EE 247B / ME 218 Discussion 10

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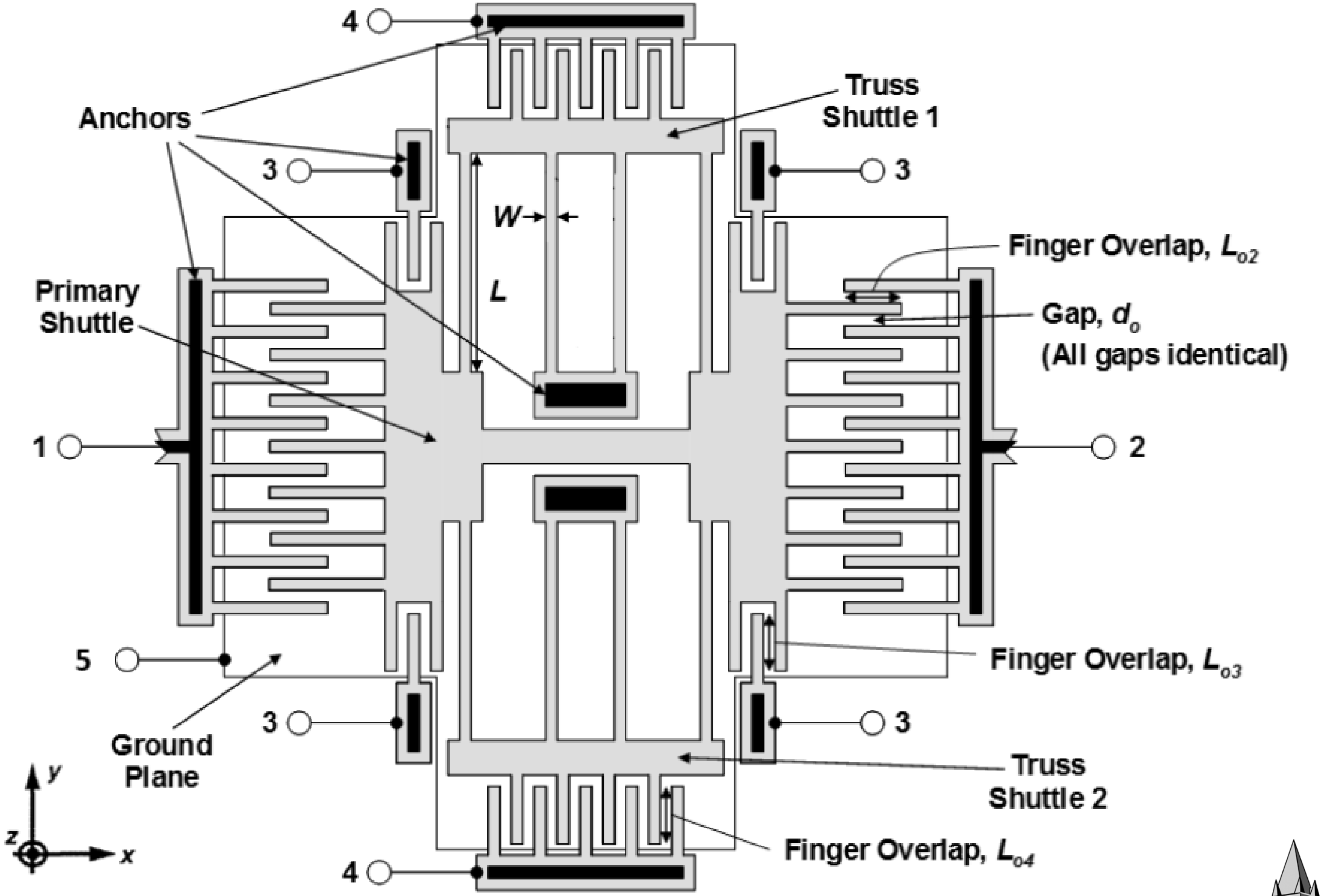
EQUIVALENT CIRCUITS & SENSING

