Lecture 13: Beam Combos I

- Announcements:
- HW#3 due Tuesday, 3/10, at 8 a.m.
- Midterm Exam about 2 weeks away, Thursday, March 19, 9:30-11:00 a.m., 521 Cory (right here)
- -----------------------------------
- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients
- -----------------------------------
- Last Time:
- Finished stress gradients
- Now, move into beam combos

over

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Bending Due to Stress/Strain Gradients

Find the radius of curvature:

Prior to release, axial stress: $\sigma = \sigma_0 - \frac{\sigma_1}{(H/2)} z$

The internal moment:

$$M_x = \int_{-\frac{H}{2}}^{\frac{H}{2}} \left[(W \cdot dz)\sigma\right] \cdot z = \int_{-\frac{H}{2}}^{\frac{H}{2}} W\left(z\sigma_0 - \frac{\sigma_1 z^2}{(H/2)}\right) dz$$

$$= W\left(\frac{1}{2}\sigma_0 z^2 - \frac{2\sigma_1 z^2}{3H}\right)\Bigg|_{-H/2}^{H/2}$$

$$= W\left(\cancel{\frac{1}{2}\sigma_0\frac{H^2}{4}} - \frac{2}{3}\sigma_1\frac{H^2}{8} - \cancel{\frac{1}{2}\sigma_0\frac{H^2}{4}} - \frac{2}{3}\sigma_1\frac{H^2}{8}\right)$$

average stress cancels out

$$\boxed{M_x = -\frac{1}{6}\sigma_1 W H^2}$$

Thus, the radius of curvature:

$$\frac{1}{R} = -\frac{M_x}{E'I} \rightarrow R = \frac{E'I}{M_x} = \frac{1}{2}\frac{E'H}{\sigma_1}$$

$[I = \frac{1}{12}WH^3]$

Biaxial Modulus (if W is large)

$$\boxed{R = \frac{1}{2}\frac{E}{(1-\nu)}\frac{H}{\sigma_1}}$$

Radius of Curvature for a Cantilever w/ a Stress Gradient

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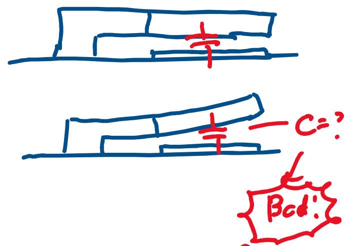
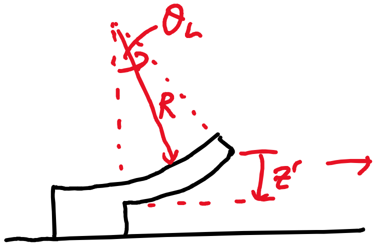
Radius of Curvature → z'



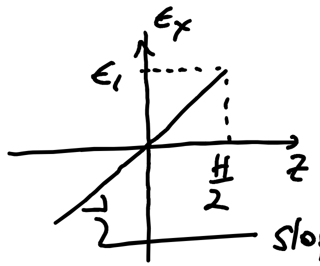


→ integrate over θ_L to get z'

↓

Do this in homework ...

Definition: Strain Gradient



slope $\triangleq$ Strain Gradient $= \Gamma$

$$R = \frac{1}{2}\frac{E}{1-\nu}\frac{H}{\sigma_1} = \frac{H}{2}\frac{E'}{\sigma_1} = \frac{(H/2)}{\epsilon_1} = \frac{1}{\Gamma} \rightarrow \boxed{\Gamma = \frac{1}{R}}$$ ✓

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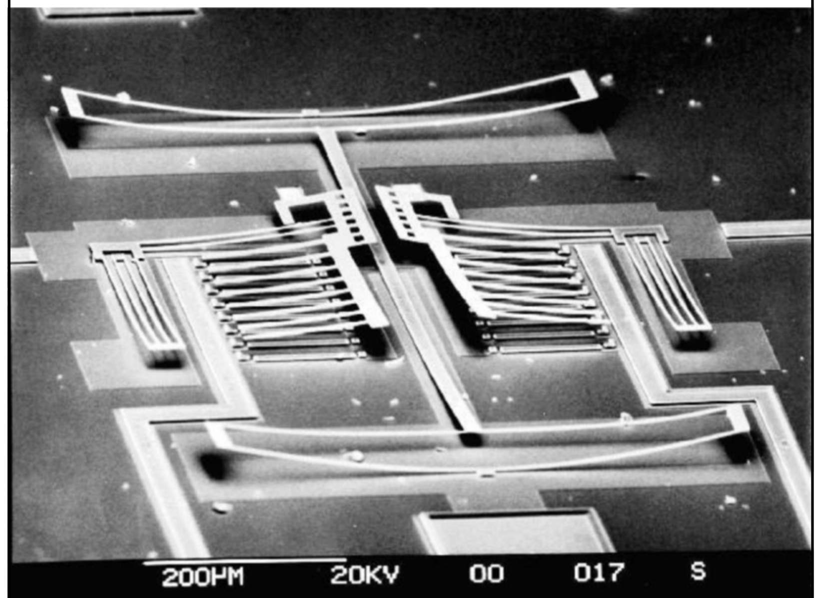
- … and the result:



- We just quantified this

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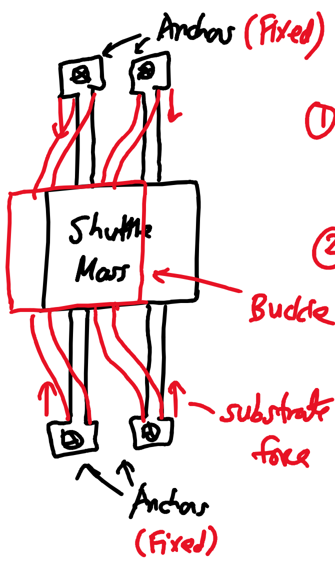
Folded-Beam Suspension

⇒ Module 8, slide 22



① During deposition @ high T → stress free

② Cool to room temp (RT) → stress

} f/ compressive stress

How to defend against this?

① Δ process parameters → e.g., the deposition recipe
→ problem: can't always do this

② Design → folded-beam!

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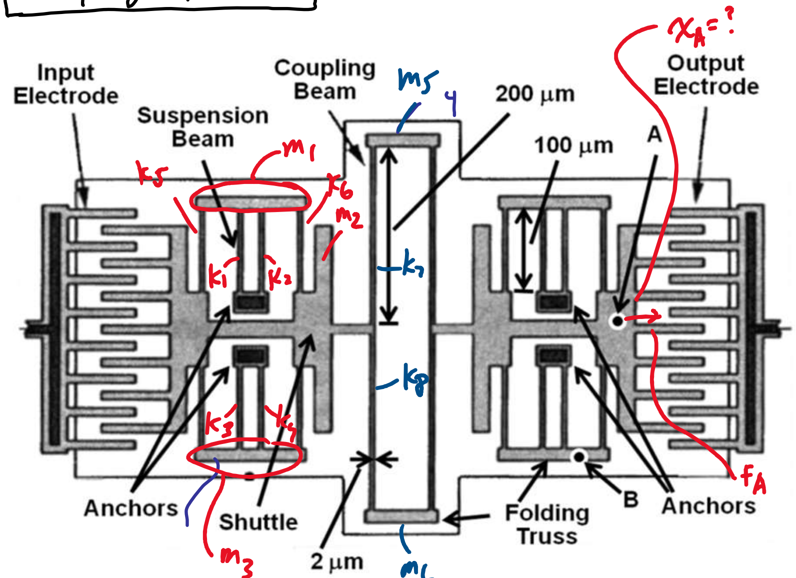
Analyzing an Interconnected Ensemble of Beams (Springs) & Masses



Typical Questions: → all demand that we know $x = f(F)$

① How does the structure move in response to a force at a specific location. ← that we know

② What is the frequency response to an AC force applied at a specific location. → the stiffness!

③ Noise?

④ Response to environmental stimuli? (e.g., rotation)

⑤ How does stress affect the behavior of the structure?

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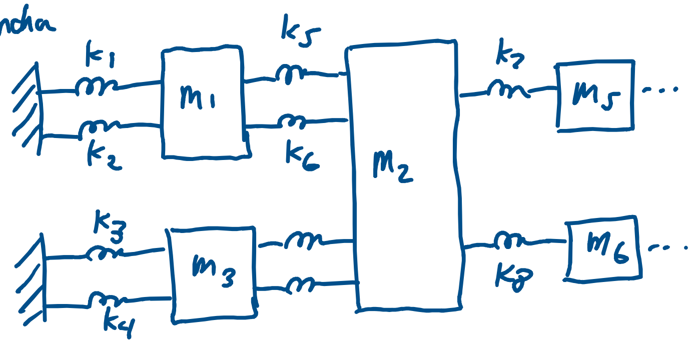
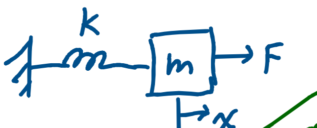
Procedure: (to get stiffness)

① Build the ckt. (Extract the ckt.) → in the x-direction (for this example)



