EE 247B / ME 218 Discussion 1

Kieran Peleaux

Introductions

Kieran Peleaux

- B.S.E.E. from University of Pittsburgh (2014)
- Worked for Powercast in Pittsburgh for ~1.5 years
- 4th year PhD student in Prof. Clark Nguyen's group
- Interests
  - Vibrating RF MEMS
  - MEMS-CMOS integration
  - MEMS filter design
  - MEMS oscillators
  - My pets
  - Cooking
  - Video games

Come introduce yourself during office hours!

Office Hours & Discussion

Office Hours

Mondays, 3:30 – 5 pm, 367 Cory Hall

Wednesdays, 10 – 11 am, 367 Cory Hall

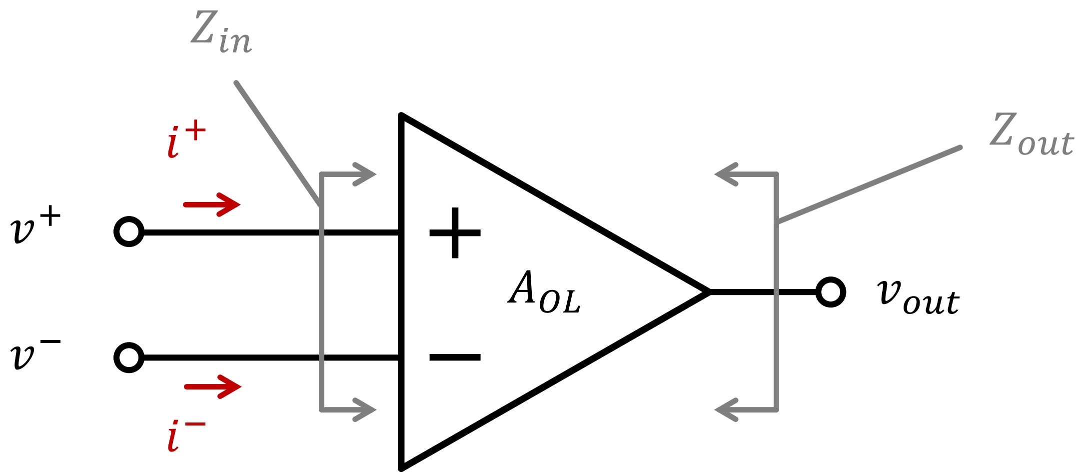
Discussion

Fridays, 10 – 11 am, 9 Evans Hall (hopefully we're here now)

