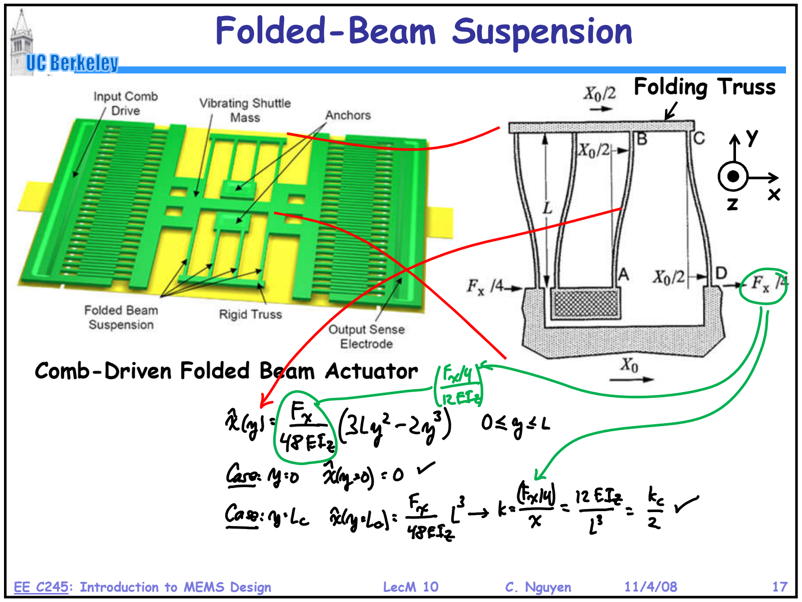
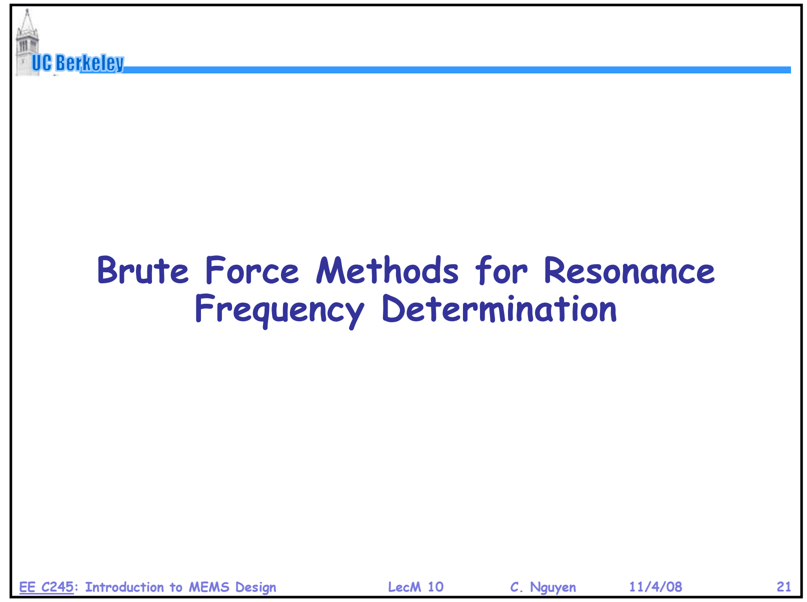

Folded-Beam Suspension
UC Berkeley
Input Comb Drive
Vibrating Shuttle Mass
Anchors
Folded Beam Suspension
Rigid Truss
Output Sense Electrode
Folding Truss
$X_0/2$
B
C
A
D
L
$F_x/4$
X_0
Y
z
x
Comb-Driven Folded Beam Actuator
EE C245: Introduction to MEMS Design
LecM 10
C. Nguyen
11/4/08
17

