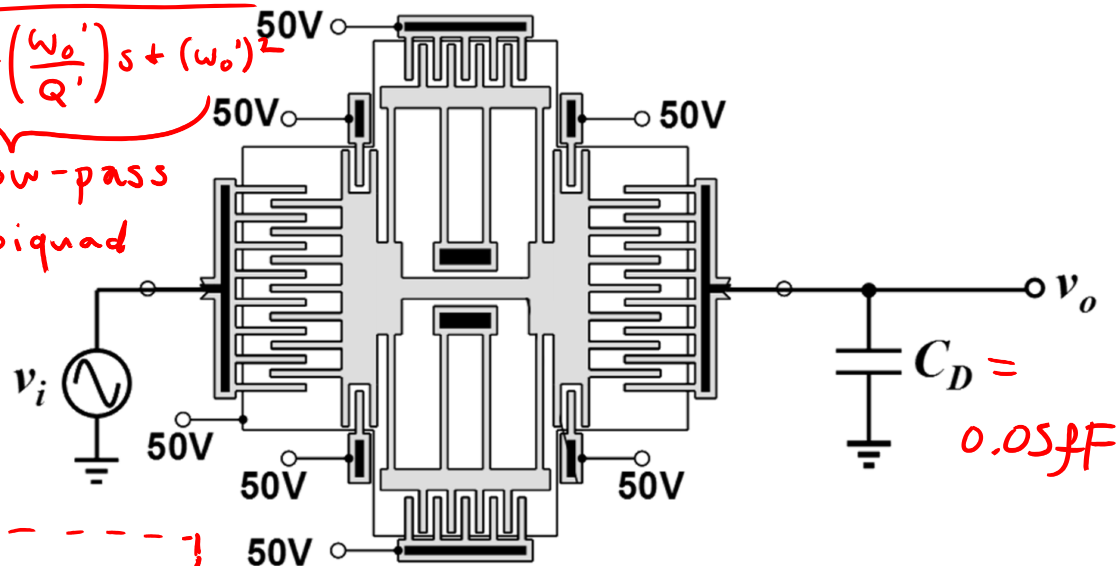


$$C_{o3} = \frac{2\overset{4}{N_{f3}}\varepsilon_o h L_o}{d_o}$$

Sensing



find $\left|\dfrac{v_o(s)}{v_i(s)}\right|$ $\qquad \dfrac{v_o(s)}{v_i(s)} = \underbrace{\dfrac{C_x/C_D}{1-C_x/C_D}}_{\text{DC gain}} \; \underbrace{\dfrac{(\omega_o')^2}{s^2+\left(\frac{\omega_o'}{Q'}\right)s+(\omega_o')^2}}_{\text{low-pass biquad}}$