② Analyze to get $x = f(F)$ — x: displacement, F: force

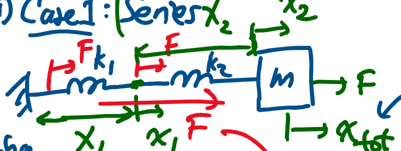


$\Rightarrow F = kx \rightarrow x = \frac{F}{k} = \left(\frac{1}{k}\right)F$ ← compliance

$x_2 = \frac{F}{k_2}$

$x_1 = \frac{F}{k_1}$

(a) Case 1: Series



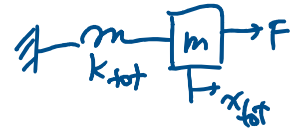
across variable

thru variable

want this ∴ need k_{tot}

$x_{tot} = \left(\frac{1}{k_{tot}}\right)F$

series because one must go thru both k_1 & k_2 to get from anchor to the forcing pt.

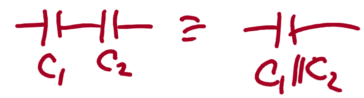


* → $\underbrace{x_{tot}}_{\text{identifies series}} = x_1 + x_2 = F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) = \frac{F}{(k_1 \| k_2)} = \frac{F}{k_{tot}}$

["$\|$" operator ≙ $A \| B = \frac{1}{\frac{1}{A} + \frac{1}{B}} = \frac{AB}{A+B}$]

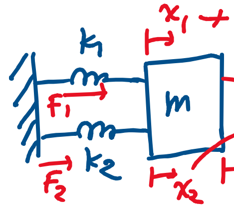
$\boxed{k_{tot} = k_1 \| k_2}$ (for k_1 & k_2 in series)

For EE's: same as capacitors (springs combine like capacitors)



(b) Case 2: Parallel Springs

$x_1 \rightarrow F_1 = k_1 x_1 = k_1 x_{tot}$



$F \rightarrow F = k_{tot} x_{tot}$ ← want k_{tot}

$F_2 = k_2 x_{tot}$

$\underbrace{x_{tot} = x_1 = x_2}_{\text{indicates parallel}}$

or → only need to go through one of the springs to get from the anchor to the forcing pt.

$$F = F_1 + F_2 = \underbrace{(k_1 + k_2)}_{k_{tot}} x_{tot}$$

$$\boxed{k_{tot} = k_1 + k_2}$$ (for k_1 & k_2 in parallel)

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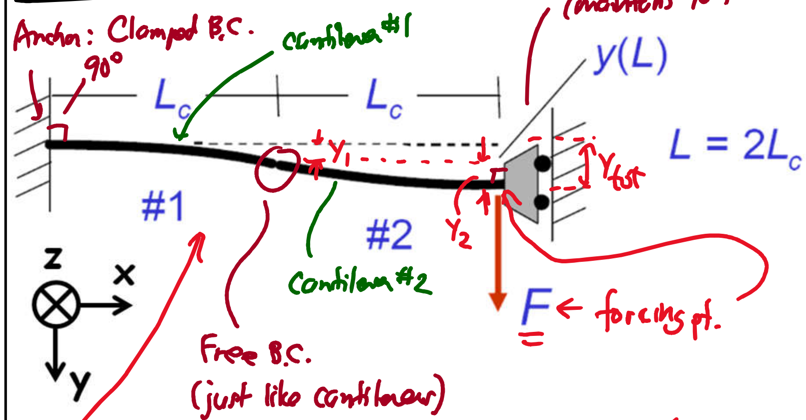
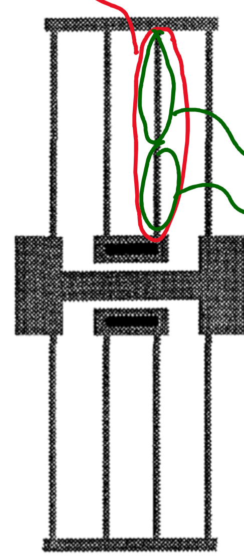
Series Combination of Springs



$Y_{tot} = Y_1 + Y_2$ ∴ series ✓

series ✓ ← Also, must go thru both springs to get from anchor to forcing pt.

$$\boxed{k_{tot} = k_1 \| k_2 = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}} = \frac{k_1 k_2}{k_1 + k_2}}$$