Let’s do some analysis on the resonance behavior of this comb-drive structure



Structural Material Properties:

Young’s Modulus, E = 150 GPa; Density, ρ = 2,300 kg/m^3

Poisson ratio, ν = 0.226; Q = 100,000

Geometric Dimensions:

L = 50μm; W = 2μm; Thickness, h = 2μm; All Finger Gaps, d_o = 1 μm

All Finger Overlaps, L_o= 10 μm, Truss Shuttle 1 Area = 300 μm^2

Truss Shuttle 2 Area = 300 μm^2, Primary Shuttle Area = 4,000 μm^2

x-DIRECTION RESONANCE FREQUENCY

First, let's find f_o when all ports are grounded

$$f_o = \sqrt{\frac{k_s}{m_s}}\frac{1}{2\pi} \quad k_s = k_c = \frac{2Ew^3h}{L^3} = \frac{2(150G)(2\mu)^4}{(50\mu m)^3} = 38.4\ N/m$$

$$m_s = M_s + \frac{1}{4}M_t + \frac{12}{35}M_b = 20.35\ ng$$

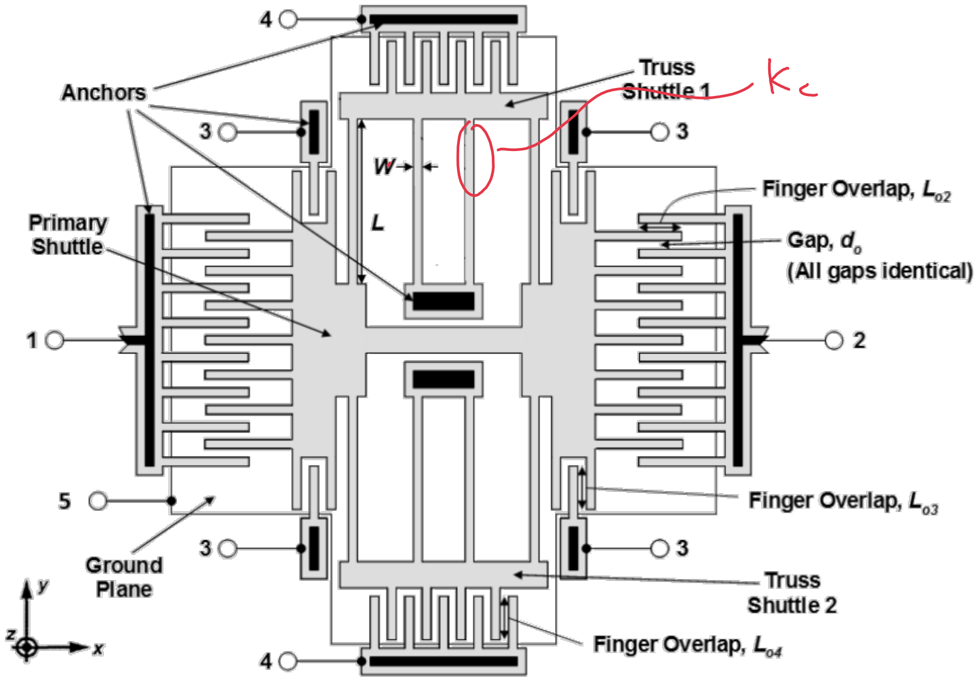
$$M_s = \rho h A_s$$

$$M_t = 2\rho h A_t$$

$$M_b = 8\rho h L W$$

$$\omega_o = \sqrt{\frac{38.4}{20.3n}} = 1.37\ \frac{Mrad}{s}$$

↳ $f_o = 218.6$ kHz



x-DIRECTION RESONANCE FREQUENCY

Now, let's find f_o when ports 1, 2 & 4 are grounded, and all other ports are biased to 50 V 8

$$f_o' = f_o\sqrt{1-\frac{k_{e4}}{k_t}} \checkmark$$

$$k_{e4} = V_p^2\frac{C_{o4}}{d_o^2} = \frac{V_p^2\varepsilon_o(2N_f h L_o)}{d_o^3} = 7\frac{N}{m}$$

