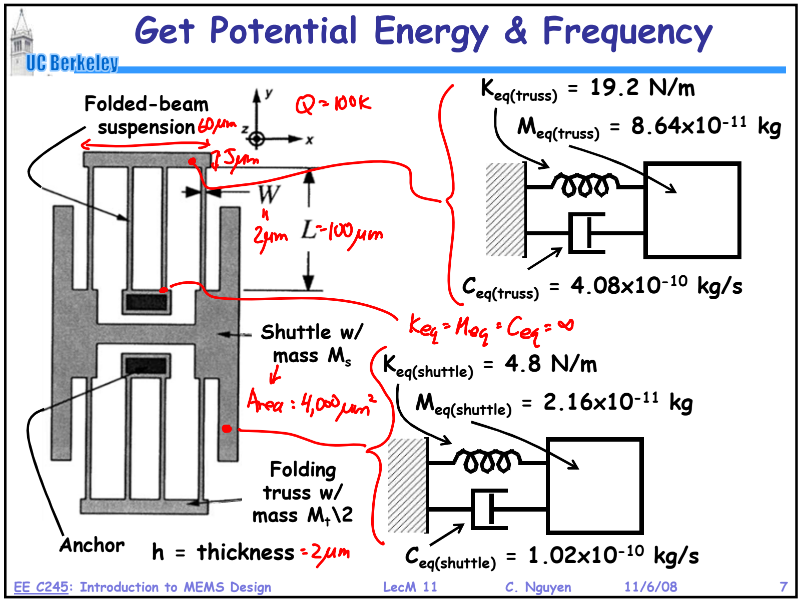
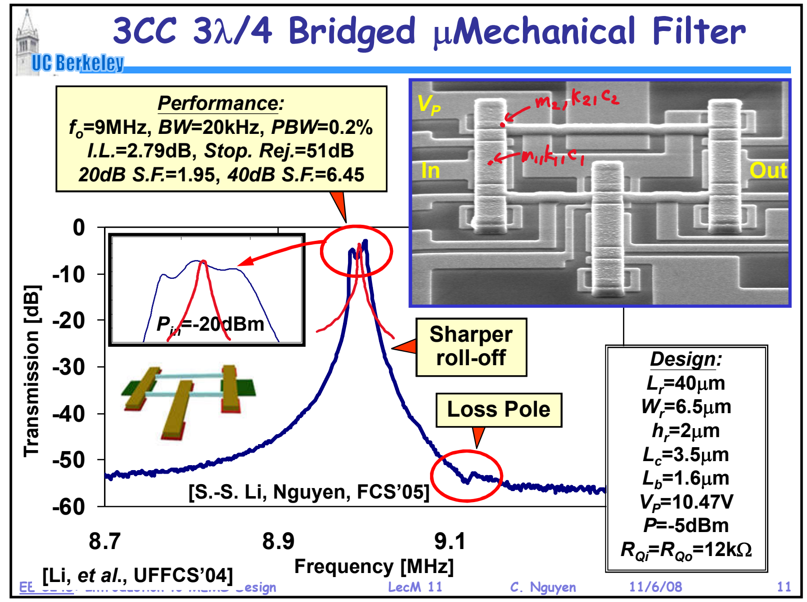

Get Potential Energy & Frequency
UC Berkeley
Folded-beam suspension
Q≈100K
W
L
Shuttle w/ mass M_s
Folding truss w/ mass M_t\2
Anchor
h = thickness
$K_{eq(truss)}$ = 19.2 N/m
$M_{eq(truss)}$ = 8.64x10^{-11} kg
$C_{eq(truss)}$ = 4.08x10^{-10} kg/s
$K_{eq(shuttle)}$ = 4.8 N/m
$M_{eq(shuttle)}$ = 2.16x10^{-11} kg
$C_{eq(shuttle)}$ = 1.02x10^{-10} kg/s
EE C245: Introduction to MEMS Design LecM 11 C. Nguyen 11/6/08 7

