EE C247B – ME C218
Introduction to MEMS Design
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Prof. Clark T.-C. Nguyen

Dept. of Electrical Engineering & Computer Sciences
University of California at Berkeley
Berkeley, CA 94720

Module 17: Noise & MDS

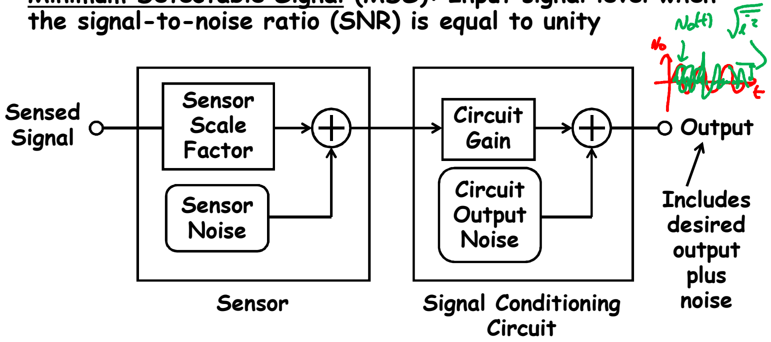
Lecture Outline

- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination

Determining Sensor Resolution

Minimum Detectable Signal (MDS)

- **Minimum Detectable Signal** (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity



- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

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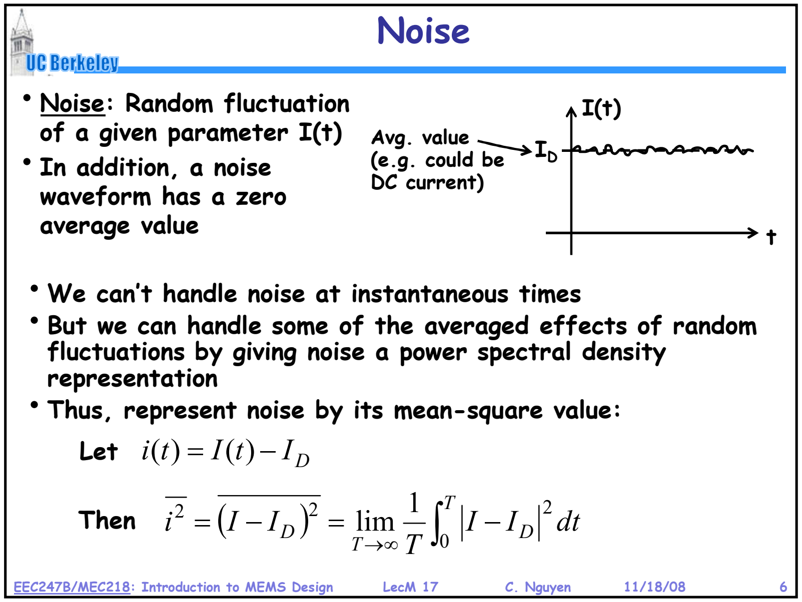
Noise



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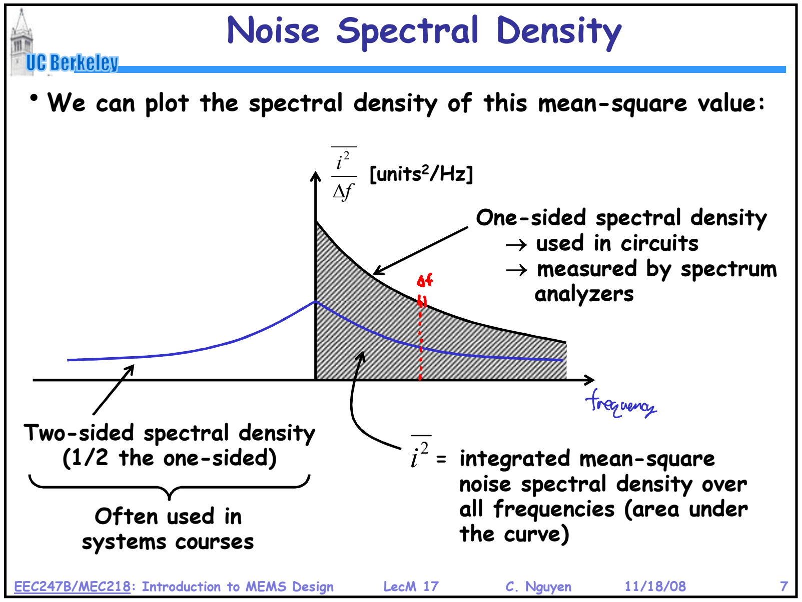
Noise

- Noise: Random fluctuation of a given parameter I(t)
- In addition, a noise waveform has a zero average value

- We can't handle noise at instantaneous times
- But we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation
- Thus, represent noise by its mean-square value:

Let $i(t) = I(t) - I_D$

Then $\overline{i^2} = \overline{(I - I_D)^2} = \lim_{T\to\infty} \frac{1}{T}\int_0^T |I - I_D|^2\, dt$



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Noise Spectral Density

- We can plot the spectral density of this mean-square value:

One-sided spectral density
→ used in circuits
→ measured by spectrum analyzers

Two-sided spectral density (1/2 the one-sided)

$\underbrace{}$

Often used in systems courses

$\overline{i^2}$ = integrated mean-square noise spectral density over all frequencies (area under the curve)

