Lecture 20: Comb Drive

- Announcements:
- Project Slide Set #1 due Friday, April 10
- HW#5 online and due Tuesday, 4/14, at 8 a.m.
- Module 13 on "Equivalent Circuits II" online
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- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis
- --
- Reading: Senturia, Chpt. 6, Chpt. 14
- Lecture Topics:
  - Input Modeling
    - Force-to-Velocity Equiv. Ckt.
    - Input Equivalent Ckt.
  - Current Modeling
    - Output Current Into Ground
    - Input Current
    - Complete Electrical-Port Equiv. Ckt.
  - Impedance & Transfer Functions
- --
- Last Time: Electrical stiffness
- Now, electrostatic comb-drive

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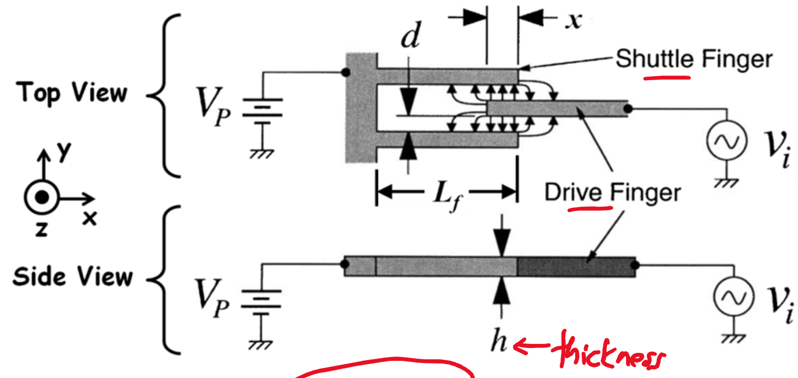
Electrostatic Comb Drive



$$F_d = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{\partial C}{\partial x}(V_P - v_i)^2$$ → Need $C(x)$.

$$\left[C(x) = \frac{2\epsilon_o x h}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2\epsilon_o h}{d}\right]$$ → not a fcn of x! (ideally)

$$F_d = \frac{1}{2}\frac{2\epsilon_o h}{d}\left(\cancel{V_P^2} - 2V_P v_i + \cancel{v_i^2}\right)$$

can balance out via symmetrically placed electrodes

$v_i \ll V_P$

$$\boxed{F_d = -2V_P\frac{\epsilon_o h}{d}v_i}$$

∴ no electrical stiffness! (no k_e!)

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- Go through remaining comb-drive slides in Module 12
- Start Module 13: go thru first few slides up to transformer definition
- Then …

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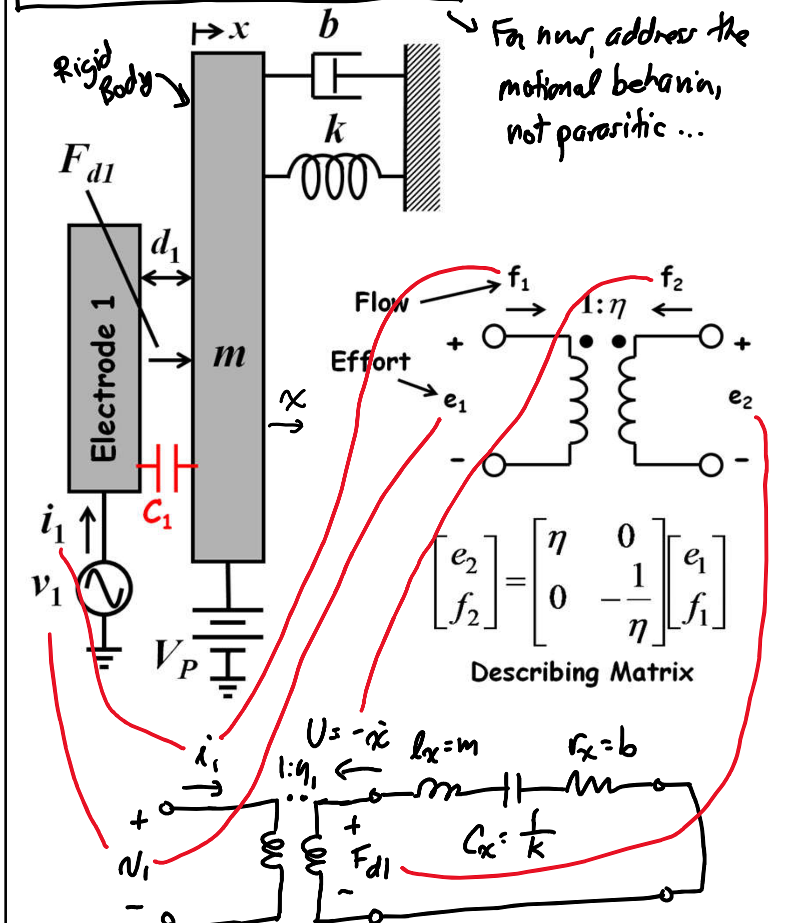
Input Electrical Equivalent Ckt.