$k_2 = k_c$
$k_1 = k_c$

$k_1 = k_2 = k_c \Rightarrow k_{tot} = \frac{k_c}{2}$

cantilever stiffness

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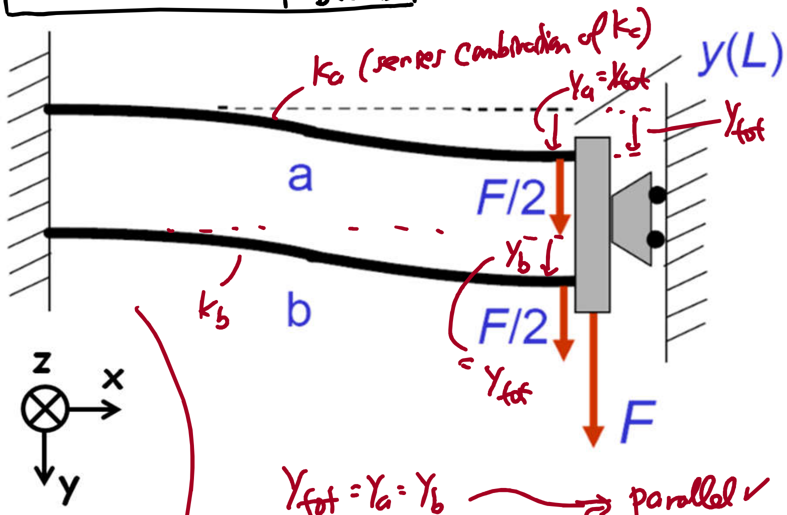
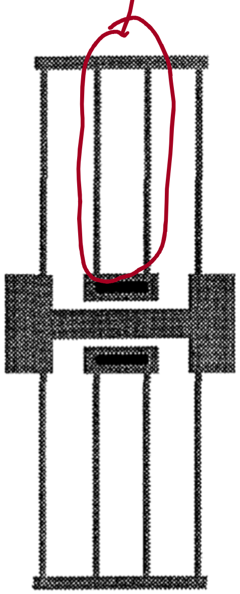
Parallel Combination of Beams



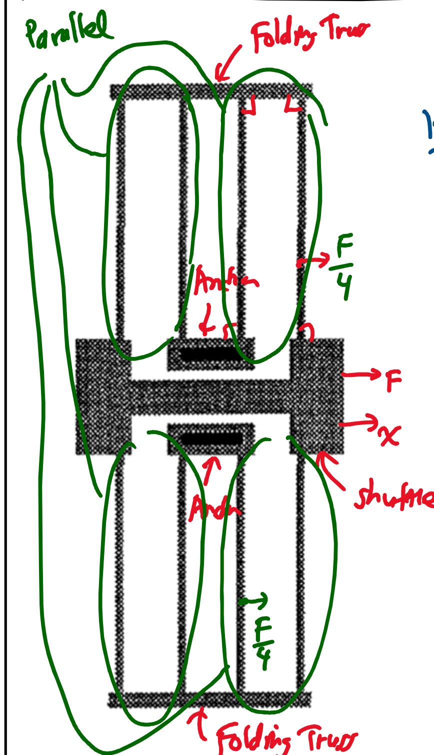
$Y_{tot} = Y_a = Y_b$ → parallel ✓

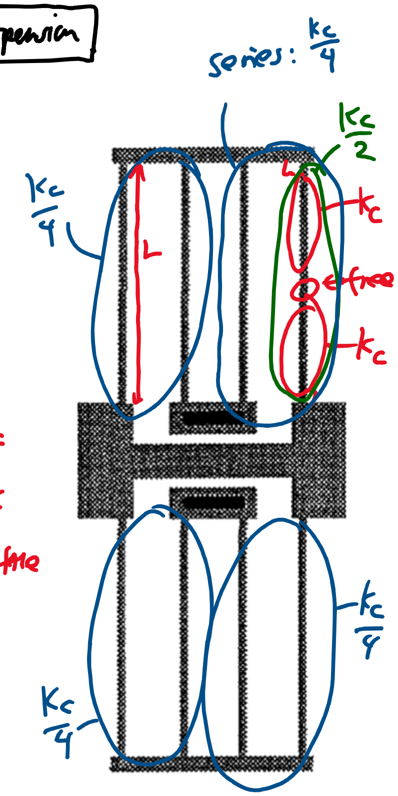
to go from anchor to forcing pt., need only go through one of the springs

