PROBLEM SET #4

Issued: Tuesday, March 10, 2020

Due: Tuesday, March 17, 2020, 8:00 am via Gradescope

1. Suppose you fabricated a clamped-free beam (i.e., a cantilever) in 2 μm-thick polysilicon, with width $W = 2$ μm and length $L = 40$ μm as shown in Figure PS4.1. Assume that the deposited polysilicon film has a vertical stress gradient as shown in Fig. PS4.2, where σ_x is proportional to $z^{1/3}$ with $\sigma_o = 10^7$ Pa at location $z = H/2$, i.e. the top surface of the beam.

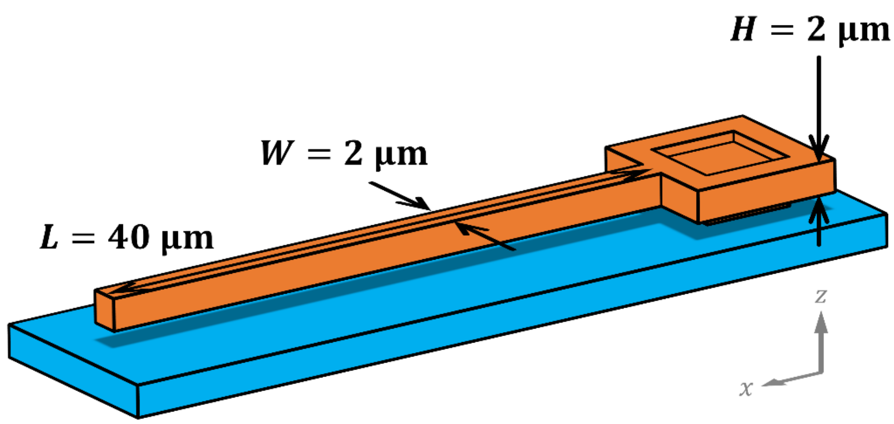

Figure PS4.1

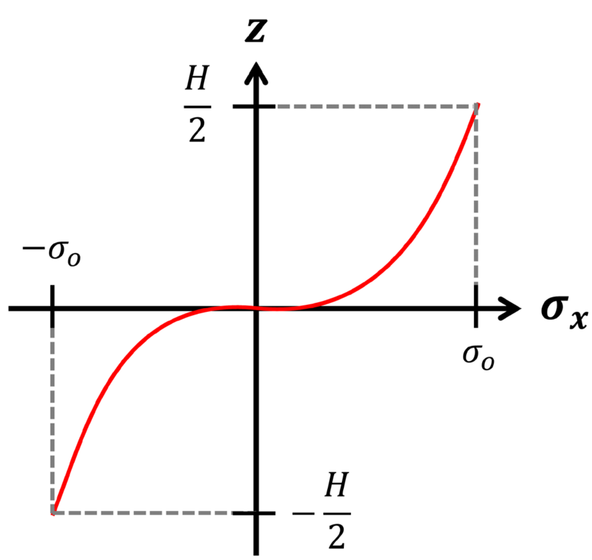

Figure PS4.2

(a) If the sacrificial layer between the underside of the beam and the substrate was 500 nm thick, how high above the substrate is the tip of the cantilever after release?

(b) How large of a point force F would you need to apply to the tip of the cantilever after release so that the deflection at the tip is zero? Explain why or why not the profile of the cantilever will be perfectly flat even when this point force is applied.

(c) One method to measure the stress gradient in a thin film involves measuring the radius of curvature by shining a laser beam straight down at the wafer surface and measuring the ensuing angle θ between the original beam and the one reflected off of the wafer. Suppose that you could use this system to measure the radius of curvature of a single cantilever beam as shown in Fig. 4.3. Write an expression for the angle θ as a function of location along the length of the cantilever, x. Then find the angle θ_{max} at the tip of the cantilever ($x = L$) as a function of the maximum stress in the film, σ_o.