Op-Amp Review

Ideal Op-amp Laws

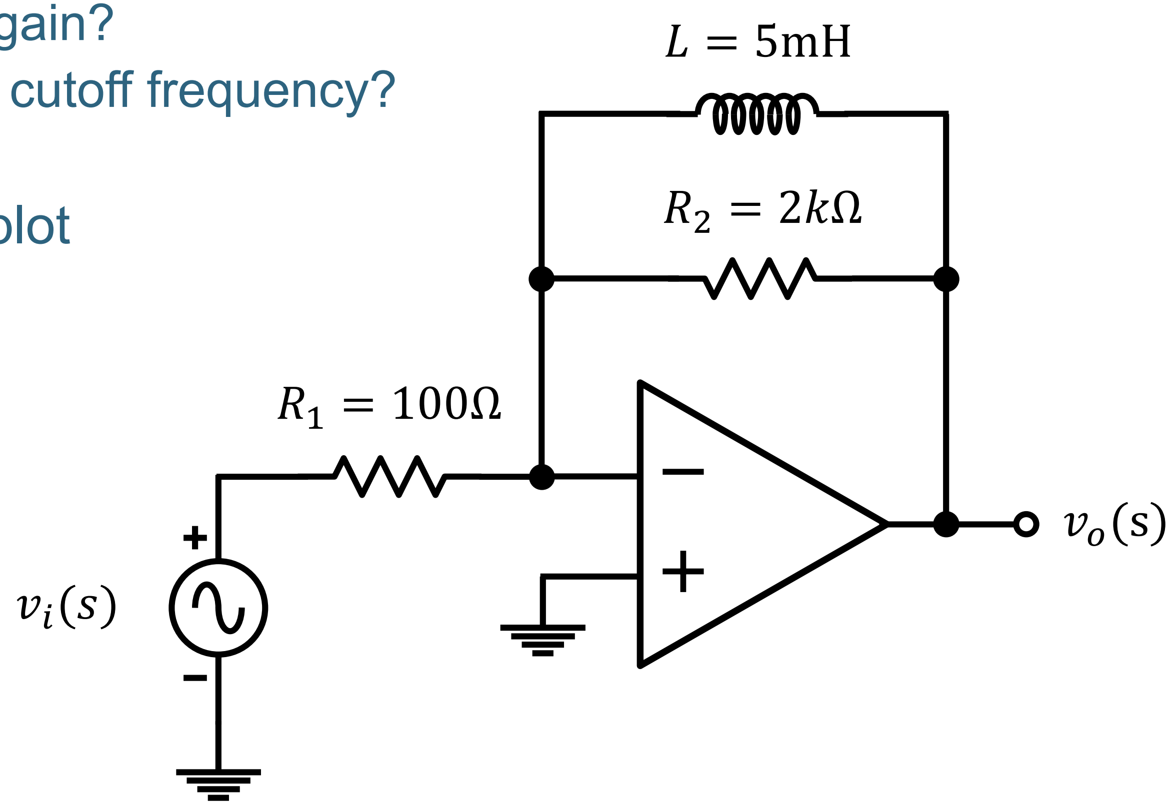
- $Z_{in} = \infty$
  - $i^+ = i^- = 0$
- $A_{OL} = \infty$
  - $v^+ = v^-$
- $Z_{out} = 0$



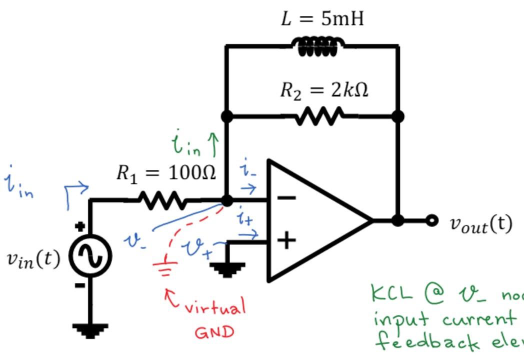
Op-Amp Example

- What is the transfer function $\frac{v_{o(s)}}{v_{i(s)}}$?
  - what's the DC gain?
  - what's the 3dB cutoff frequency?

- Sketch the Bode plot
  - magnitude
  - phase



Op-Amp Example (cont.)



$A_{OL} = \infty$

$v_o = A_{OL}(v_+ - v_-)$

$\therefore$ if $v_o < \infty$, $(v_+ - v_-) = 0$

$v_+ = v_-$

since v_+ is grounded,

$v_+ = v_- = 0\,V$

$Z_{in} = \infty$

$i_+ = \frac{v_+}{Z_{in}} = \frac{v_+}{\infty} = 0$

$i_- = \frac{v_-}{Z_{in}} = \frac{v_-}{\infty} = 0$

KCL @ v_- node shows all input current flows thru feedback elements.

KVL btwn. v_i & v_-:

$v_i - i_{in} \cdot R_1 = v_-$

$i_{in} = \frac{(v_i - v_-)}{R_1}$

$i_{in} = \frac{v_i}{R_1}$ (1)

KVL btwn. v_- & v_o:

$v_- - i_{in} \cdot Z_{fb} = v_o$

$v_o = -i_{in} \cdot Z_{fb}$

plug in (1) & (2)

$v_o = -\left(\frac{v_i}{R_1}\right)\left(\frac{R_2 \cdot sL}{R_2 + sL}\right)$

$\frac{v_o}{v_i} = -\left(\frac{R_2}{R_1}\right) \cdot \frac{s\frac{L}{R_2}}{s\frac{L}{R_2} + 1}$

one zero @ $s = 0$

$\omega_z = 0\ \text{rad/sec}$

$f_z = 0\ \text{Hz}$

one pole: $s\frac{L}{R_2} = 1$

$\omega_p = \frac{R_2}{L} = \frac{2k\Omega}{5mH}$

$\omega_p = 400\ \text{krad/sec}$

$f_p = \frac{\omega_p}{2\pi} = 63.7\ \text{kHz}$

this is 3dB cutoff frequency

max gain (not @ DC!):

