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EE C247B - ME C218 Introduction to MEMS Design Spring 2020

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Lecture Module 13: Equivalent Circuits II

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Lecture Outline

- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
 - ↪ Input Modeling
 - Force-to-Velocity Equiv. Ckt.
 - Input Equivalent Ckt.
 - ↪ Current Modeling
 - Output Current Into Ground
 - Input Current
 - Complete Electrical-Port Equiv. Ckt.
 - ↪ Impedance & Transfer Functions

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Input Modeling

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Electromechanical Analogies

$F(t) = F \cos(\omega t) \rightarrow x(t) = X \cos \omega t$
 Equation of Motion:
 $m_{eq} \ddot{x} + c_{eq} \dot{x} + k_{eq} x = F(t)$
 \Rightarrow using phasor concepts:
 $F = j\omega m_{eq} \dot{x} + \frac{k_{eq}}{j\omega} x + c_{eq} \dot{x}$
 \Rightarrow by analogy:

$F \rightarrow N$	$m_{eq} \rightarrow l_x$	$c_{eq} \rightarrow r_x$
$\dot{x} \rightarrow i$	$k_{eq} \rightarrow \frac{1}{c_x}$	

[Parameter Relationships in the Current Analogy]

$N(t) = V \cos \omega t \rightarrow i(t) = I \cos \omega t$
 Impedance looking in:
 $\frac{N}{i} = j\omega l_x + \frac{1}{j\omega c_x} + r_x$
 $N = j\omega l_x i + \frac{(Vc_x)}{j\omega} i + r_x i$

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Bandpass Biquad Transfer Function

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$F = j\omega m_{eq} \dot{x} + \frac{k_{eq}}{j\omega} x + C_{eq} \dot{x}$

\Rightarrow Converting to full phasor form:

$F = (j\omega)(j\omega x) m_{eq} + \frac{k_{eq}}{j\omega} (j\omega x) + C_{eq} (j\omega x)$

$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}} \left[-\omega^2 \frac{m_{eq}}{k_{eq}} + 1 + j \frac{C_{eq}\omega}{k_{eq}} \right]^{-1} = \frac{1}{k_{eq}} \left[-\left(\frac{\omega}{\omega_0}\right)^2 + 1 + j \frac{\omega}{Q\omega_0} \right]^{-1}$

$\left[\frac{k_{eq}}{m_{eq}} = \omega_0^2, Q = \frac{m_{eq}\omega_0}{C_{eq}} = \frac{k_{eq}}{\omega_0 C_{eq}} \rightarrow \frac{k_{eq}}{C_{eq}} = Q\omega_0 \right]$

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Force-to-Velocity Relationship

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- The relationship between input voltage v_1 and force F_{d1} :

$$F_{d1} \approx -V_P \frac{\partial C_1}{\partial x} v_1$$
- When displacement x is the mechanical output variable:

$$\frac{X(s)}{F_{d1}(s)} = \frac{1}{k} \frac{s^2 \omega_o^2}{s^2 + (\omega_o/Q)s + \omega_o^2}$$
- When velocity v is the mechanical output variable:

$$\frac{v(s)}{F_{d1}(s)} = \frac{sX(s)}{F_{d1}(s)} = \frac{1}{k} \frac{\omega_o^2 s}{s^2 + (\omega_o/Q)s + \omega_o^2}$$

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Force-to-Velocity Equiv. Ckt.

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- Combine the previous lumped LCR mechanical equivalent circuit with a circuit modeling the capacitive transducer \rightarrow circuit model for voltage-to-velocity

Linear Two-Port Element

Electrical: Current I_1 , Voltage V_1

Mechanical: Velocity $U = -\dot{x}$, Force F_{d1}

$I_x = m$, $r_x = b$, $c_x = 1/k$

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Equiv. Circuit for a Linear Transducer

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- A transducer ...
 - converts energy from one domain (e.g., electrical) to another (e.g., mechanical)
 - has at least two ports
 - is not generally linear, but is virtually linear when operated with small signals (i.e., small displacements)

Linear Two-Port Element

Electrical: Current I , Voltage V

Mechanical: Velocity $U = -\dot{x}$, Force F

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Equiv. Circuit for a Linear Transducer

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Current $\rightarrow I$
 Voltage $\rightarrow V$
 Linear Two-Port Element
 $U = -\dot{x}$ Velocity
 Force $\leftarrow F$
 Electrical | Mechanical

- For physical consistency, use a transformer equivalent circuit to model the energy conversion from the electrical domain to mechanical domain

Flow $\rightarrow f_1$
 Effort $\rightarrow e_1$
 $1:\eta$
 f_2
 e_2
 Describing Matrix

$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

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