7


3CC 3λ/4 Bridged μMechanical Filter
UC Berkeley
Performance:
f_o=9MHz, BW=20kHz, PBW=0.2%
I.L.=2.79dB, Stop. Rej.=51dB
20dB S.F.=1.95, 40dB S.F.=6.45
V_P
In
Out
P_{in}=-20dBm
Sharper roll-off
Loss Pole
[S.-S. Li, Nguyen, FCS'05]
Design:
L_r=40μm
W_r=6.5μm
h_r=2μm
L_c=3.5μm
L_b=1.6μm
V_P=10.47V
P=-5dBm
R_{Qi}=R_{Qo}=12kΩ
Transmission [dB]
0
-10
-20
-30
-40
-50
-60
8.7
8.9
9.1
Frequency [MHz]
[Li, et al., UFFCS'04]
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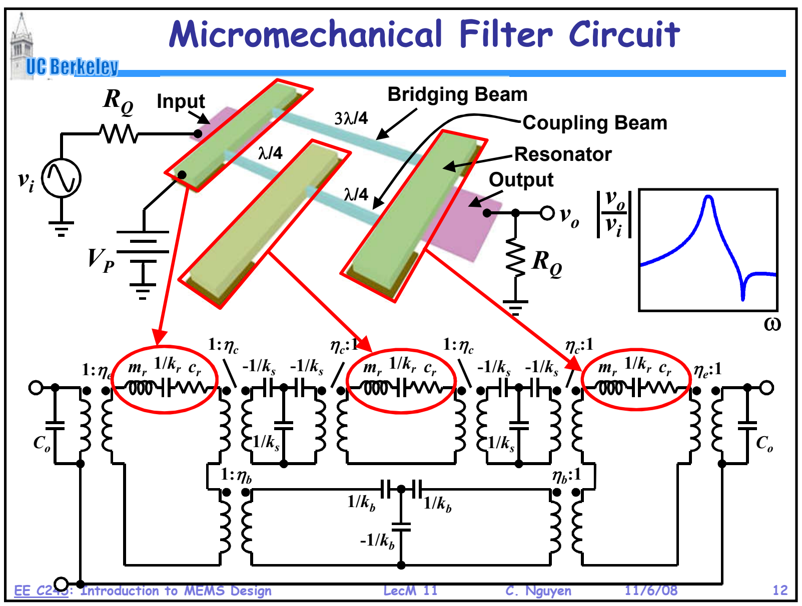
11


Micromechanical Filter Circuit
UC Berkeley
R_Q
Input
Bridging Beam
3λ/4
Coupling Beam
λ/4
Resonator
Output
v_i
V_P
v_o
$\left|\frac{v_o}{v_i}\right|$
ω
m_r $1/k_r$ c_r
$-1/k_s$
$1/k_s$
$1:\eta_e$
$1:\eta_c$
$\eta_c:1$
$\eta_e:1$
$1:\eta_b$
$\eta_b:1$
$1/k_b$
$-1/k_b$
C_o
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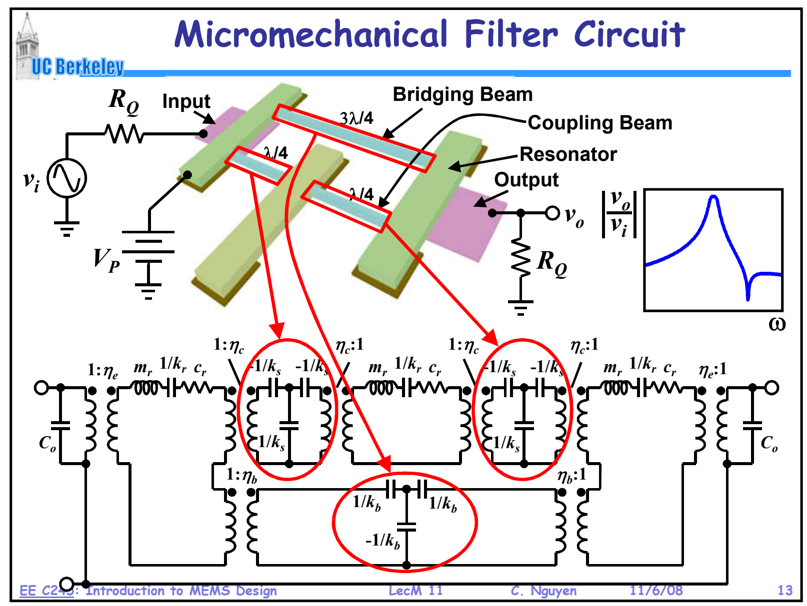
12


Micromechanical Filter Circuit
UC Berkeley
R_Q
Input
Bridging Beam
3λ/4
Coupling Beam
λ/4
Resonator
Output
v_i
V_P
v_o
$\left|\frac{v_o}{v_i}\right|$
ω
m_r $1/k_r$ c_r
$-1/k_s$
$1/k_s$
$1:\eta_e$
$1:\eta_c$
$\eta_c:1$
$\eta_e:1$
$1:\eta_b$
$\eta_b:1$
$1/k_b$
$-1/k_b$
C_o
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13