$|A_{CL,max}| = \frac{R_2}{R_1} = \frac{2k\Omega}{100\Omega}$

$|A_{CL,max}| = 20\ V/V$

$A_{CL,max,dB} = 20\log(20\ V/V)$

$A_{CL,max,dB} = 26\ dB$

$|A_{DC}| = 0\ V/V$

Z_{fb}:

$Z_{fb} = (Z_{R2} \| Z_L)$

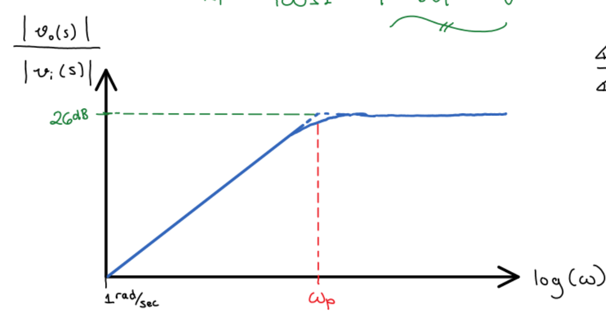
$= \frac{(Z_{R2} \cdot Z_L)}{(Z_{R2} + Z_L)}$

$Z_{fb} = \frac{R_2 \cdot sL}{R_2 + sL}$ (2)

Remember:

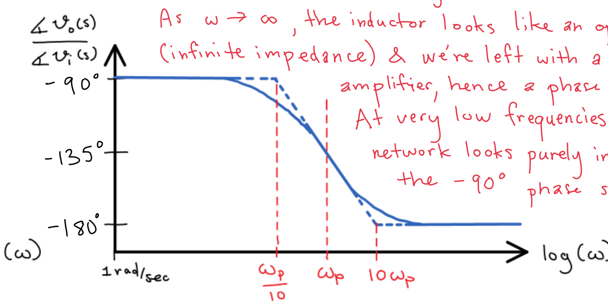
$Z_L = sL$ where

$Z_C = \frac{1}{sC}$ $s = j\omega$



$V = (j\omega L)I$

★ To determine the phase at low/high frequencies (i.e. $\omega_p/10$ & $10\omega_p$) analyze the circuit as $\omega \to 0$ or ∞. As $\omega \to \infty$, the inductor looks like an open-circuit (infinite impedance) & we're left with a standard inverting amplifier, hence a phase shift of $-180°$. At very low frequencies, the feedback network looks purely inductive, hence the $-90°$ phase shift.

