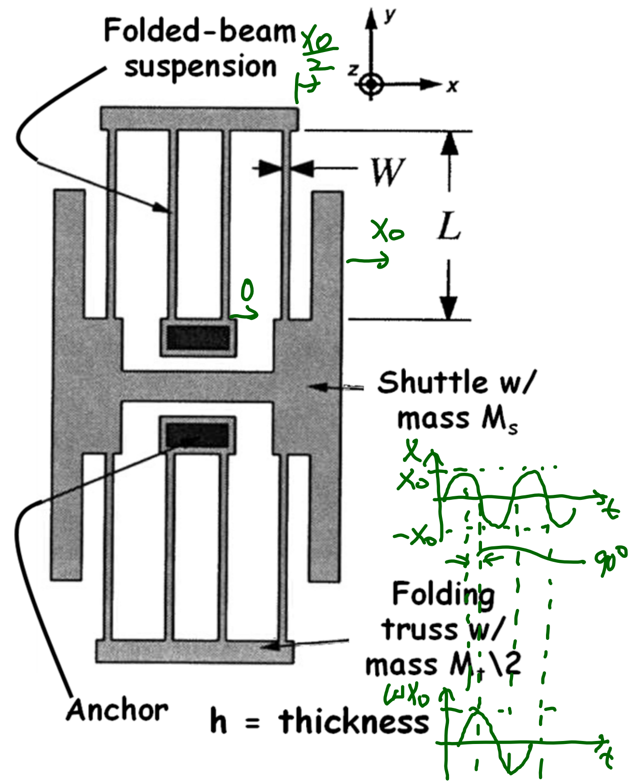
Lecture 17: Equivalent Circuits I

- Announcements:
- HW#5 online soon; due Tuesday, 4/14
- Module 11 on Equivalent Circuits I online
- Module 12 on Capacitive Transducers online
- Will discuss graded Midterm Exam w/ solutions
  - You can see your graded exam on Gradescope
  - I will email your Z-score
- Project Definition online (and will discuss today)
- ----------------------------------
- Reading: Senturia, Chpt. 5
- Lecture Topics:
  - Lumped Mechanical Equivalent Circuits
  - Electromechanical Analogies
- ----------------------------------
- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
- ----------------------------------
- Last Time:
- Determined resonance frequency of a distributed micromechanical structure

1

Resonance Freq. of a Folded Beam Structure



- Derive an expression for the resonance frequency of the above structure

Approximation)

$\Rightarrow m =$ shuttle mass
$\Rightarrow k = k_c$

$\omega_o = \sqrt{\frac{k_c}{m}}$

But not accurate enough for some applications.

$\Rightarrow$ for better accuracy, must integrate

2

... and after integrating over beam velocity...

and

$$K_{max} = X_o^2 \omega_o^2 \left[\frac{1}{2} M_s + \frac{1}{8} M_t + \frac{6}{35} M_b \right]$$

← for the total mechanical ckt.

both trusses (M_t) — all beams (M_b)

W_{max} → max. potential energy → equal to the work done to achieve maximum deflection

$$W_{max} = \frac{1}{2} k_x X_o^2$$

Then, using Rayleigh-Ritz:

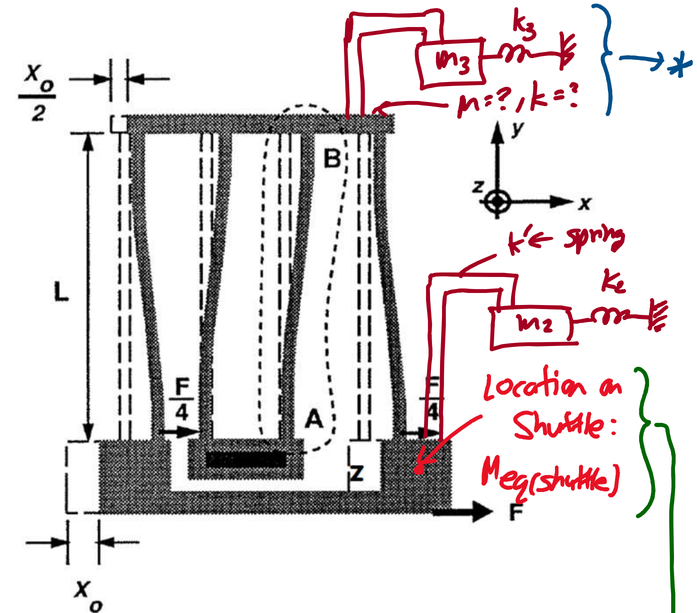
$$K_{max} = W_{max}$$

$$\cancel{X_o^2} \omega_o^2 \left[\frac{1}{2} M_s + \frac{1}{8} M_t + \frac{6}{35} M_b \right] = \frac{1}{2} \cancel{k_x}^{\,k_c} \cancel{X_o^2}$$

$$\boxed{\omega_o = \left[\frac{k_c}{M_{eq}} \right]^{1/2} \quad \text{where } M_{eq} = M_s + \frac{1}{4} M_t + \frac{12}{35} M_b}$$

(Resonance Freq. of a Folded-Beam Suspended Shuttle)

Equivalent Dynamic Mass



Location on Shuttle: $M_{eq(shuttle)}$

Equivalent Mass:

$$\text{Equiv. Mass} = M_{eq,x} = \frac{K_{max}}{\frac{1}{2} V_x^2} = \frac{\frac{1}{2} \rho A \int_0^l V^2(x)\,dx}{\frac{1}{2} V_x^2}$$

↑ velocity @ location x

$$M_{eq(shuttle)} : \frac{K_{max}}{\frac{1}{2} V_{shuttle}^2} = \frac{\omega_o^2 \cancel{X_o^2} \cancel{\left(\frac{1}{2}\right)} \left(M_s + \frac{1}{4} M_t + \frac{12}{35} M_b \right)}{\cancel{\frac{1}{2} \omega_o^2 X_o^2}}$$

$$\boxed{M_{eq(shuttle)} = M_s + \frac{1}{4} M_t + \frac{12}{35} M_b}$$

ρ(Volume)
"
static masses

*—

$$M_{eq}(truss) = \frac{K_{max}}{\frac{1}{2}V_{truss}^2} = \frac{\cancel{\omega_0^2 X_0^2 (\frac{1}{2})}\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]}{\cancel{\frac{1}{2}} \underbrace{\cancel{\omega_0^2 X_0^2}\left(\frac{1}{4}\right)}_{V_{truss}^2}}$$

$$M_{eq}(truss) = 4\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]$$

Equiv. Dynamic Mass @ the Truss location
↳ 4x that of the shuttle!

Equiv. Dynamic Stiffness

$$\omega_0 = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \rightarrow \boxed{K_{eq}(x) = \omega_0^2 M_{eq}(x)}$$

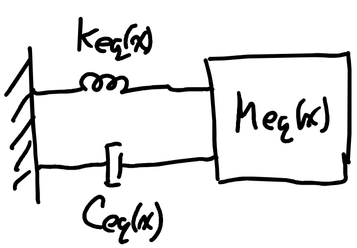
⇒ large equiv. mass & large equiv. stiffness go hand-in-hand