17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Brute Force Methods for Resonance Frequency Determination
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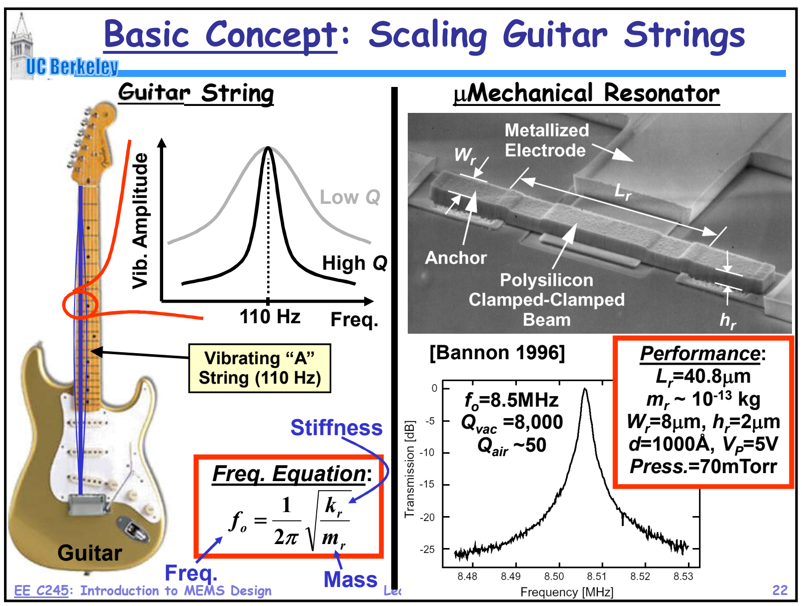
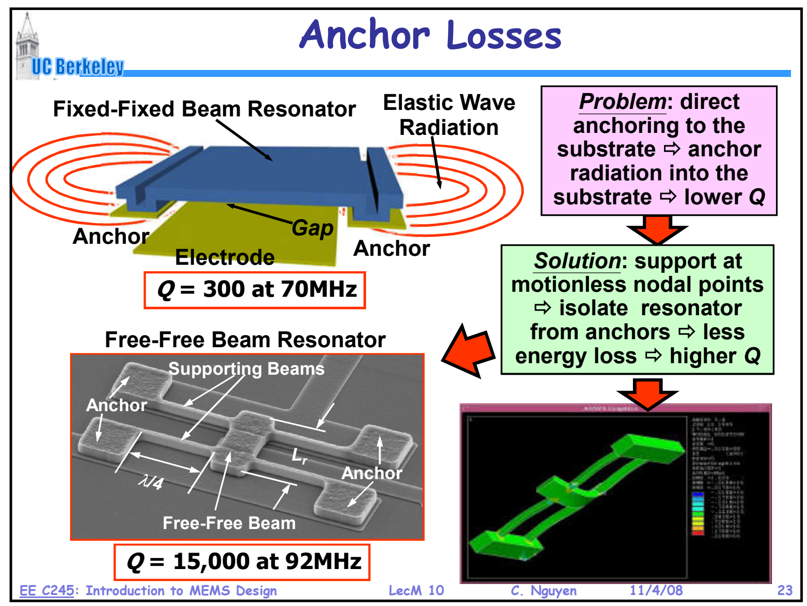

Basic Concept: Scaling Guitar Strings
UC Berkeley
Guitar String
µMechanical Resonator
Vib. Amplitude
Low Q
High Q
110 Hz
Freq.
Vibrating "A" String (110 Hz)
Guitar
Stiffness
Freq. Equation:
$f_o = \frac{1}{2\pi}\sqrt{\frac{k_r}{m_r}}$
Freq.
Mass
Metallized Electrode
W_r
L_r
Anchor
Polysilicon Clamped-Clamped Beam
h_r
[Bannon 1996]
f_o=8.5MHz
Q_{vac} =8,000
Q_{air} ~50
Performance:
L_r=40.8µm
m_r ~ 10^{-13} kg
W_r=8µm, h_r=2µm
d=1000Å, V_P=5V
Press.=70mTorr
Transmission [dB]
Frequency [MHz]
EE C245: Introduction to MEMS Design
22

22


Anchor Losses
UC Berkeley
Fixed-Fixed Beam Resonator
Elastic Wave Radiation
Anchor
Gap
Anchor
Electrode
Q = 300 at 70MHz
Problem: direct anchoring to the substrate ⇨ anchor radiation into the substrate ⇨ lower Q
Solution: support at motionless nodal points ⇨ isolate resonator from anchors ⇨ less energy loss ⇨ higher Q
Free-Free Beam Resonator
Supporting Beams
Anchor
L_r
λ/4
Anchor
Free-Free Beam
Q = 15,000 at 92MHz
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92 MHz Free-Free Beam μResonator