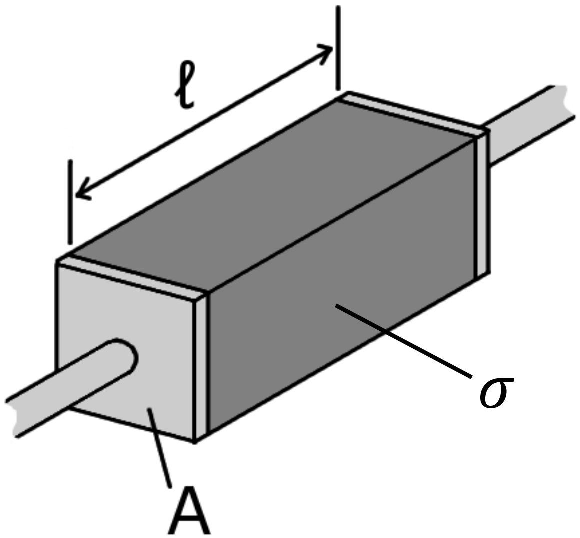


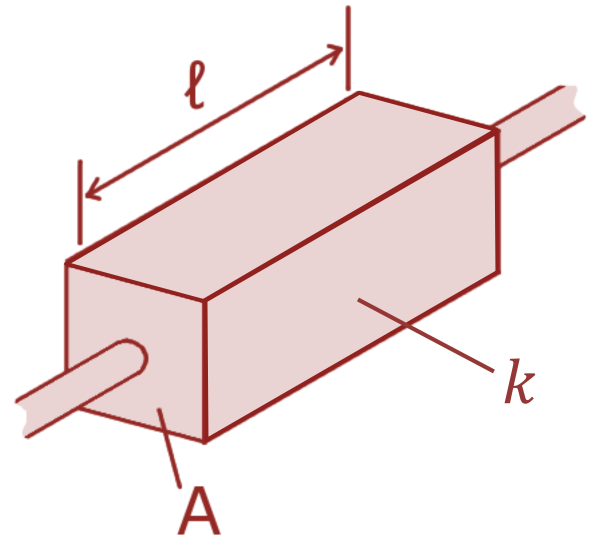
Intro to Thermal Circuits

- Modeling a thermal body/excitation as circuit elements
- Can then apply circuit analysis techniques to non-electrical problems—or hybrid electromechanical problems
- Common theme in this course, will see more complex modeling

Electrical	Thermal
Voltage, V (V)	Temperature, T (°C or K)
Current, I (A)	Power, P (W)
Resistance, R (Ω or V/A)	Thermal Resistance, R_{th} (K/W)
Capacitance, C (F or C/V)	Heat Capacity, C_{th} (J/K)

Resistance : Electrical vs. Thermal





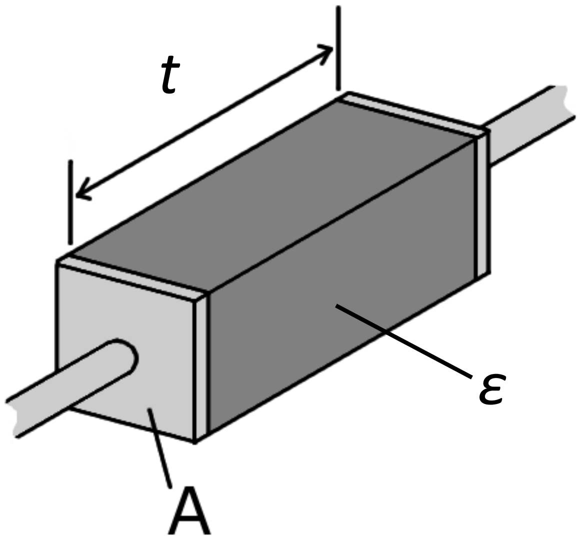
$$R = \frac{l}{\sigma \cdot A}$$

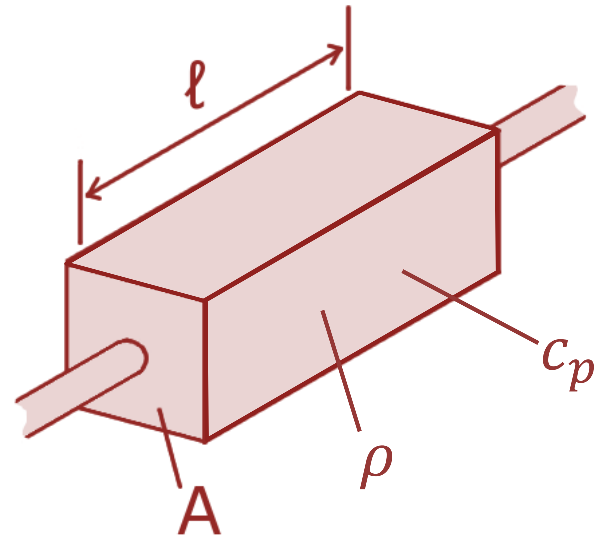
- resistance (Ω)
- length (m)
- conductivity ($(\Omega \cdot m)^{-1}$)
- cross-sectional area (m^2)

$$R_{th} = \frac{l}{k \cdot A}$$

- thermal resistance (K/W)
- length (m)
- thermal conductivity ($\frac{W}{K \cdot m}$)
- cross-sectional area (m^2)

