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EE C247B – ME C218
Introduction to MEMS Design
Spring 2020

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Lecture Module 7: Mechanics of Materials

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Outline

- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics

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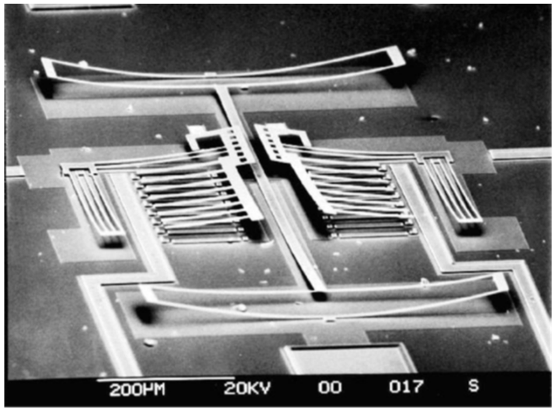
Vertical Stress Gradients

- Variation of residual stress in the direction of film growth
- Can warp released structures in z-direction



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Elasticity

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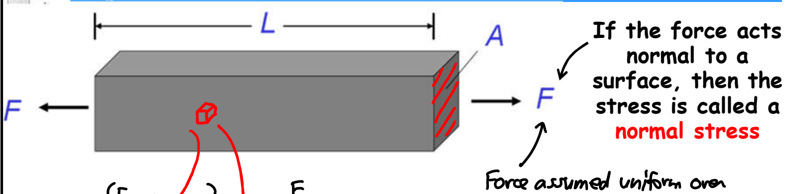
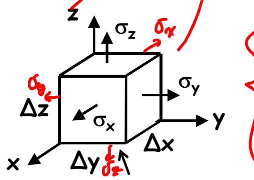
Normal Stress (1D)



Stress = {Force per unit area} = $\sigma = \frac{F}{A}$ [N/m² = Pa]
↖ standard mks unit



⇒ Microscopic Definition: force per unit area acting on the surface of a differential volume element of a solid body

⇒ Note: assume stress acts uniformly across the entire surface of the element, not at just a point

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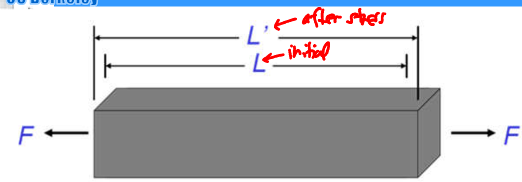
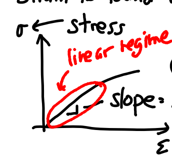
Strain (1D)



Sometimes a unit called the "microstrain" is used, where $1\mu\varepsilon = \frac{\Delta L}{L}$ of 1 part in 10^6 ($=10^{-6}$)

Strain = {Fractional Change in Length} = $\varepsilon = \frac{L'-L}{L} = \frac{\Delta L}{L}$ [unitless]

In the elastic regime (i.e., for "small" stresses at "low" temperatures), strain is found to be proportional to stress



slope = E = Young's modulus of elasticity

For solids: MPa → GPa

$\sigma = \varepsilon E$ → $\varepsilon = \frac{\sigma}{E}$ [unitless]

Thus, the units of E are the same as σ → Pa

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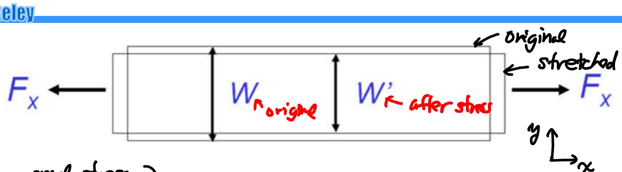
The Poisson Ratio



Apply normal stress to a free-standing object → uniaxial strain
→ but also get contraction in directions transverse to the uniaxial strain

⇒ contraction creates a (−) strain:

$$\varepsilon_y = \frac{W'-W}{W} = \frac{\Delta W}{W} = -\nu\varepsilon_x$$

ν = Poisson ratio [unitless]

Typical values: 0 → 0.5
- ⇒ inorganic solids: 0.2 → 0.3
- ⇒ elastomers (e.g., rubber): ~0.5

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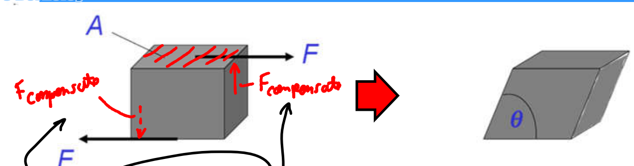
Shear Stress & Strain (1D)



Note: Assume compensating forces are applied to the vertical faces to avoid a net torque. (This by convention.)

