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# EE C247B - ME C218 Introduction to MEMS Design Spring 2020

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Lecture Module 9: Energy Methods

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## Lecture Outline

- Reading: Senturia, Chpt. 10
- Lecture Topics:
  - ↳ Energy Methods
    - ↳ Virtual Work
    - ↳ Energy Formulations
    - ↳ Tapered Beam Example

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## Energy Methods

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## More General Geometries

- Euler-Bernoulli beam theory works well for simple geometries
- But how can we handle more complicated ones?
- **Example:** tapered cantilever beam
- **Objective:** Find an expression for displacement as a function of location  $x$  under a point load  $F$  applied at the tip of the free end of a cantilever with tapered width  $W(x)$

Top view of cantilever's  $W(x)$

$W(x) = W(1 - \frac{x}{2L_c})$


50% taper

$x = L_c$

$F$

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 **Solution: Use Principle of Virtual Work**

- In an energy-conserving system (i.e., elastic materials), the energy stored in a body due to the quasi-static (i.e., slow) action of surface and body forces is equal to the work done by these forces ...
- **Implication:** if we can formulate **stored energy** as a function of the deformation of a mechanical object, then we can determine how an object responds to a force by determining the shape the object must take in order to **minimize the difference  $U$**  between the stored energy and the work done by the forces:

$$U = \text{Stored Energy} - \text{Work Done}$$

- **Key idea:** we don't have to reach  $U = 0$  to produce a very useful, approximate *analytical* result for load-deflection

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