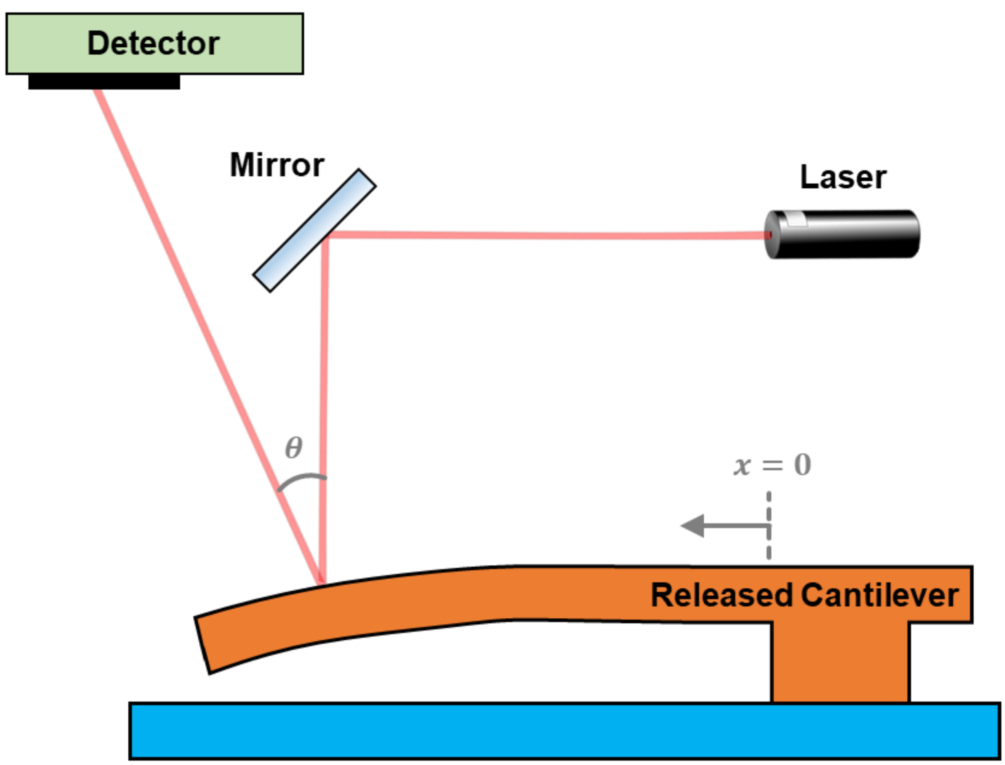

Figure PS4.3

2. Fig. PS4.4 presents a small micromechanical filter constructed in a 2 μm-thick structural layer. Here everything is suspended 2 μm above the substrate except for the darkly shaded anchor regions. Data on the structural material are given in Table PS4.1 below. Assume that all folding trusses and shuttles are perfectly rigid in all directions, including the vertical (i.e., z) direction. Also, assume that all suspension and coupling beam widths are 2 μm.

(a) Write an expression for the static spring constant in the x-direction at location A in the figure and calculate its numerical value with units.

(b) Write an expression for the static spring constant in the x-direction at location B in the figure and calculate its numerical value with units.

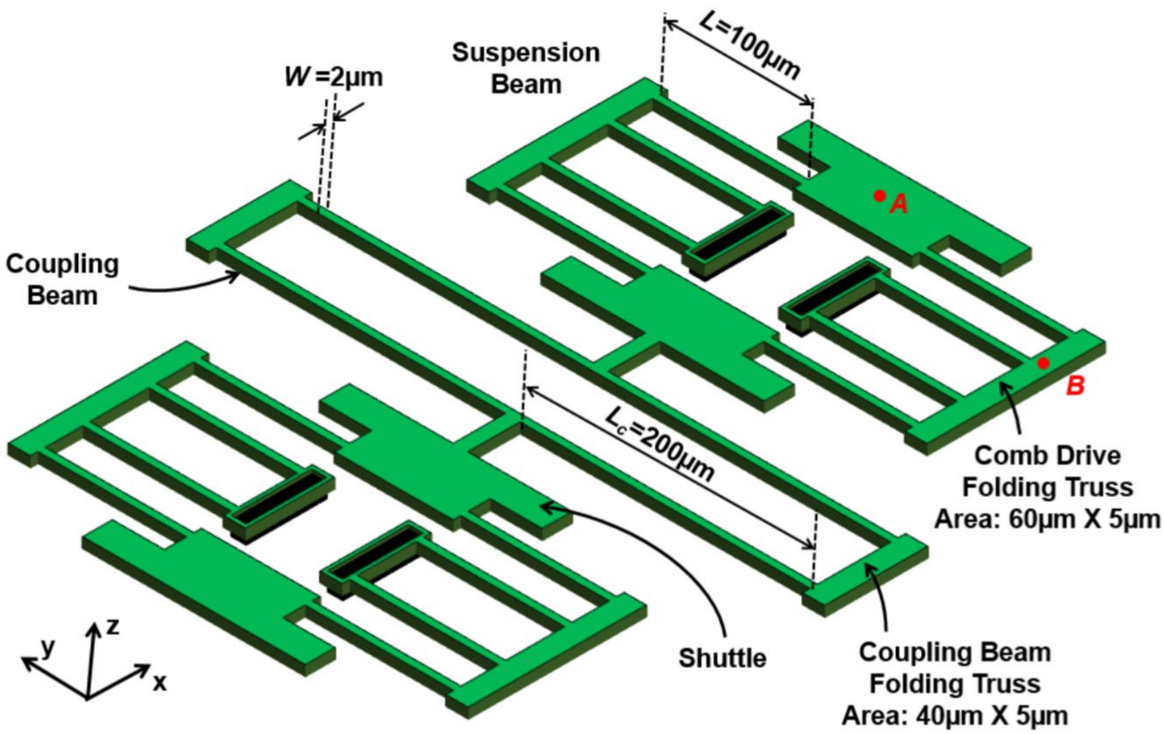

Figure PS4.4

PARAMETER	*VALUE*	*UNIT*
Young's Modulus, E	150	GPa
Density, ρ	2,300	kg/m^3
Poisson's Ratio, ν	0.226	-

Table PS4.1