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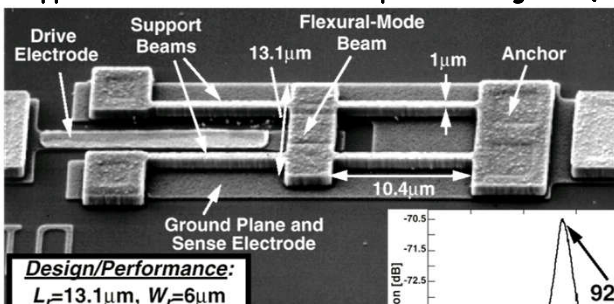
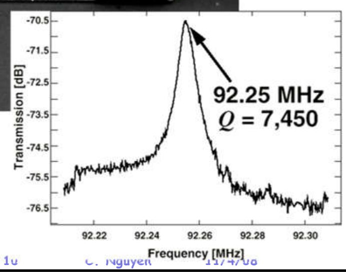
- Free-free beam μmechanical resonator with non-intrusive supports ⇨ reduce anchor dissipation ⇨ higher *Q*



Design/Performance:
L_r=13.1μm, W_r=6μm
h=2μm, d=1000Å
V_P=28-76V, W_e=2.8μm
f_o~92.25MHz
Q~7,450 @ 10mTorr



[Wang, Yu, Nguyen 1998]

Higher Order Modes for Higher Freq.

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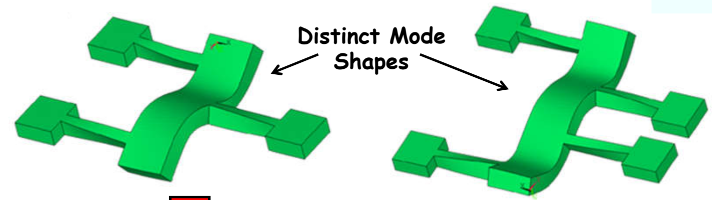
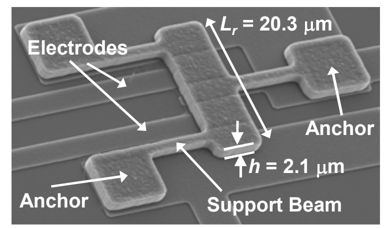
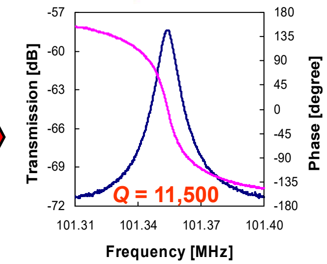
2nd Mode Free-Free Beam

3rd Mode Free Free Beam







Flexural-Mode Beam Wave Equation

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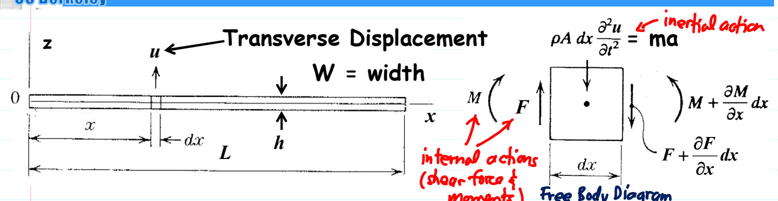

- Derive the wave equation for transverse vibration:

Dynamic Equilibrium Condition for Forces in the y-direction: $M + F\frac{dx}{2} + F\frac{dx}{2} + \frac{dF}{dx}\frac{dx}{2} - M - \frac{\partial M}{\partial x}dx = 0$ (neg.)

$$F - \left(F + \frac{\partial F}{\partial x}dx\right) - \rho A dx \frac{\partial^2 u}{\partial t^2} = 0 \quad (1)$$

neglect the $\frac{\partial F}{\partial x}dx$ term

and the moment equilibrium condition: $-F dx + \frac{\partial M}{\partial x}dx \approx 0 \quad (2)$

Combining (1) & (2):

$$\frac{\partial^2 M}{\partial x^2}dx = -\rho A dx \frac{\partial^2 u}{\partial t^2} \rightarrow \frac{\partial^2}{\partial x^2}\left(-EI\frac{\partial^2 u}{\partial x^2}\right) = -\rho A \frac{\partial^2 u}{\partial t^2} \rightarrow \boxed{\frac{\partial^2 u}{\partial t^2} = \left(\frac{EI}{\rho A}\right)\frac{\partial^4 u}{\partial x^4}}$$

$$\left[\frac{\partial^2 u}{\partial x^2} = -\frac{M}{EI} \rightarrow M = -EI\frac{\partial^2 u}{\partial x^2}\right] \qquad I_y = \frac{Wh^3}{12}$$