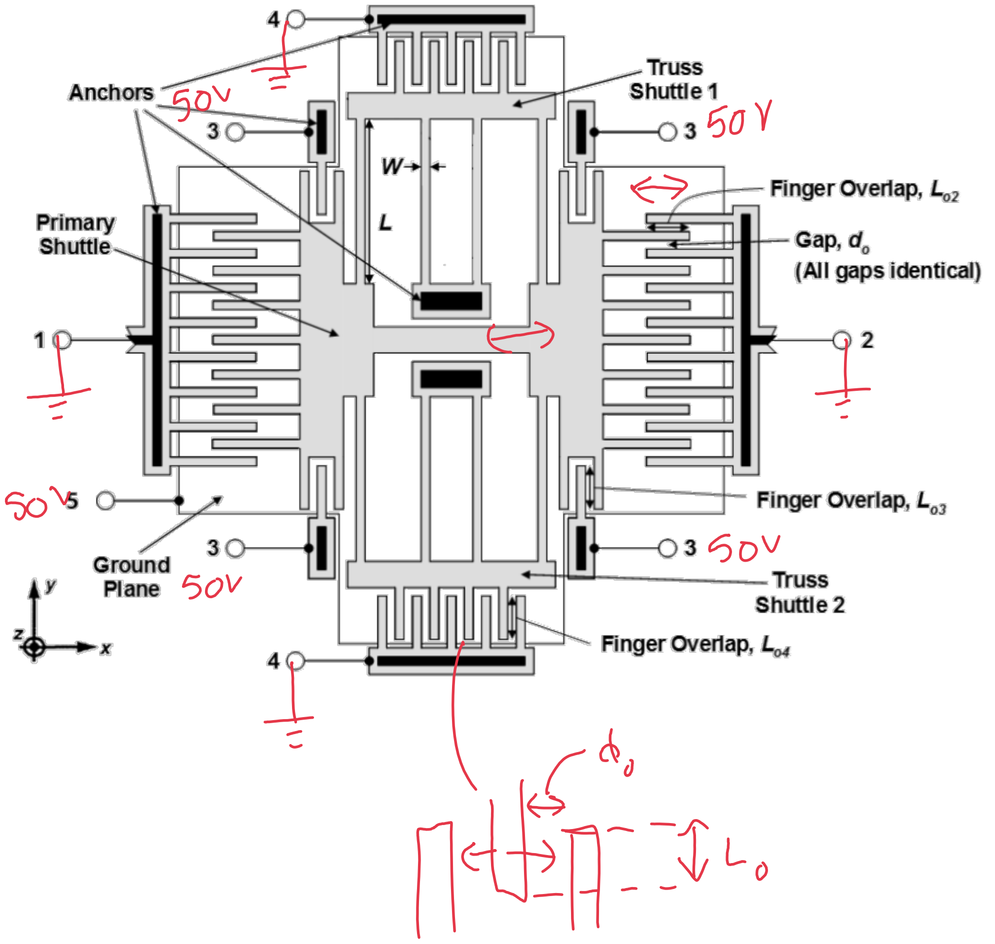
$$\frac{1}{2}m_t v_t^2 = \frac{1}{2}m_s v_s^2$$

$$m_t = \left(\frac{v_s}{v_t}\right)^2 m_s = 4m_s \Rightarrow k_t = 4k_s = 154\frac{N}{m}$$