Capacitance : Electrical vs. Thermal





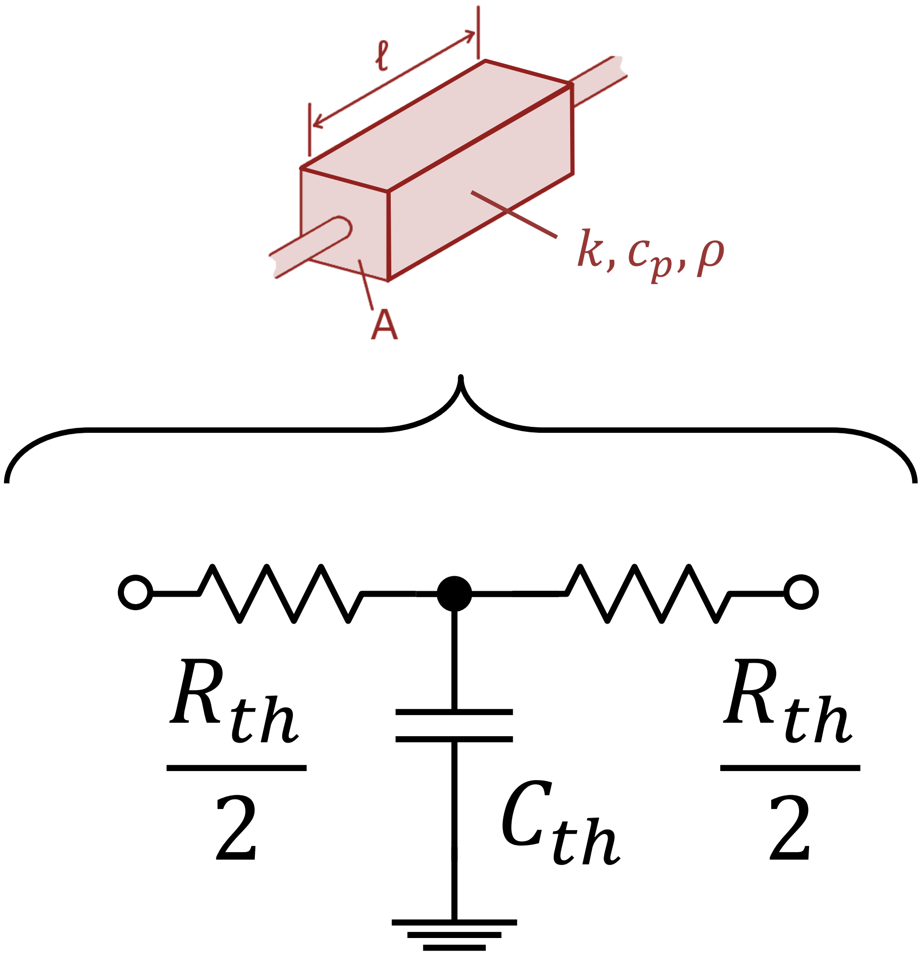
$$C = \varepsilon \cdot \frac{A}{t}$$

- capacitance (F)
- permittivity of dielectric $\left(\frac{\mathrm{F}}{\mathrm{m}}\right)$
- area of plate (m^2)
- thickness of dielectric (m)