Shear Stress = {Force Per Unit Area Parallel to the Surfaces} = $\tau = \frac{F}{A}$ [Pa]

→ Generates a shear strain:

Shear Strain = $\theta = \frac{\tau}{G}$ ← $G \triangleq$ shear modulus

$$G = \frac{E}{2(1+\nu)}$$

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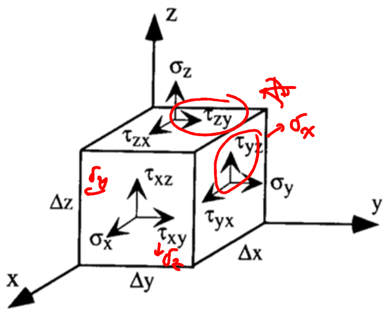
2D and 3D Considerations

- Important assumption: the differential volume element is in static equilibrium → no net forces or torques (i.e., rotational movements)
  - Every σ must have an equal σ in the opposite direction on the other side of the element
  - For no net torque, the shear forces on different faces must also be matched as follows:



Stresses acting on a differential volume element

$$\tau_{xy} = \tau_{yx} \qquad \tau_{xz} = \tau_{zx} \qquad \tau_{yz} = \tau_{zy}$$

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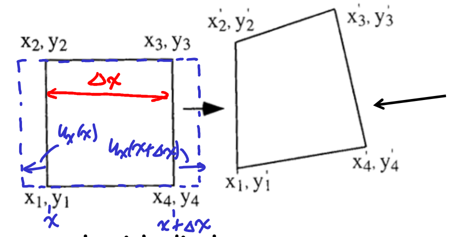
2D Strain

- In general, motion consists of
  - rigid-body displacement (motion of the center of mass)
  - rigid-body rotation (rotation about the center of mass)
  - Deformation relative to displacement and rotation



Area element experiences both displacement and deformation

- Must work with displacement vectors
- Differential definition of axial strain: $\longrightarrow$ $\varepsilon_x = \dfrac{u_x(x+\Delta x) - u_x(x)}{\Delta x} = \dfrac{\partial u_x}{\partial x}$

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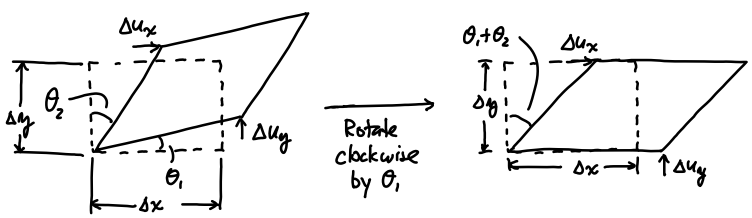
2D Shear Strain



⇒ For shear strains, must remove any rigid body rotation that accompanies the deformation
↳ use a symmetric definition of shear strain:

$$\gamma_{xy} = \theta_2 + \theta_1 \approx \left(\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x}\right) = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

↑ For small amplitude deformations.

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Isotropic Elasticity in 3D

- Isotropic = same in all directions
- The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke's Law)

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right] \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_z + \sigma_x\right)\right] \qquad \gamma_{yz} = \frac{1}{G}\tau_{yz}$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \underbrace{\nu\left(\sigma_x + \sigma_y\right)}\right] \qquad \gamma_{zx} = \frac{1}{G}\tau_{zx}$$

Basically, add in off-axis strains from normal stresses in other directions

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α_T As a Function of Temperature

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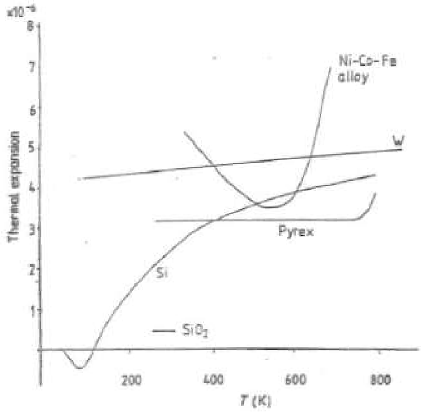

[Madou, Fundamentals of Microfabrication, CRC Press, 1998]

- Cubic symmetry implies that α is independent of direction

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