$$x_t = \frac{1}{2}x_s$$

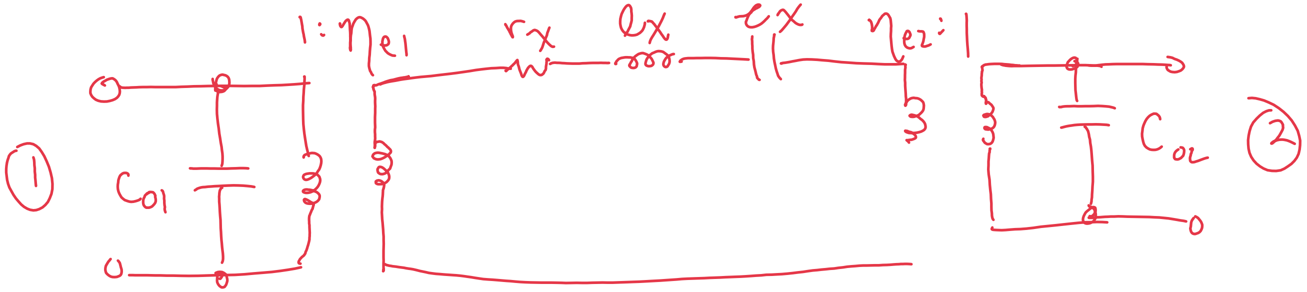
$$v_t = \frac{1}{2}v_s$$

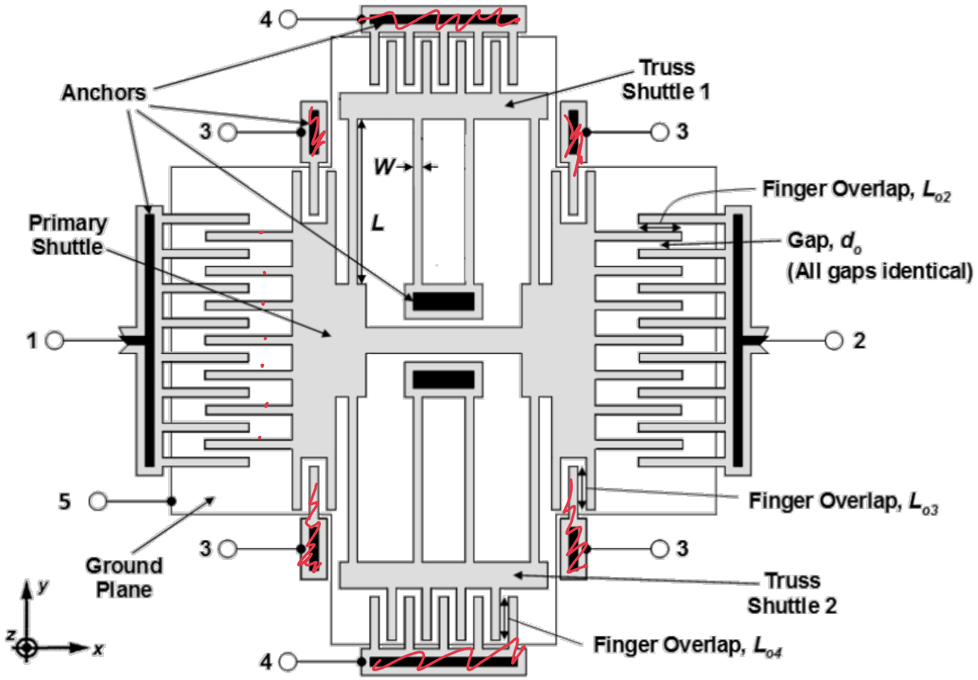
$$f_o' = \underbrace{0.977}_{\sqrt{1-k_{e4}/k_t}} f_o = \underline{\underline{213.5\ \text{kHz}}}$$



EQUIVALENT CIRCUIT

Now draw the transformer-based equivalent circuit (between ports 1 & 2) with other ports biased to 50 V





$l_x = m_s = 2.035 \times 10^{-11}$ kg (H)

$c_x = \frac{1}{k_s} = 0.026 \frac{m}{N}$ (F)

$r_x = b_{re} = \frac{\sqrt{k_s m_s}}{Q} = 0.28 \frac{nkg}{s}$ (nΩ)

