

### Measurement of Stress Gradient

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- Use cantilever beams
  - Strain gradient ( $\Gamma$  = slope of strain-thickness curve) causes beams to deflect up or down
  - Assuming linear strain gradient  $\Gamma$ ,  $z = \Gamma L^2/2$

■ compressive  
■ tensile

[P. Krulvitch Ph.D.]

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### Folded-Flexure Suspensions

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### Folded-Beam Suspension

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- Use of folded-beam suspension brings many benefits
  - Stress relief: folding truss is free to move in y-direction, so beams can expand and contract more readily to relieve stress
  - High y-axis to x-axis stiffness ratio

*Folding Truss*

Comb-Driven Folded Beam Actuator

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### Beam End Conditions

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TABLE 4.1  
Types of commonly used support conditions for beams and frames

Type of support	Displacement boundary conditions	Force boundary conditions
 FREE	None	All, as specified
 PINNED	$u = 0$ $w = 0$	Moment is specified
 ROLLER (vertical)	$u = 0$	Transverse force and moment are specified
 ROLLER (horizontal)	$w = 0$	Horizontal force and bending moment are specified
 FIXED or CLAMPED	$u = 0$ $w = 0$ $dw/dx = 0$	None specified

[From Reddy, Finite Element Method]

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**Common Loading & Boundary Conditions**  
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- Displacement equations derived for various beams with concentrated load  $F$  or distributed load  $f$
- Gary Fedder Ph.D. Thesis, EECS, UC Berkeley, 1994

cantilever	guided-end	fixed-fixed
$x = \frac{F_x L}{E h w}$	$x = \frac{F_x L}{E h w}$	$x = \frac{F_x L}{4 E h w}$
$y = 4 \frac{F_y L^2}{E h w^3}$	$y = \frac{F_y L^2}{E h w^3}$	$y = \frac{1}{16} \frac{F_y L^2}{E h w^3}$
$z = 4 \frac{F_z L^2}{E w h^3}$	$z = \frac{F_z L^2}{E w h^3}$	$z = \frac{1}{16} \frac{F_z L^2}{E w h^3}$

(a) Concentrated load.

cantilever	guided-end	fixed-fixed
$x = \frac{f_x L}{E}$	$x = \frac{f_x L}{E}$	$x = \frac{f_x L}{4 E}$
$y = \frac{3}{2} \frac{f_y L^4}{E h w^3}$	$y = \frac{1}{2} \frac{f_y L^4}{E h w^3}$	$y = \frac{1}{32} \frac{f_y L^4}{E h w^3}$
$z = \frac{3}{2} \frac{f_z L^4}{E w h^3}$	$z = \frac{1}{2} \frac{f_z L^4}{E w h^3}$	$z = \frac{1}{32} \frac{f_z L^4}{E w h^3}$

(b) Distributed load.

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