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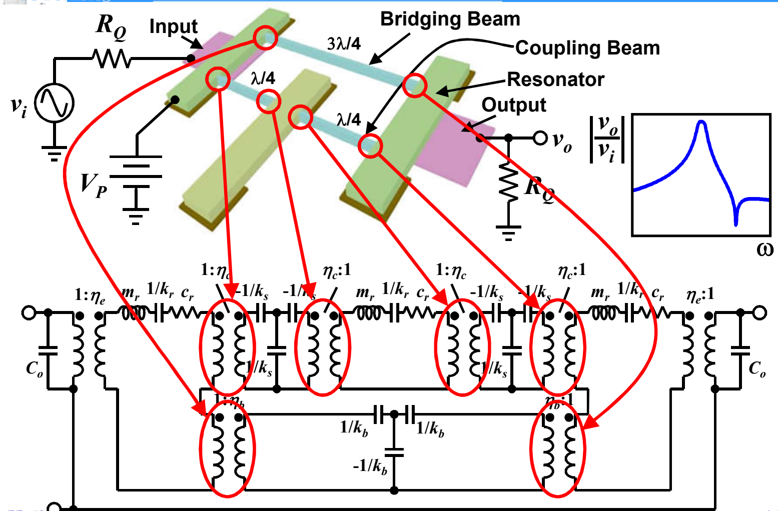
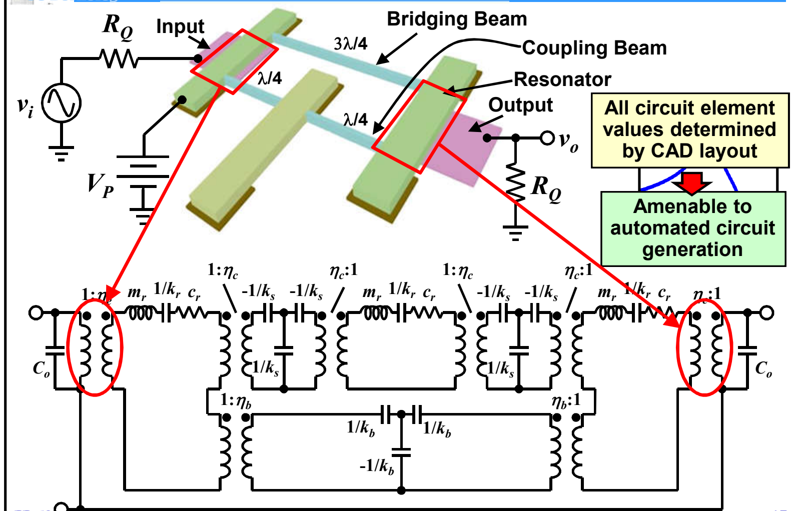
Micromechanical Filter Circuit



14

UC Berkeley

Micromechanical Filter Circuit



15

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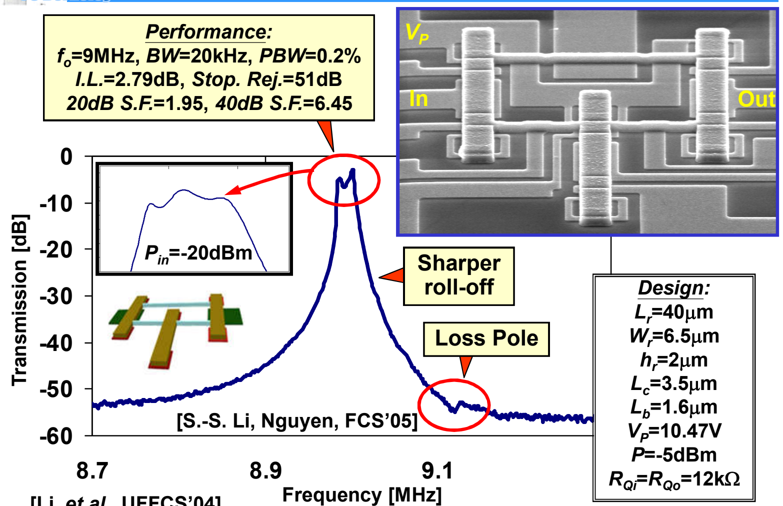
3CC 3λ/4 Bridged μMechanical Filter