$\eta_{e1} = \eta_{e2} = V_p \frac{\partial C}{\partial x}$

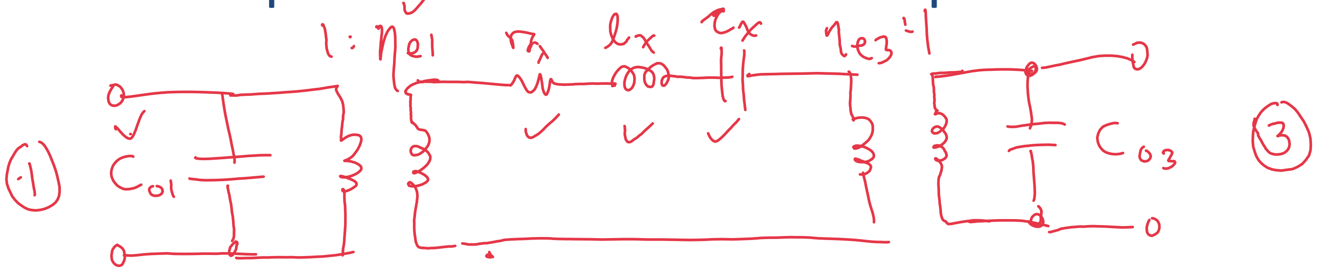
$= V_p \frac{2 \overset{7}{N_{f1}} \varepsilon_o h}{d_o}$

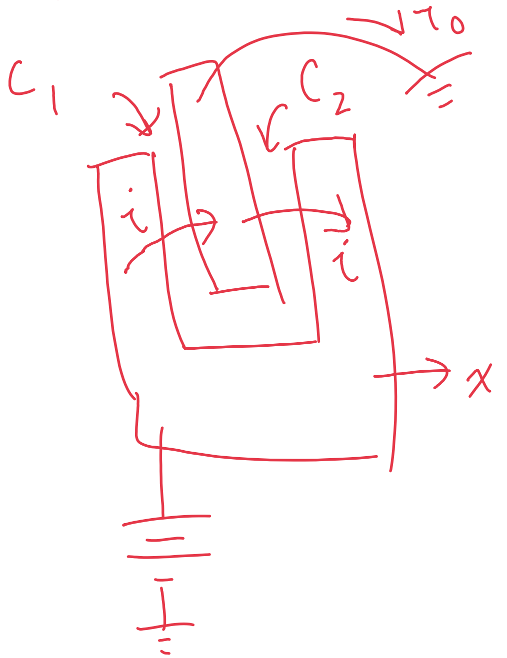
$= 1.24 \times 10^{-8}$ C/m

$C_{o1} = C_{o2} = \frac{\varepsilon_o h L_o (2 N_{f1})}{d_o} = 2.48$ fF

EQUIVALENT CIRCUIT

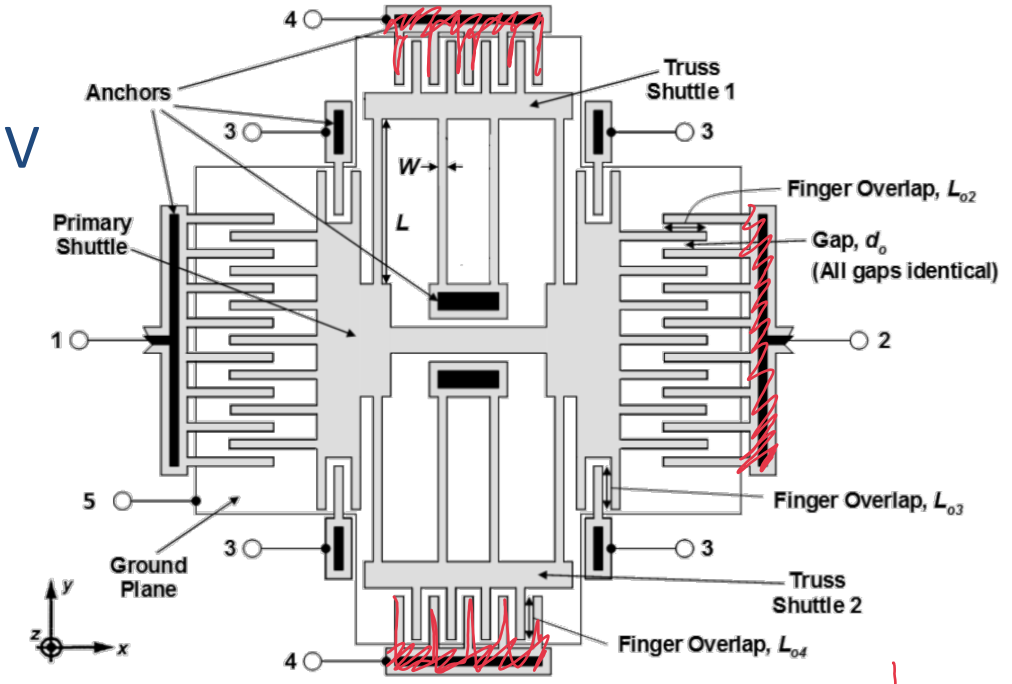
Next, draw the transformer-based equivalent circuit between ports 1 & 3 with all other ports are biased to 50 V