Example: Free-Free Beam

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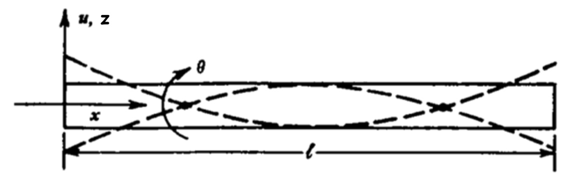

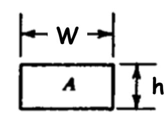

- Determine the resonance frequency of the beam
- Specify the lumped parameter mechanical equivalent circuit
- Transform to a lumped parameter electrical equivalent circuit
- Start with the flexural-mode beam equation:

$$\frac{\partial^2 u}{\partial t^2} = \left(\frac{EI}{\rho A}\right)\frac{\partial^4 u}{\partial x^4}$$

Free-Free Beam Frequency

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- Substitute $u = u_1 e^{j\omega t}$ into the wave equation:

$$\frac{\partial^4 u}{\partial x^4} = \left(\omega^2 \frac{\rho A}{EI}\right) u \qquad (1)$$

- This is a 4th order differential equation with solution:

$$u(x) = \mathscr{A}\cosh kx + \mathscr{B}\sinh kx + \mathscr{C}\cos kx + \mathscr{D}\sin kx \qquad (2)$$

Gives the mode shape during resonance vibration.

- Boundary Conditions:

At $x = 0$	At $x = \ell$	
$\frac{\partial^2 u}{\partial x^2} = 0$	$\frac{\partial^2 u}{\partial x^2} = 0$	$M = 0$ (Bending moment)
$\frac{\partial^3 u}{\partial x^3} = 0$	$\frac{\partial^3 u}{\partial x^3} = 0$	$\frac{\partial M}{\partial x} = 0$ (Shearing force)

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Free-Free Beam Frequency (cont)

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- Applying B.C.'s, get A=C and B=D, and

$$\begin{bmatrix} (\cosh k\ell - \cos k\ell) & (\sinh k\ell - \sin k\ell) \\ (\sinh k\ell + \sin k\ell) & (\cosh k\ell - \cos k\ell) \end{bmatrix} \begin{bmatrix} \mathscr{A} \\ \mathscr{B} \end{bmatrix} = 0 \qquad (3)$$

- Setting the determinant = 0 yields

$$\cos k\ell = \frac{1}{\cosh k\ell}$$

- Which has roots at

$$k_1\ell = 4.730 \qquad k_2\ell = 7.853 \qquad k_3\ell = 10.996$$

These values of $k_n\ell$ correspond to the different modes of vibration!

- Substituting (2) into (1) finally yields:

$$k^4 = \frac{\rho A}{EI}\omega^2 \longrightarrow f_n = \frac{(k_n\ell)^2}{2\pi\ell^2}\sqrt{\frac{EI}{\rho A}}$$

Free-Free Beam Frequency Equation

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Higher Order Free-Free Beam Modes

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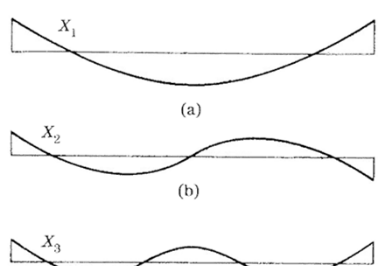
Mode	n	Nodal Points	$k_n\ell$	f_n/f_1
Fundamental (f_1)	1	2	4.730	1.000
1st Harmonic	2	3	7.853	2.757
2nd Harmonic	3	4	10.996	5.404
3rd Harmonic	4	5	14.137	8.932
4th Harmonic	5	6	17.279	13.344

← More than 10x increase



Fundamental Mode (n=1)

1st Harmonic (n=2)

2nd Harmonic (n=3)

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Mode Shape Expression

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- The mode shape expression can be obtained by using the fact that A=C and B=D into (2), yielding

$$u_x = \mathscr{B}\left[\left(\frac{\mathscr{A}}{\mathscr{B}}\right)(\cosh kx + \cos kx) + (\sinh kx + \sin kx)\right]$$

- Get the amplitude ratio by expanding (3) [the matrix] and solving, which yields

$$\frac{\mathscr{A}}{\mathscr{B}} = \frac{\sin k\ell - \sinh k\ell}{\cosh k\ell - \cos k\ell}$$

- Then just substitute the roots for each mode to get the expression for mode shape

Fundamental Mode (n=1)
[Substitute $k_1\ell = 4.730$]

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