16

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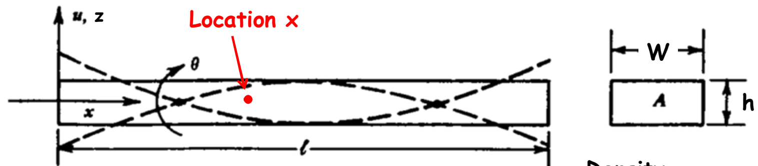
Beam Resonator Equivalent Circuits (Pretty Much the Same Stuff)

17

Equivalent Dynamic Mass

UC Berkeley

- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location x using knowledge of kinetic energy and velocity



Maximum Kinetic Energy → Density

$$\text{Equivalent Mass} = M_{eq\,x} = \frac{K.E.}{\frac{1}{2}V_x^2} = \frac{\frac{1}{2}\rho A\int_0^l V^2(x)\,dx}{\frac{1}{2}V_x^2}$$

Maximum Velocity @ location x — Maximum Velocity Function

18

Equivalent Dynamic Mass

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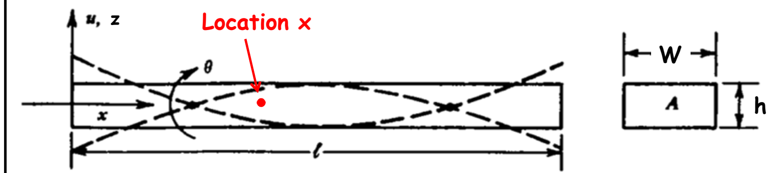

- We know the mode shape, so we can write expressions for displacement and velocity at resonance

Displacement: $u(x) = B\left[\varsigma(\cosh kx + \cos kx) + (\sinh kx + \sin kx)\right]$, $\varsigma = \frac{A}{B}$

$[V(x) = \omega_0 u(x)] \Rightarrow M_{eq}(x) = \frac{KE_{max}}{\frac{1}{2}[V(x)]^2} = \frac{\frac{1}{2}\rho A\int_0^l \omega_0^2[u(x')]^2dx'}{\frac{1}{2}\omega_0^2[u(x)]^2}$

$$M_{eq}(x) = \frac{\rho A\int_0^l B^2[\varsigma(\cosh kx' + \cos kx') + (\sinh kx' + \sin kx')]^2dx'}{B^2[\varsigma(\cosh kx + \cos kx) + (\sinh kx + \sin kx)]^2}$$

19

Equivalent Dynamic Stiffness & Damping

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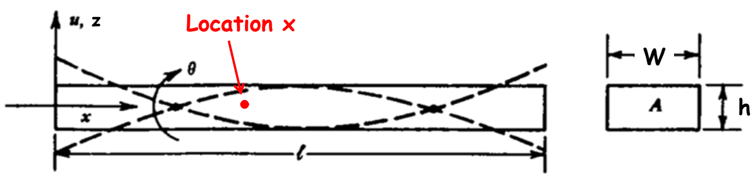

- Stiffness then follows directly from knowledge of mass and resonance frequency

$$\omega_0 = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \rightarrow K_{eq}(x) = \omega_0^2 M_{eq}(x)$$

- And damping also follows readily

$$Q = \frac{\omega_0 M_{eq}(x)}{C_{eq}(x)} \rightarrow C_{eq}(x) = \frac{\omega_0 M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x)M_{eq}(x)}}{Q}$$

($C_{eq}(x)$ ↳ damping)

20

Equivalent Lumped Mechanical Circuit

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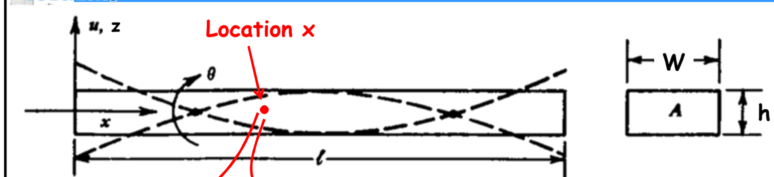
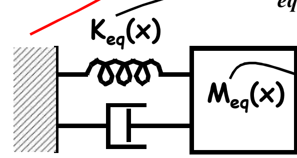


$$K_{eq}(x) = \omega_o^2 M_{eq}(x)$$



$$M_{eq}(x) = \frac{\rho A\int_o^l [u(x')]^2dx'}{[u(x)]^2}$$

$$C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q}$$

21

Equivalent Lumped Mechanical Circuit