$$\frac{\partial C_1}{\partial x} \approx \frac{C_{o3}}{8d_o} \qquad \frac{\partial C_2}{\partial x} \approx -\frac{C_{o3}}{8d_o}$$

$$i_o = \underbrace{V_p\left(\frac{\partial C_1}{\partial x} + \frac{\partial C_2}{\partial x}\right)}_{\eta_{e3} = 0}\frac{dx}{dt} = 0$$



$$C_{o3} = \varepsilon_o \frac{h L_{o3} 2 N_f}{d_o} \cdot 4$$

SENSING

$$\left|\frac{v_o(s)}{v_i(s)}\right| = \frac{C_x/C_D}{\underbrace{1-C_x/C_D}_{\text{DC gain}}}\;\frac{(\omega_o')^2}{\underbrace{s^2+\left(\frac{\omega_o'}{Q'}\right)s+(\omega_o')^2}_{\text{low-pass biquad}}}$$

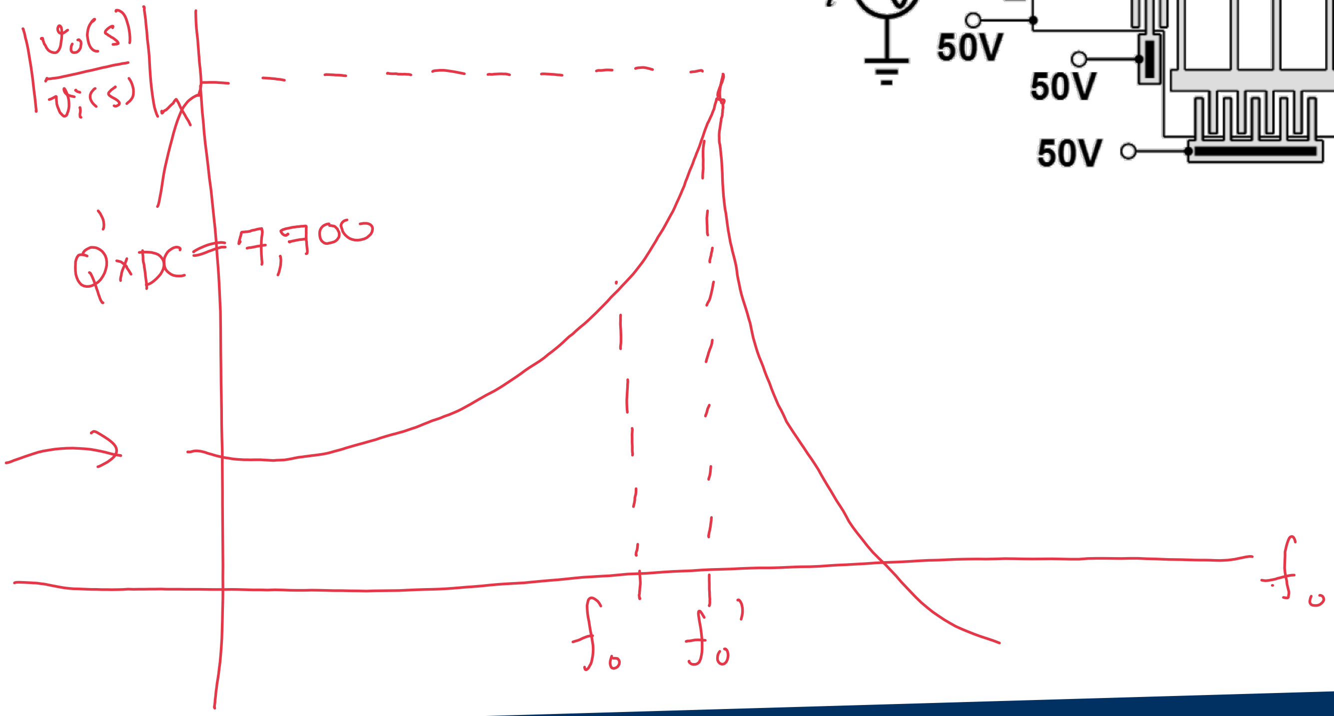
$$\boxed{\omega_o'} = \omega_o\sqrt{1+\frac{C_x}{C_D}}$$