$$C_{th} = \rho \cdot l \cdot A \cdot c_p$$

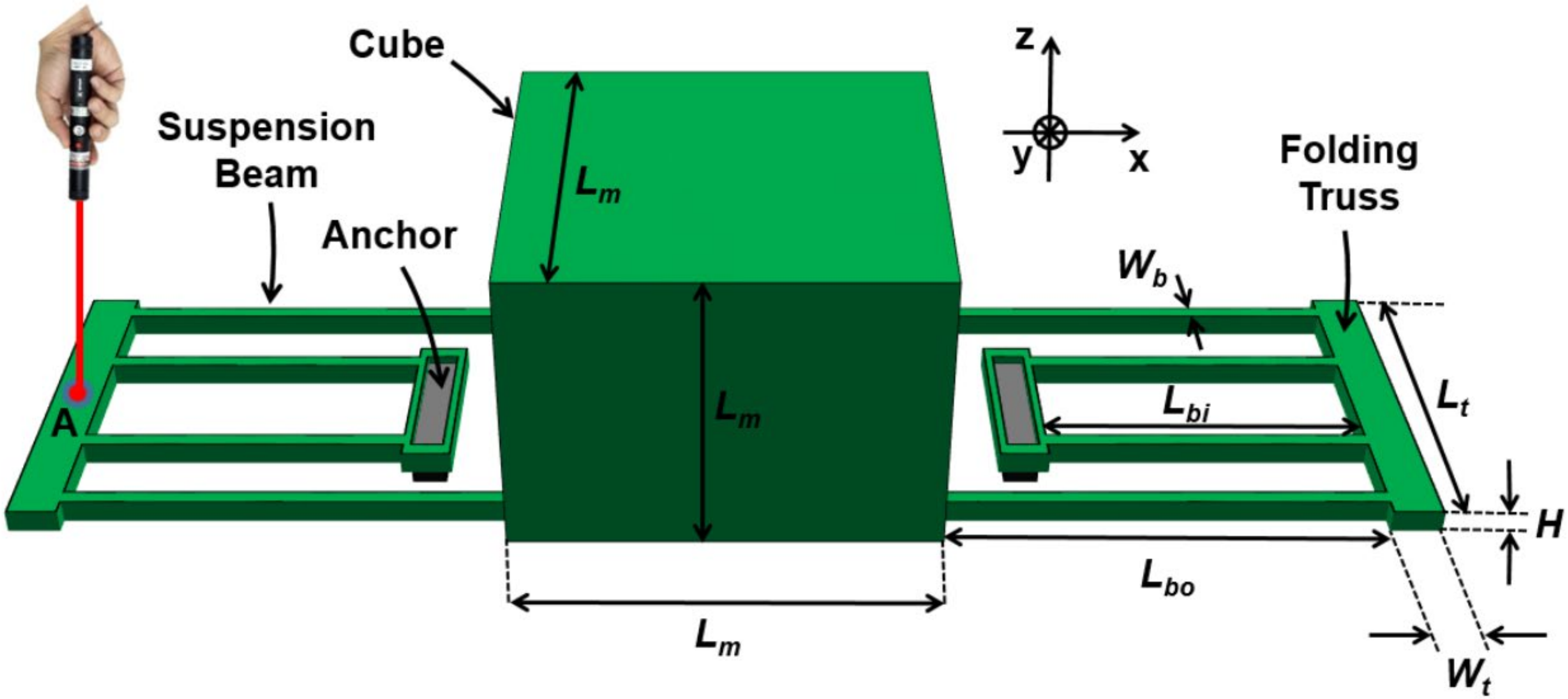
- thermal capacitance $\left(\frac{\mathrm{J}}{\mathrm{K}}\right)$
- density $\left(\frac{\mathrm{kg}}{\mathrm{m}^3}\right)$
- length (m)
- cross-sectional area (m^2)
- specific heat $\left(\frac{\mathrm{J}}{\mathrm{kg \cdot K}}\right)$

Distributed Thermal Model

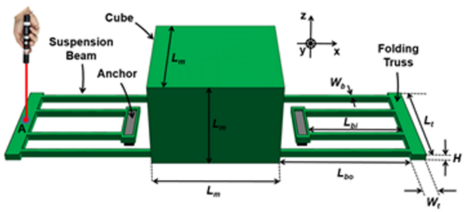


Thermal Ckt. Example

- Draw the full thermal circuit model (include effects of laser!)
- What approximations can you make?



Thermal Ckt. Example (cont.)



$L_{bi} = 50\,\mu m$ $\quad L_t = 50\,\mu m$ $\quad k = 30\,W/m\cdot K$

$L_{bo} = 75\,\mu m$ $\quad W_t = 6\,\mu m$ $\quad c_p = 770\,J/kg\cdot K$

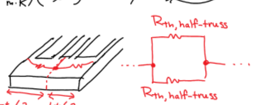
$W_b = 2\,\mu m$ $\quad L_m = 50\,\mu m$ $\quad \rho = 2{,}300\,kg/m^3$