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} \begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

$$\underbrace{F_{d1}}_{e_2} \cong \underbrace{-V_P \frac{\partial C_1}{\partial x}}_{\eta_1} \underbrace{v_1}_{e_1} \rightarrow \boxed{\eta_1 = \left| V_P \frac{\partial C_1}{\partial x} \right|}$$

$\frac{\epsilon_0 A}{d^2}$ → gap ↓ → η_1 ↑↑

V_P ↑ → η_1 ↑

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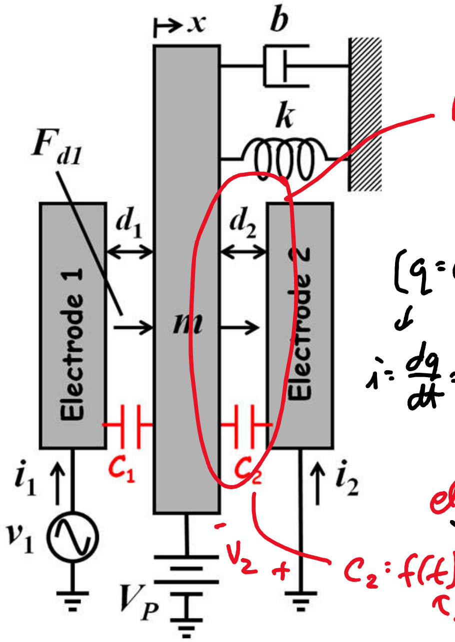
Output Current Into Ground



Want this ckt. model.

$C_2 = f(t)$ ← time

$[q = CV]$

$$i = \frac{dq}{dt} = \underbrace{C\frac{dV}{dt}}_{\text{electronics}} + \underbrace{V\frac{dC}{dt}}_{\text{MEMS}}$$

$$i_2 = C_2(x,t)\cancel{\frac{dV_2(t)}{dt}}^{\,0} + V_2(t)\frac{dC_2(x,t)}{dt}$$

$[V_2(t) = -V_P] \Rightarrow \quad i_2 = -V_P\frac{dC_2}{dt} = -V_P\frac{\partial C_2}{\partial x}\frac{\partial x}{\partial t}$

In phasor form: $\quad I_2(j\omega) = -V_P\frac{\partial C_2}{\partial x}(j\omega X)$

$$\boxed{I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X}$$

↑ motional current

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$I_2(j\omega) = -j\omega V_P\frac{\partial C_2}{\partial x}X = -V_P\frac{\partial C_2}{\partial x}\dot{x}$ ← velocity

90° phase lag (+) (+) → $I_2 = (-)$ when $x = (+)$ ✓

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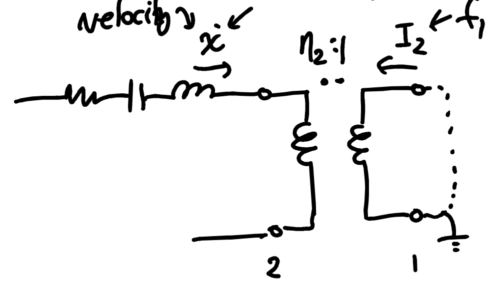
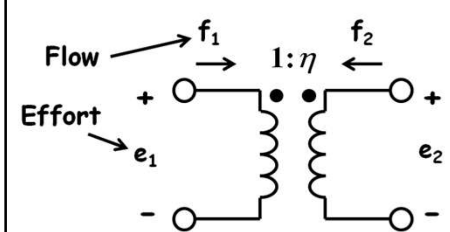


$$f_2 = -\frac{1}{\eta_2}f_1 \rightarrow f_1 = -\eta_2 f_2$$

$[f_1 = I_2,\ f_2 = \dot{x}] \Rightarrow \quad I_2 = -\eta_2\dot{x}$

$$\therefore \boxed{\eta_2 = \left|V_P\frac{\partial C_2}{\partial x}\right|}$$



$$\begin{bmatrix} e_2 \\ f_2 \end{bmatrix} = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix}\begin{bmatrix} e_1 \\ f_1 \end{bmatrix}$$

Describing Matrix

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