Equiv. Dynamic Damping

$$Q = \frac{\omega_0 M_{eq}(x)}{c_{eq}(x)} \quad {\sim}L,\ {\sim}R$$

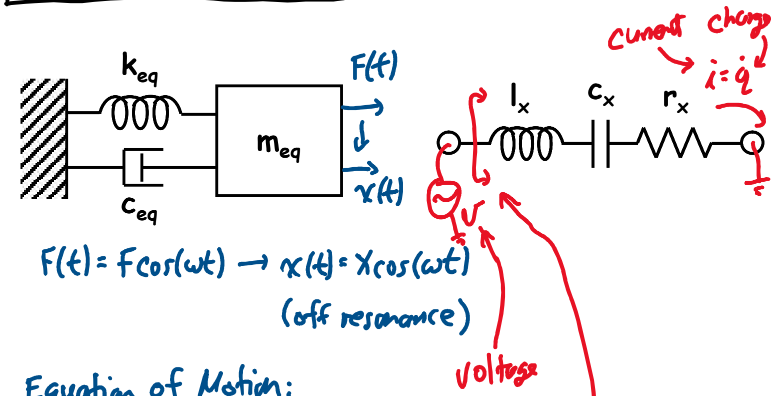
damping

$$\rightarrow c_{eq}(x) = \frac{\omega_0 M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x) M_{eq}(x)}}{Q}$$



} Lumped Parameter Mechanical Equiv. Ckt. @ location x

Electromechanical Analogies



$F(t) = F\cos(\omega t) \rightarrow x(t) = X\cos(\omega t)$
(off resonance)

Equation of Motion:

$$m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x = F(t)$$

⇒ using phasor concept:

$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

Impedance Looking into

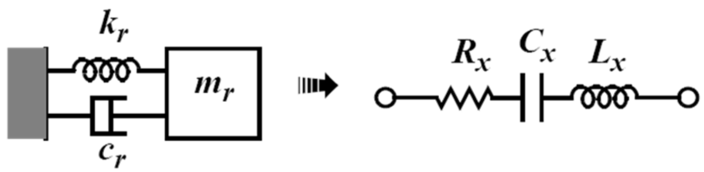
$$\frac{v}{i} = j\omega l_x + \frac{1}{j\omega c_x} + r_x$$

$$v = j\omega l_x i + \frac{(1/c_x)}{j\omega} i + r_x i$$

⇒ by analogy:

$$\boxed{\begin{array}{lll} F \rightarrow v & m_{eq} \rightarrow l_x & c_{eq} \rightarrow r_x \\ \dot{x} = i = \dot{q} & k_{eq} \rightarrow \frac{1}{c_x} & \end{array}}$$

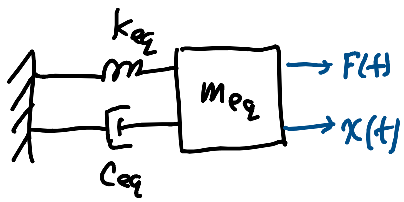
[Parameter Relationships in the Current Analogy]



- Mechanical-to-electrical correspondence in the current analogy:

Mechanical Variable	Electrical Variable
Damping, c	Resistance, R
Stiffness^{-1}, k^{-1}	Capacitance, C
Mass, m	Inductance, L
Force, f	Voltage, V
Velocity, v	Current, I

$\boxed{\text{Lowpass Biquad Transfer Function}}$



$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

⇒ convert to full phasor form:

$$F = (j\omega)(j\omega X)m_{eq} + \frac{k_{eq}}{\cancel{j\omega}}(\cancel{j\omega}X) + c_{eq}(j\omega X)$$

$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\omega^2\frac{m_{eq}}{k_{eq}} + 1 + j\frac{c_{eq}}{k_{eq}}\right]^{-1} \text{ — } *$$

$$\left[\frac{k_{eq}}{m_{eq}} = \omega_0^2,\quad Q = \frac{m_{eq}\omega_0}{c_{eq}} = \frac{k_{eq}}{\omega_0 c_{eq}} \rightarrow \frac{k_{eq}}{c_{eq}} = Q\omega_0\right]$$

$$* \quad \frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\left(\frac{\omega}{\omega_0}\right)^2 + 1 + j\frac{\omega}{Q\omega_0}\right]$$

$$\frac{X}{F}(j\omega) = \frac{k_{eq}^{-1}}{1-\left(\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{Q\omega_0}}$$