$k_{tot} = k_a + k_b = \frac{k_c}{2} + \frac{k_c}{2} = k_c$

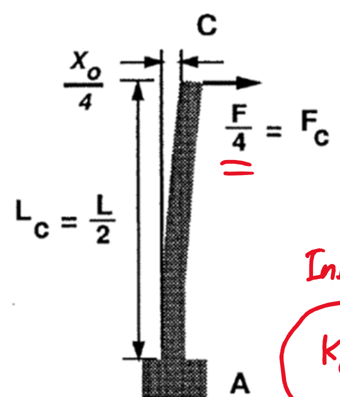


Stiffness of Folded-Beam Suspension





$k_{tot} = 4\left(\frac{k_c}{4}\right) = k_c$



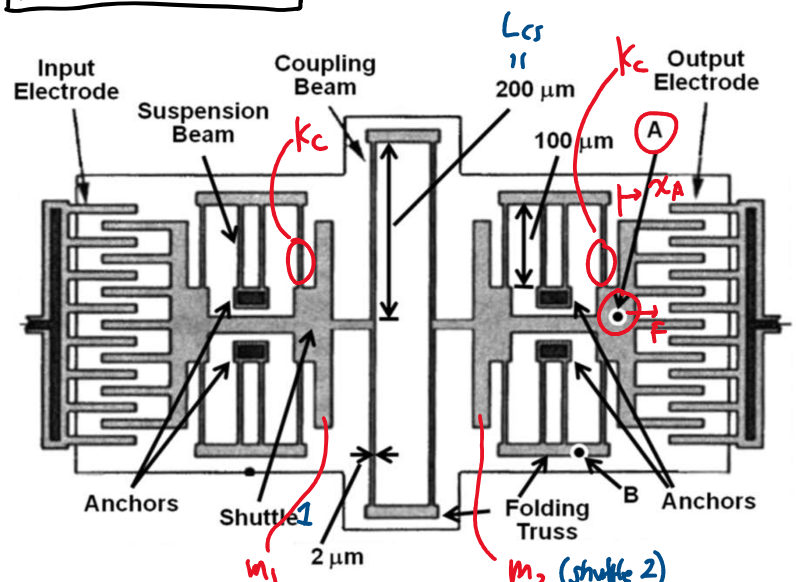
$$\boxed{k_c = \frac{3EI_z}{L_c^3} = \frac{1}{4}Eh\left(\frac{W}{L_c}\right)^3}$$

Insert $L_c = \frac{L}{2}$ $\qquad \frac{1}{2}hW^3$

$$\boxed{k_c = \frac{3EI_z}{(L/2)^3} = \frac{24EI_z}{L^3}}$$

full beam length

Micromechanical Filter



⇒ Find the stiffness at point A.

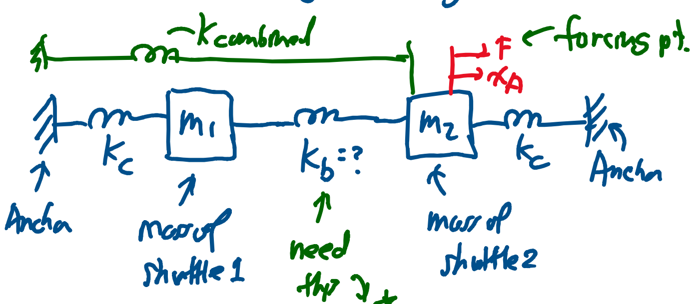
↳ apply F@A → what is x_A? } $x_A = \frac{F}{k_A}$

$k_A \triangleq$ stiffness @ point A

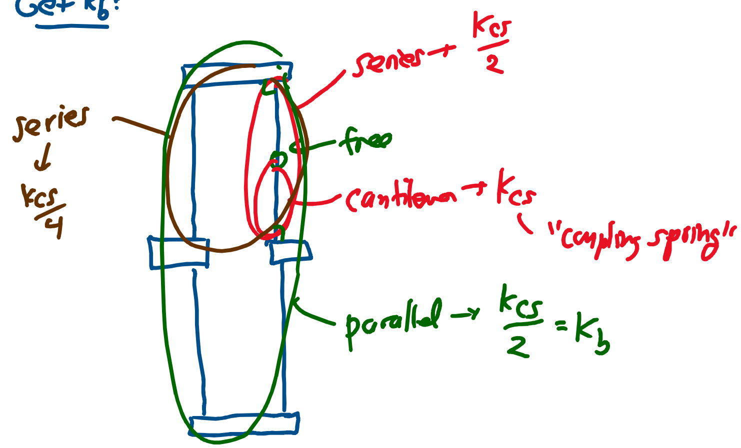
↖ want this

Assume: shuttles & folding trusses are rigid.

beams are small ∴ neglect their mass



Get k_b:



$\therefore k_A = k_c + k_{combined}$

$= k_c + k_c \| k_b = \boxed{k_c + k_c \| \frac{k_{cs}}{2} = k_A}$

where $k_c = \frac{24EI_z}{L^3}$

$k_{cs} = \frac{24EI_z}{L_{cs}^3}$