$$\omega_o' = \omega_o\sqrt{1+\frac{C_x}{C_D}}$$

$$Q' = Q\sqrt{1+\frac{C_x}{C_D}}$$

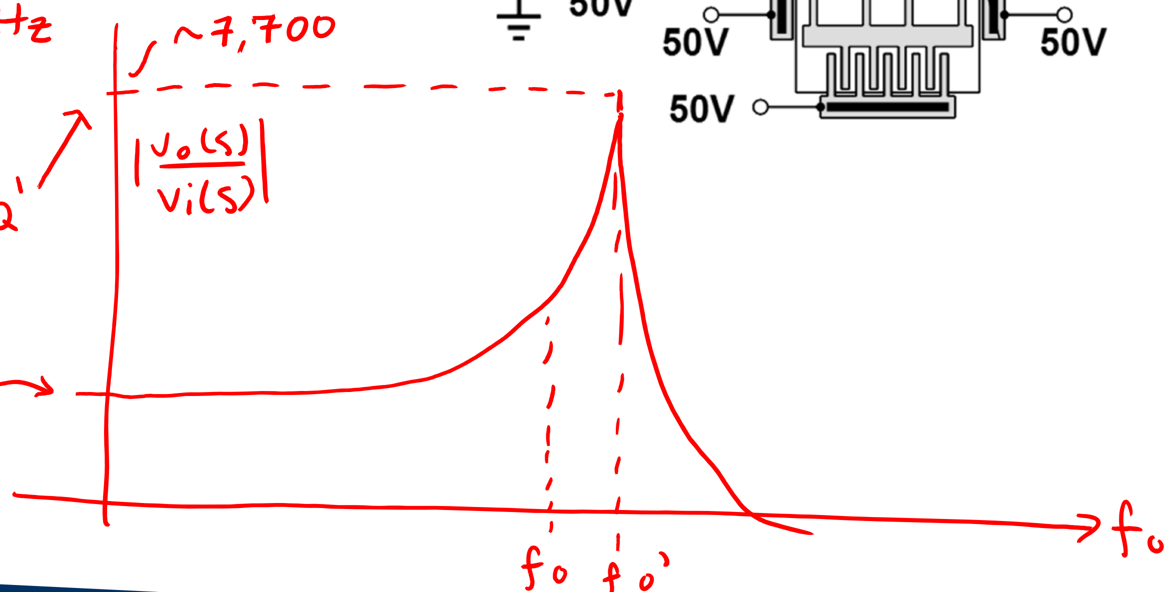
$$C_x = \eta_{e1}^2 c_x$$
$$= 4\,aF$$

$$\omega_o' = 227.2\,kHz$$

$$Q' = 104{,}000$$

$\times Q'$

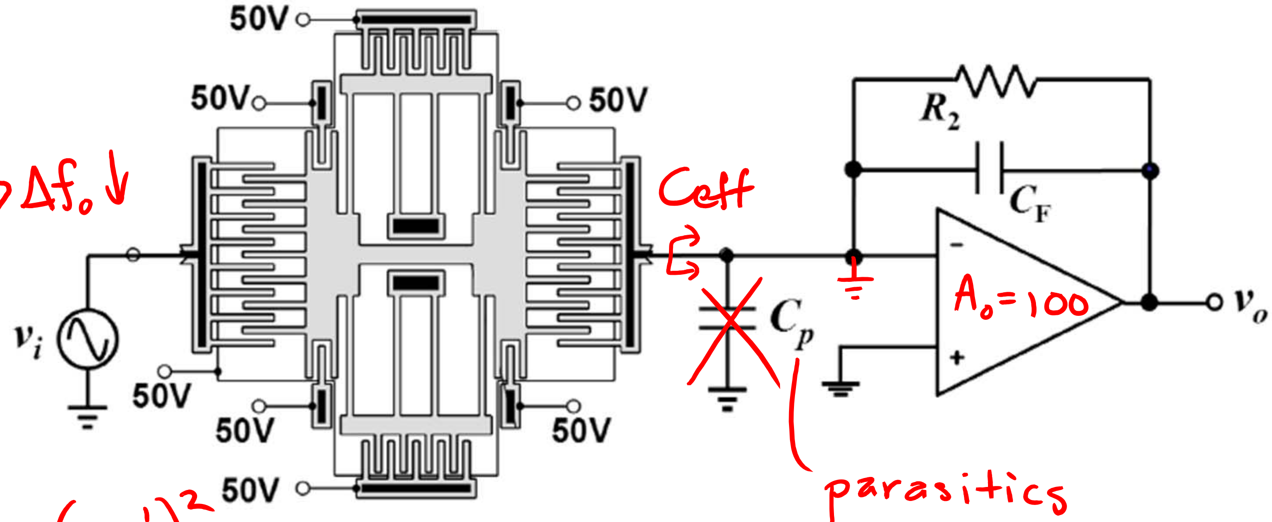
$$\frac{C_x/C_D}{1-C_x/C_D} = 0.074$$



Sensing

$$C_{eff} = C_p + (1+A_o)C_f$$

$$f_o' = f_o\left(1 + \frac{C_x}{C_{eff}}\right) \uparrow \Rightarrow \Delta f_o \downarrow$$



$$\left|\frac{v_o(s)}{v_i(s)}\right| = \frac{C_x/C_{eff}}{1 + C_x/C_{eff}} \cdot \frac{(\omega_o')^2}{s^2 + \left(\frac{\omega_o'}{Q'}\right)s + (\omega_o')^2}$$