$H = 2\,\mu m$

$$R_{th,bi} = \frac{L_{bi}}{k(W_b \cdot H)} = \frac{50\,\mu m}{(30\frac{W}{m\cdot K})(2\,\mu m)^2} = 4.17\times10^5\ K/W$$

$$R_{th,bo} = \frac{L_{bo}}{k(W_b \cdot H)} = \frac{75\,\mu m}{(30\frac{W}{m\cdot K})(2\,\mu m)^2} = 6.25\times10^5\ K/W$$

modeling R_{th} of folding truss:



$$R_{th,t} = R_{th,half\text{-}truss} \,\|\, R_{th,half\text{-}truss} = \frac{R_{th,half\text{-}truss}}{2}$$

$$= \frac{1}{2}\frac{L_t/2}{k(W_t\cdot H)} = \frac{(50\,\mu m)}{(4)(30\frac{W}{m\cdot K})(6\,\mu m)(2\,\mu m)}$$

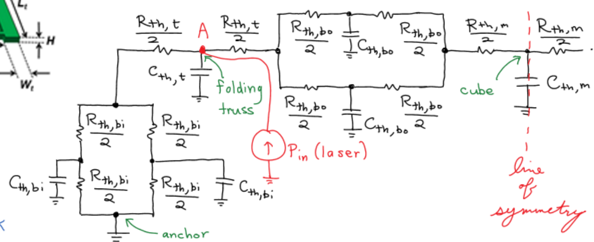
$$R_{th,t} = 3.47\times10^4\ K/W \ll R_{th,bi}, R_{th,bo}$$

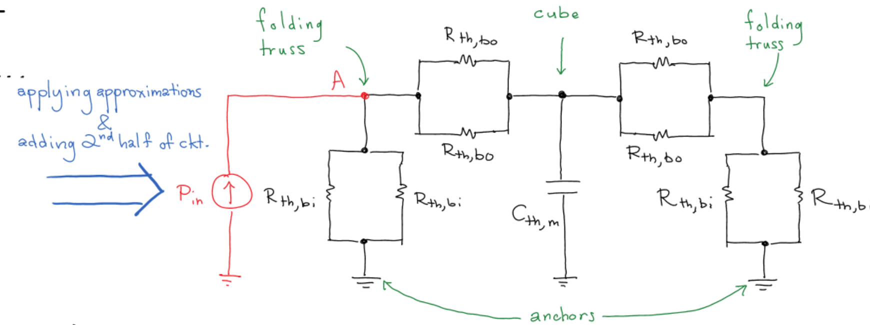
$\therefore$ can neglect $R_{th,t}$!

$$R_{th,m} = \frac{L_m}{k(L_m)^2} = \frac{50\,\mu m}{(30\frac{W}{m\cdot K})(50\,\mu m)^2} = 667\ K/W$$

↳ can also neglect $R_{th,m}$!

Can start w/ ½ of thermal ckt. due to symmetry:





$$C_{th,bi} = \rho(L_{bi}\cdot W_b \cdot H)c_p = (2.3\times10^3\tfrac{kg}{m^3})(50\,\mu m)(2\,\mu m)^2(770\,J/kg\cdot K)$$

$$C_{th,bi} = 3.54\times10^{-10}\ J/K$$

$$C_{th,bo} = \rho(L_{bo}\cdot W_b \cdot H)c_p = (2.3\times10^3\tfrac{kg}{m^3})(75\,\mu m)(2\,\mu m)^2(770\,J/kg\cdot K)$$

$$C_{th,bo} = 5.31\times10^{-10}\ J/K$$

$$C_{th,t} = \rho(L_t\cdot W_t \cdot H)c_p = (2.3\times10^3\tfrac{kg}{m^3})(50\,\mu m)(6\,\mu m)(2\,\mu m)(770\,J/kg\cdot K)$$

$$C_{th,t} = 1.06\times10^{-9}\ J/K$$

$$C_{th,m} = \rho(L_m)^3 c_p = (2.3\times10^3\tfrac{kg}{m^3})(50\,\mu m)^3(770\,J/kg\cdot K)$$

$$C_{th,m} = 2.21\times10^{-7}\ J/K \gg C_{th,bi}, C_{th,bo}, C_{th,t}$$

$\therefore$ can neglect all C_{th}'s except $C_{th,m}$!

further reducing parallel R's

Fully simplified final circuit: