EE 247B / ME 218 Discussion 2

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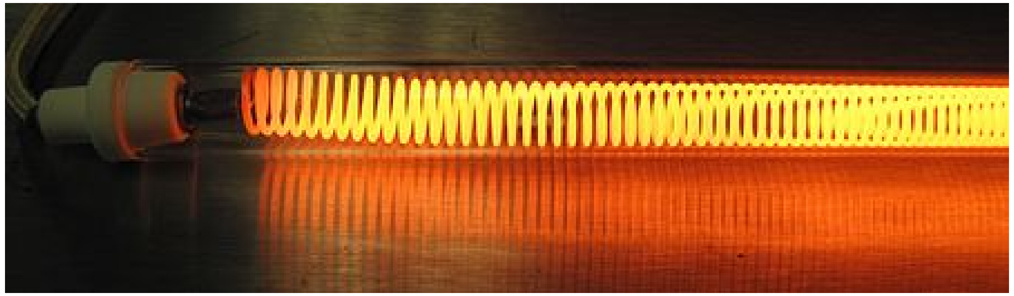
February 7th, 2020

Thermal/Electrical Hybrid Circuits

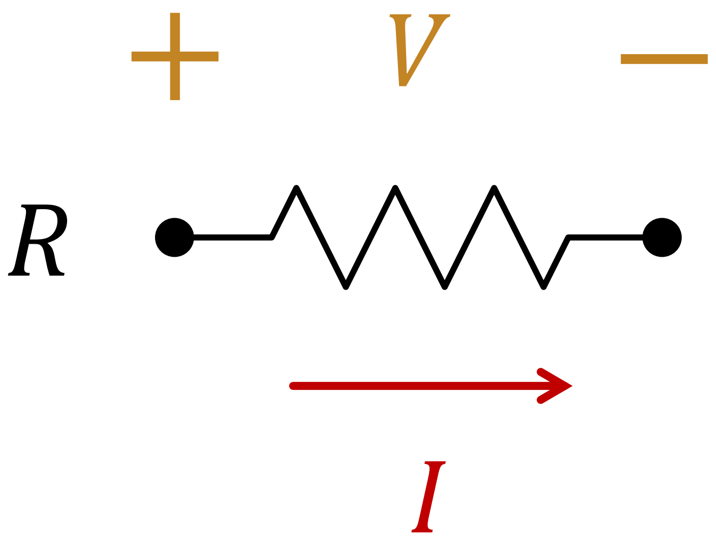
- The circuit problems we've looked at have been mostly thermal
- Inputs have been convenient power sources (like lasers)
- Is this actually useful? How is thermal energy typically coupled into a thermal body/MEMS?



Electrical power through resistive heating!

POWER DISSIPATED IN A RESISTOR

- Remember Ohm's law!



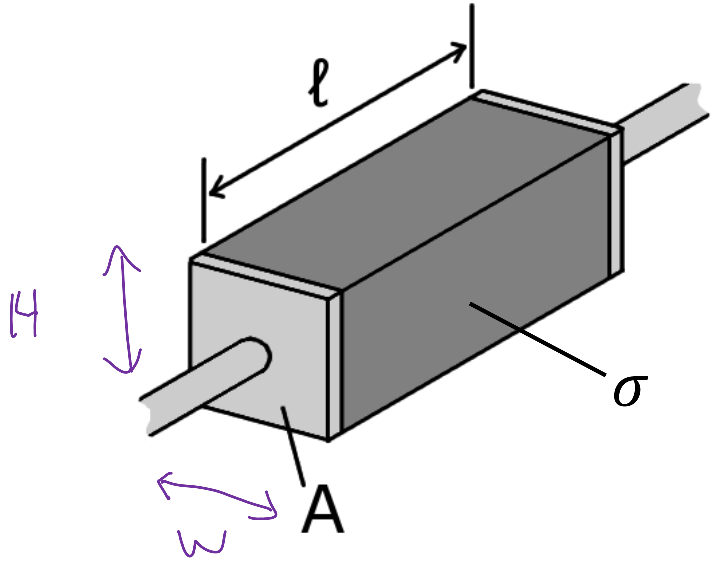
$$V = I \cdot R \;,\quad I = \frac{V}{R}$$

$$P = I \cdot V$$

$$P = \frac{V^2}{R} = I^2 R$$

Most of this energy is thermal!

Refresher: Electrical Resistance



$$\rho = \frac{1}{\sigma}$$

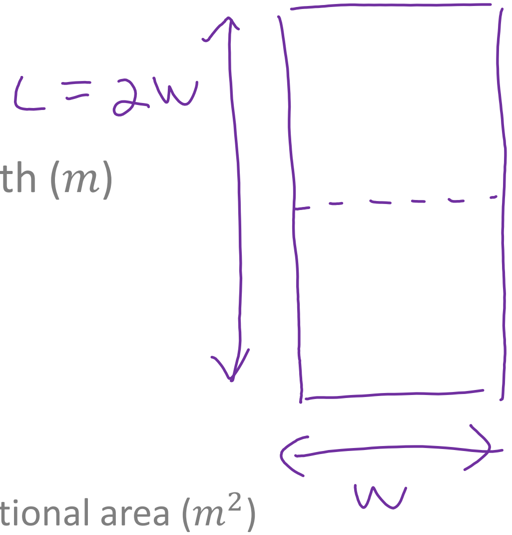
resistance (Ω)

length (m)

$$R = \frac{l}{\sigma \cdot A}$$

conductivity ($(\Omega \cdot m)^{-1}$)

cross-sectional area (m^2)



$$R = \rho \frac{L}{WH} = \frac{\rho}{H} \frac{L}{W}$$

resistivity

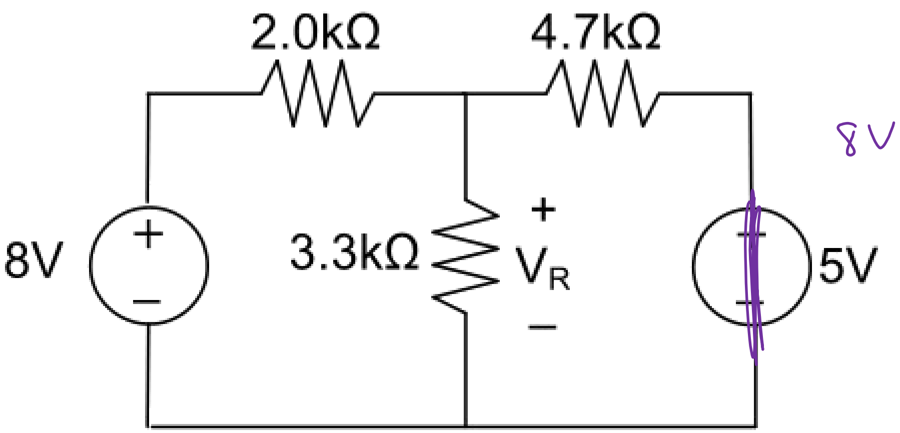
R_s [Ω/□]

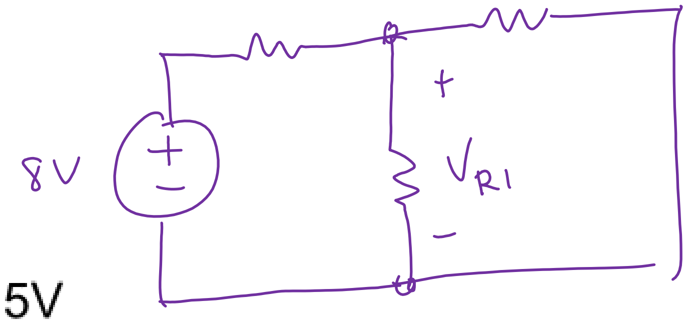
number of squares

Refresher: Superposition

How do we handle circuits with multiple inputs (i.e., sources)?

1. Pick a source to analyze
2. Suppress (turn off) all other independent sources
   - Set either V or I to zero
     - short-circuit voltage sources & open-circuit current sources
3. Find the output of interest for the modified circuit
4. Repeat steps 1-3 for all sources
5. Sum all resultant outputs to find total output due to all sources

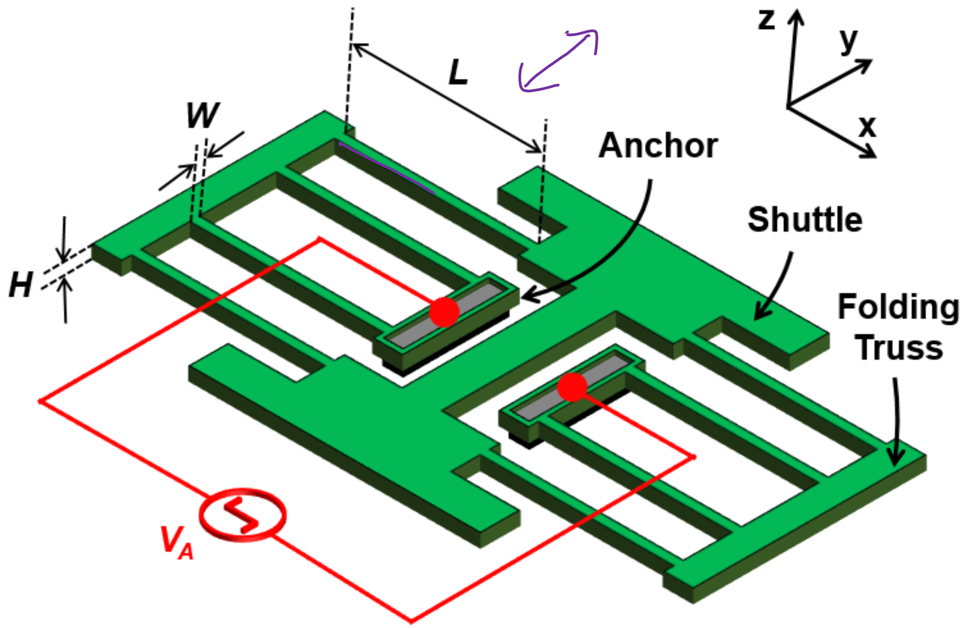




$V_R = V_{R1} + V_{R2}$

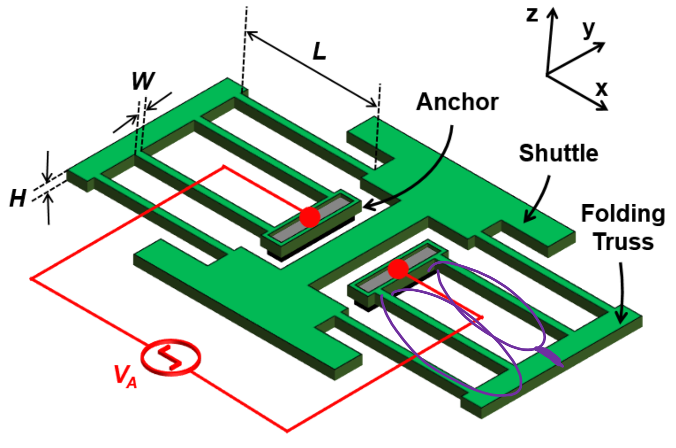
THERMAL CKT. EXAMPLE

- V_A represents a step function voltage source (can think of it as switching the voltage on at some time, t)
- Find the time-constant of the circuit
- Find the steady-state temperature on the shuttle if the final value of V_A is 1V



PARAMETER	VALUE	UNIT
Young's Modulus, *E*	150	GPa
Density, ρ	2,300	kg/m^3
Poisson Ratio, ν	0.226	-
Sheet Resistance, R_s	10	Ω/□
Specific Heat, c_p	770	J/kg.K
Thermal Conductivity, *k*	30	W/m.K
Beam Length/Width/Thickness, *L*/*W*/*H*	50/2/2	μm/μm/μm
Folding Truss Area, A_f	250	μm^2
Shuttle Area, A_s	8,000	μm^2

THERMAL CKT. EXAMPLE



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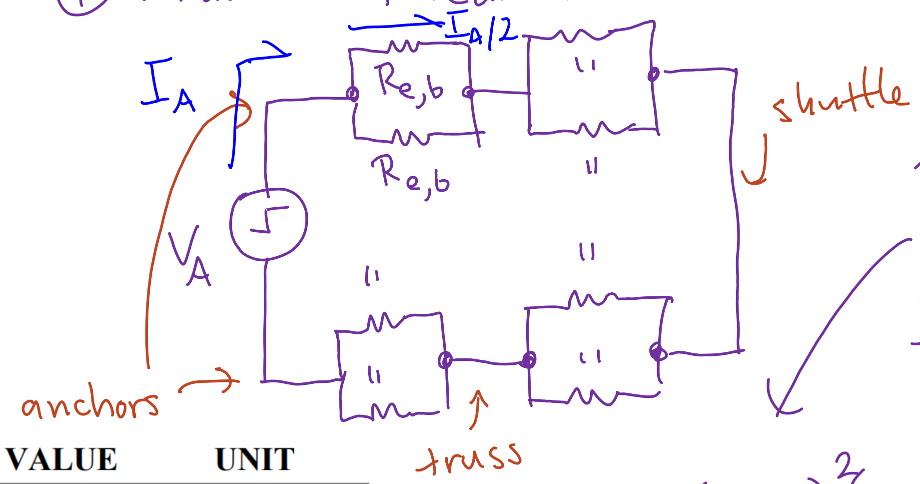
① Draw electrical ckt.



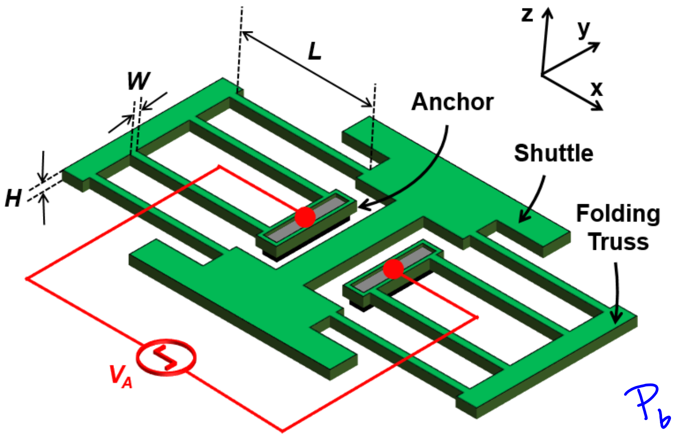
$$R_{e,b} = R_s \frac{L}{W} = \left(10 \frac{\Omega}{\square}\right)\left(\frac{50\mu m}{2\mu m}\right) = 250\,\Omega$$

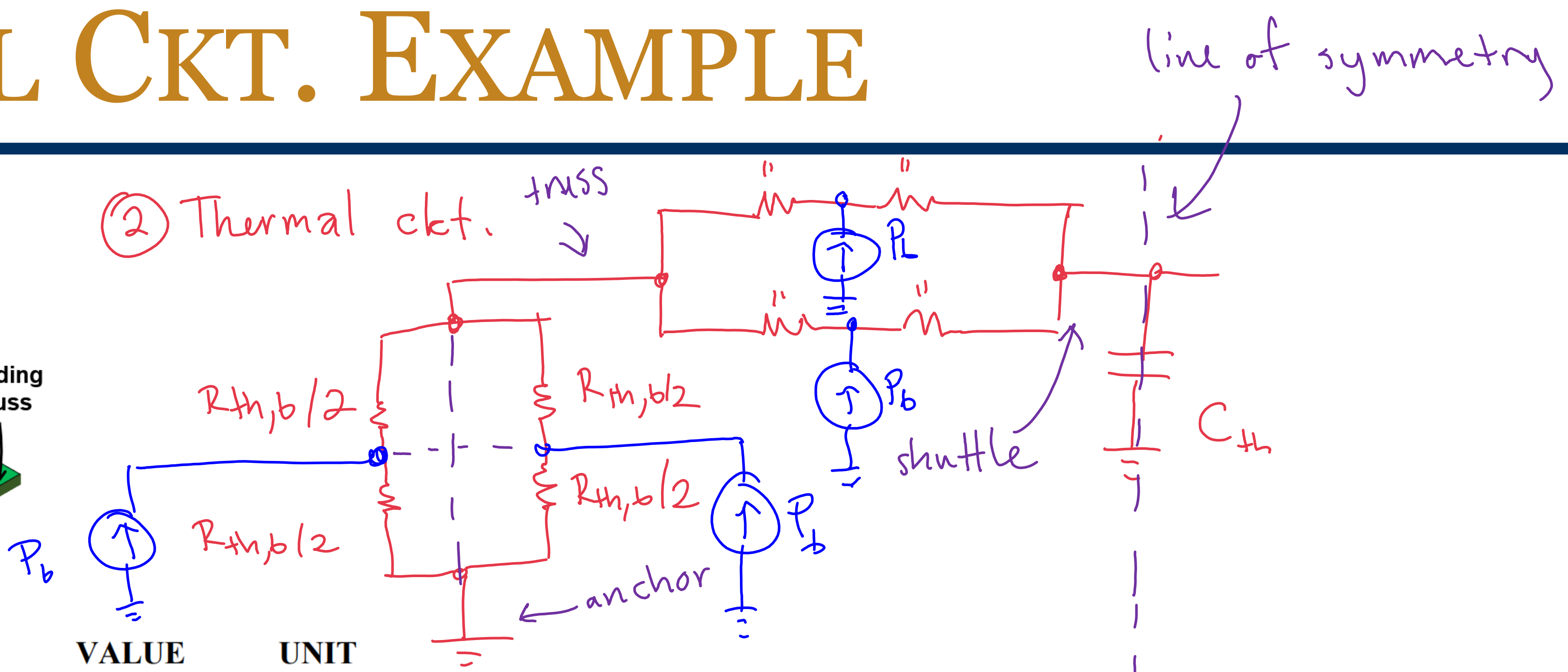
$$P_b = I_b \cdot V_b = \left(\frac{I_A}{2}\right)^2 R_{e,b}$$

$$I_A = \frac{V_A}{2R_{e,b}} = \frac{(1V)}{2(250\,\Omega)} = 2mA$$

$$P_b = (1mA)^2(250\Omega) = 250\,\mu W$$

THERMAL CKT. EXAMPLE





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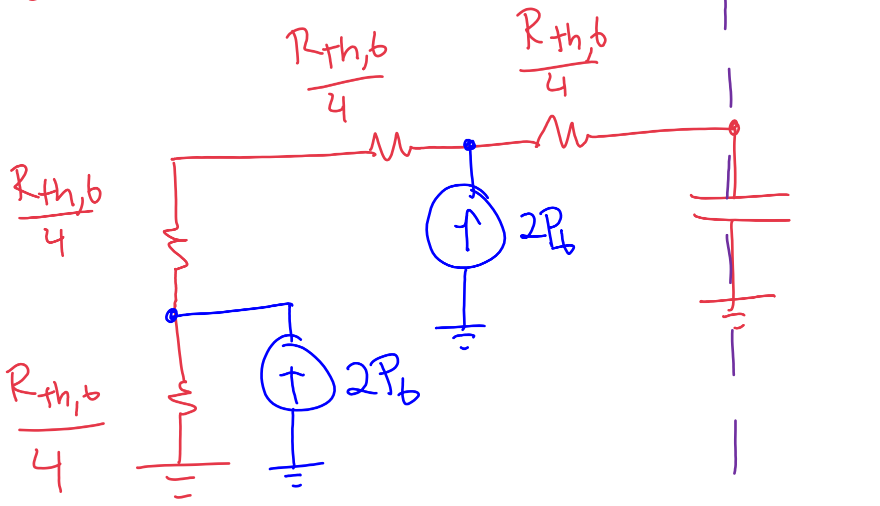
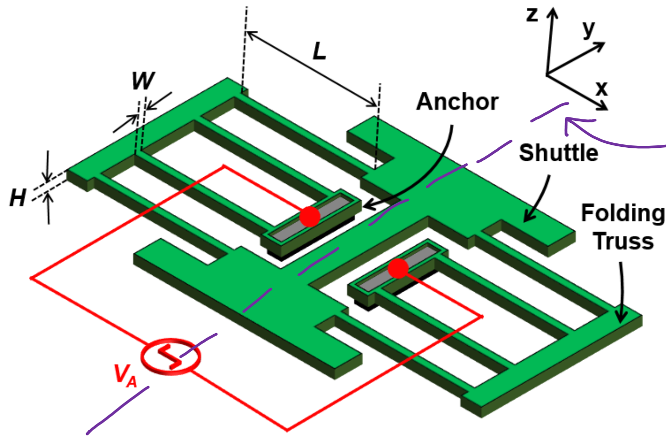
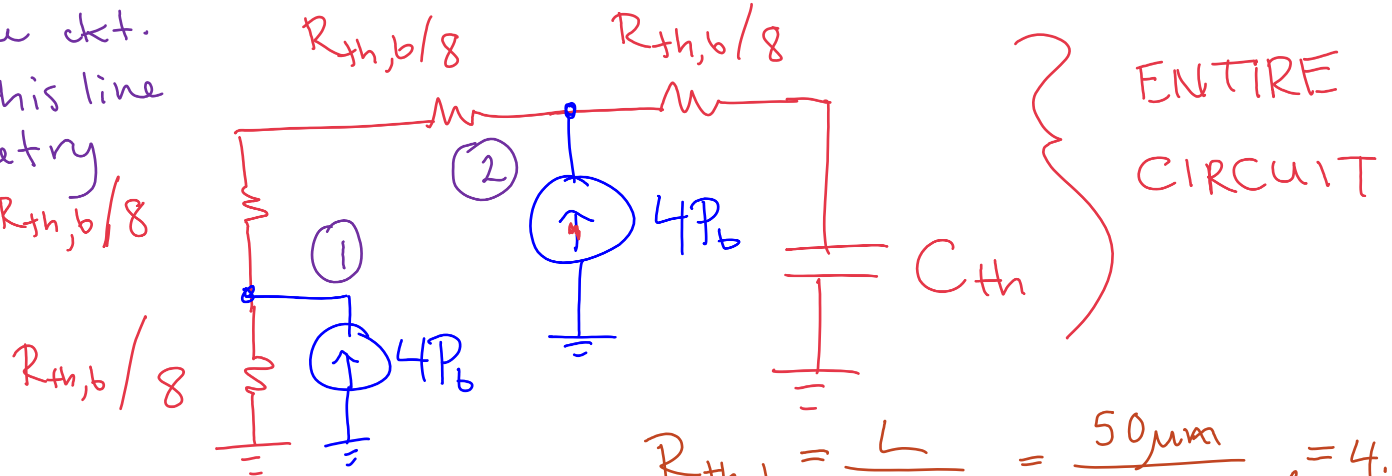


THERMAL CKT. EXAMPLE



"Fold" the ckt. across this line of symmetry



$$R_{th,b} = \frac{L}{k(W \cdot H)} = \frac{50\mu m}{\left(30\frac{W}{m \cdot K}\right)(2\mu m)^2} = 4.17\times10^5 \ K/W$$

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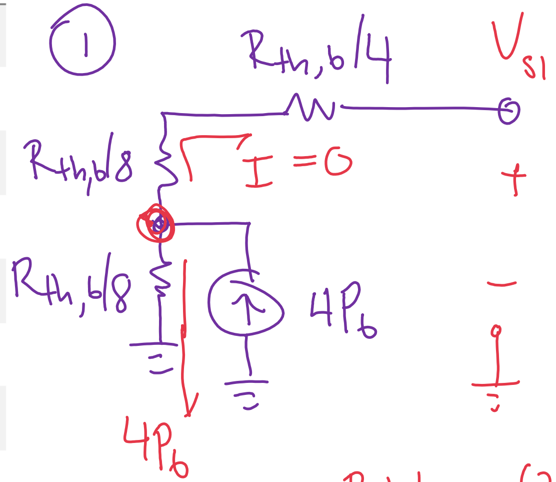
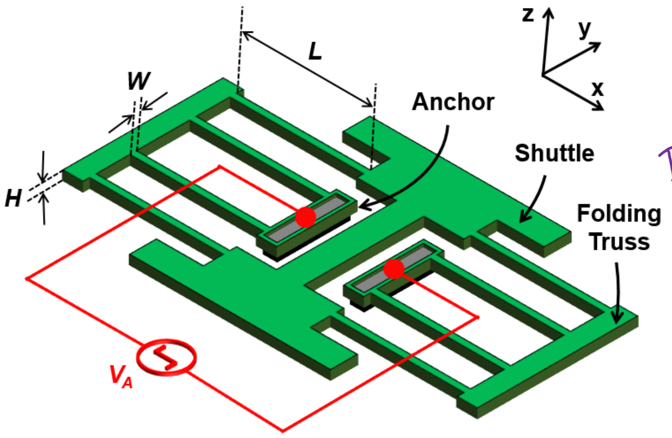
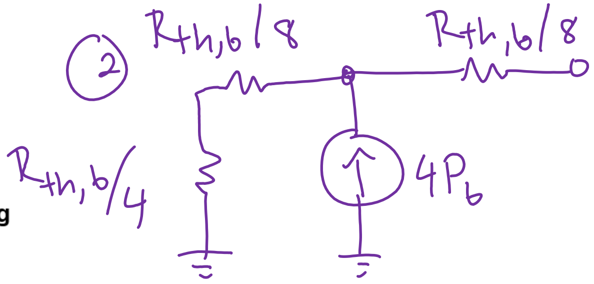


$$V_{S1} = 4P_b \cdot \frac{R_{th,b}}{8} = \frac{(250\mu W)\left(4.17\times10^5 \frac{K}{W}\right)}{2} = 52.1K = \Delta T_1$$

$I \cdot R$

THERMAL CKT. EXAMPLE





$$\Delta T_2 = 4P_b \cdot \frac{3}{8} R_{th,b} = 156.3\,K$$

$$T_{shuttle,ss} = 298K + \Delta T_1 + \Delta T_2 = 506.3K$$

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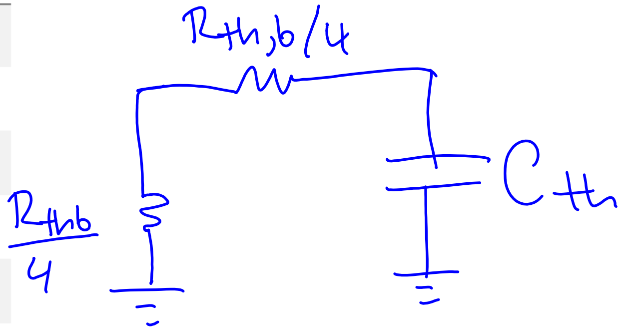
Time constant



$$\tau = C_{th} \cdot \frac{R_{th,b}}{2}$$

$$= \left(2.83\times10^{-8}\,\frac{J}{K}\right)\left(\frac{4.17\times10^{5}\,\frac{K}{W}}{2}\right)$$

$$= 5.9\,ms$$

$$C_{th} = \rho(A_s \cdot H)c_p$$

$$= 2.83\times10^{-8}\,J/K$$