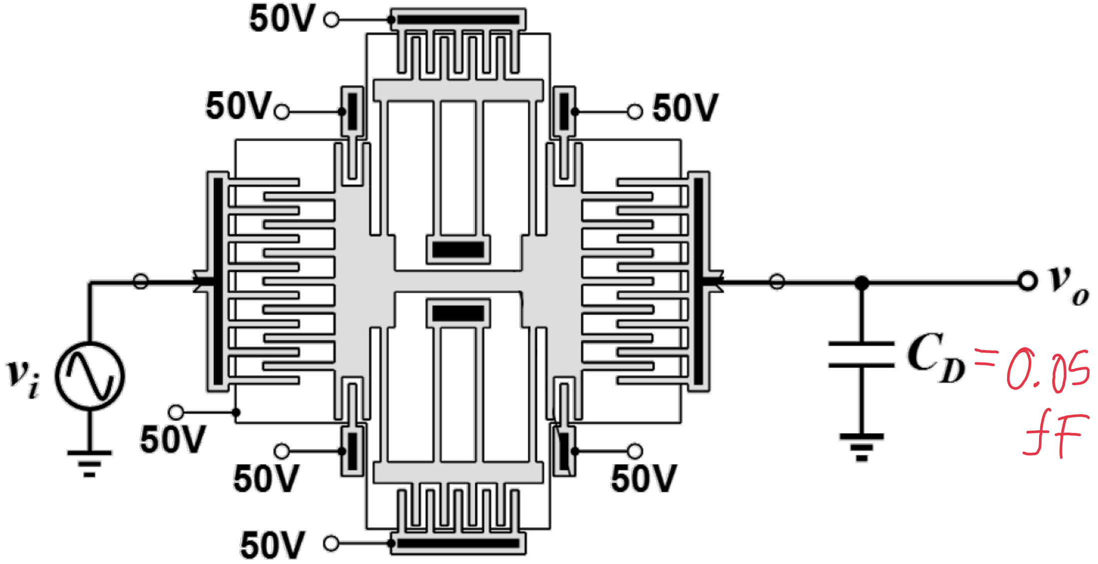
$$Q' = Q\sqrt{1+\frac{C_x}{C_D}}$$

$$C_x = \eta_{e1}^2 c_x \simeq 4\,aF$$

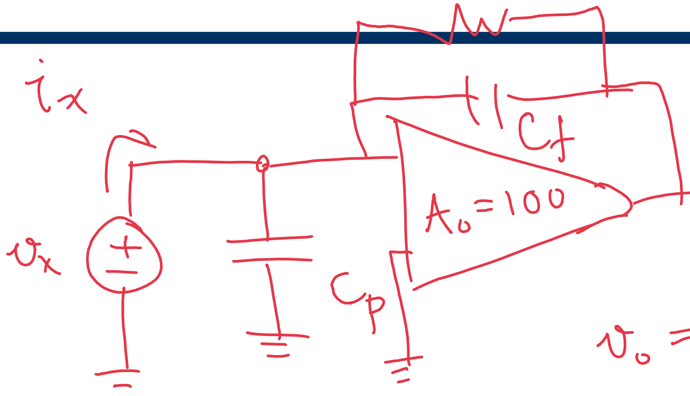
$Q' \times DC = 7,700$

DC gain = 0.074 →



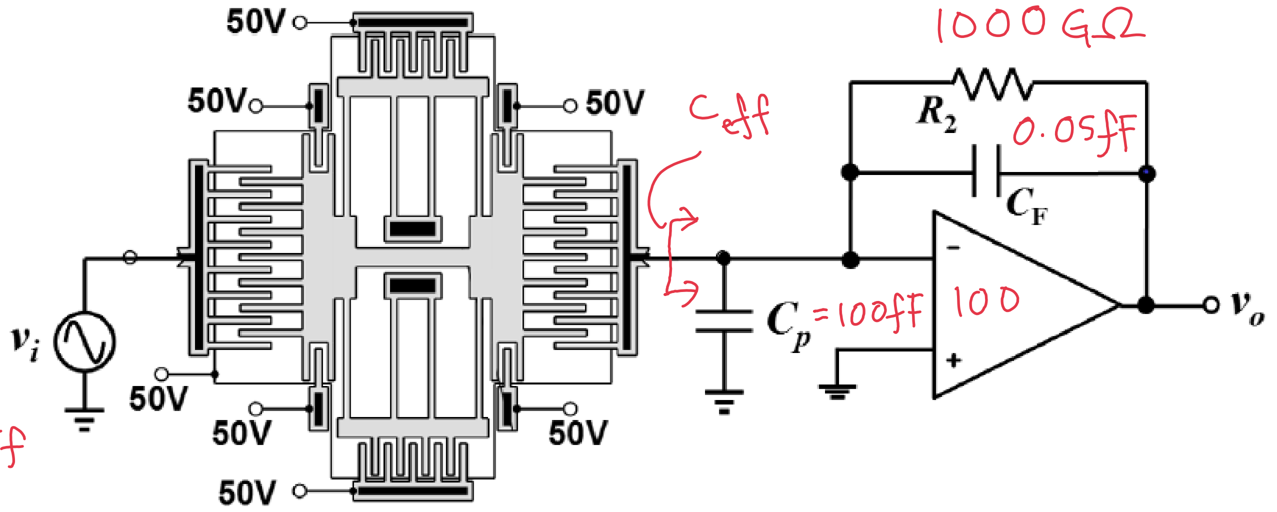


SENSING



$$Z_{eff} = \frac{v_x}{i_x}$$

$$v_o = A_o(v^+ - v^-)$$



$$Z_{eff} = \frac{1}{s[C_p + (1+A_o)C_f]} \rightarrow C_{eff} = C_p + (1+A_o)C_f$$

$$C_{eff} \approx 105fF$$

$$f_o' = f_o\left(1 + \frac{4aF}{105fF}\right)^{1/2} = 1.00004 f_o$$

↖ much closer to f_o

$$v_o = A_o(v^+ - v^-)$$

$$= -A_o v^-$$

$$\frac{v_o}{v^-} = -A_o$$

$$\frac{v_o(s)}{v_i(s)} = \underbrace{\frac{C_x/C_{eff}}{1 + C_x/C_{eff}} \frac{(\omega_o')^2}{s^2 + \left(\frac{\omega_o'}{Q'}\right)s + (\omega_o')^2}}_{\frac{v^-}{v_i}} \underbrace{(-A_o)}_{\frac{v_o}{v^-}}$$