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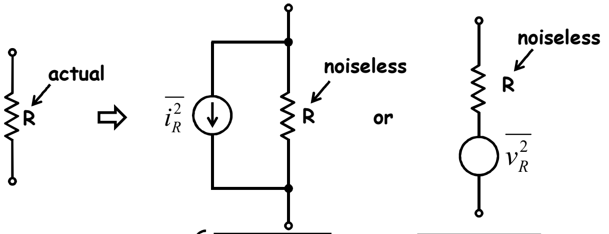
Noise Sources

Thermal Noise

- Thermal Noise in Electronics: (Johnson noise, Nyquist noise)
  - ↳ Produced as a result of the thermally excited random motion of free e⁻'s in a conducting medium
  - ↳ Path of e⁻'s randomly oriented due to collisions
- Thermal Noise in Mechanics: (Brownian motion noise)
  - ↳ Thermal noise is associated with all dissipative processes that couple to the thermal domain
  - ↳ Any damping generates thermal noise, including gas damping, internal losses, etc.
- Properties:
  - ↳ Thermal noise is white (i.e., constant w/ frequency)
  - ↳ Proportional to temperature
  - ↳ Not associated with current
  - ↳ Present in any real physical resistor

Circuit Representation of Thermal Noise

- Thermal Noise can be shown to be represented by a series voltage generator $\overline{v_R^2}$ or a shunt current generator $\overline{i_R^2}$



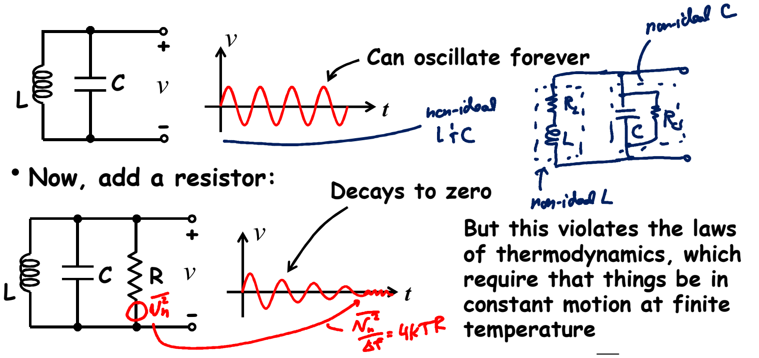
Note: These are one-sided mean-square spectral densities! To make them 2-sided, must divide by 2.

$$\boxed{\frac{\overline{i_R^2}}{\Delta f}=\frac{4kT}{R}} \qquad \boxed{\frac{\overline{v_R^2}}{\Delta f}=4kTR}$$

where $4kT = 1.66x10^{-20} V\cdot C$
and where these are spectral densities.

Noise in Capacitors and Inductors?

- Resistors generate thermal noise
- Capacitors and inductors are noiseless → why?



- Now, add a resistor:

But this violates the laws of thermodynamics, which require that things be in constant motion at finite temperature

Need to add a forcing function, like a noise voltage $\overline{v_R^2}$ to keep the motion going → and this noise source is associated with R

Why 4kTR?

- Why is $\overline{v_R^2}=4kTR\Delta f$ (a heuristic argument)
- The Equipartition Theorem of Statistical Thermodynamics says that there is a mean energy (1/2)kT associated w/ each degree of freedom in a given system
- An electronic circuit possesses two degrees of freedom:
  - ↳ Current, i, and voltage, v
  - ↳ Thus, we can write:

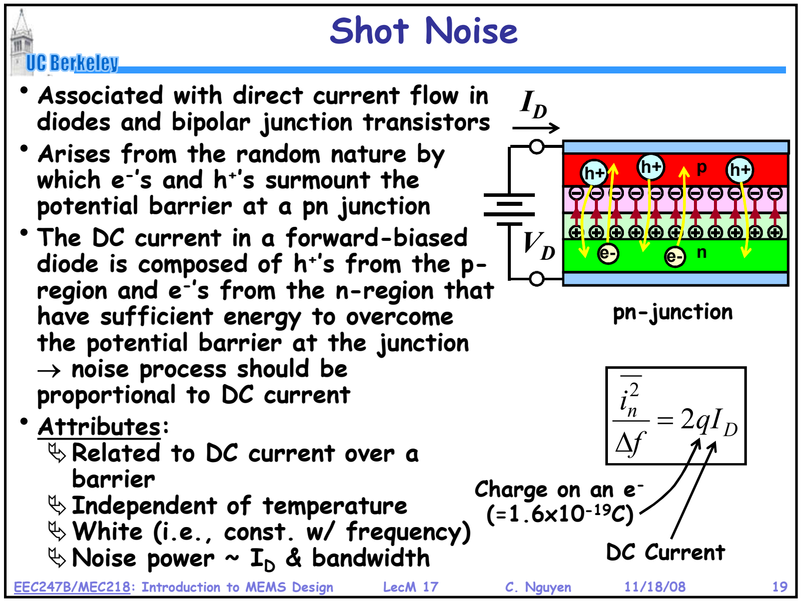
$$\frac{1}{2}L\overline{i^2}=\frac{1}{2}k_BT \quad , \quad \overbrace{\frac{1}{2}C\overline{v^2}}^{\text{Energy}}=\frac{1}{2}k_BT$$

- Similar expressions can be written for mechanical systems
  - ↳ For example: for displacement, x

Spring constant

$$\frac{1}{2}k\overline{x^2}=\frac{1}{2}k_BT$$

$\frac{1}{2}\left(\frac{1}{k}\right)\overline{F^2}$



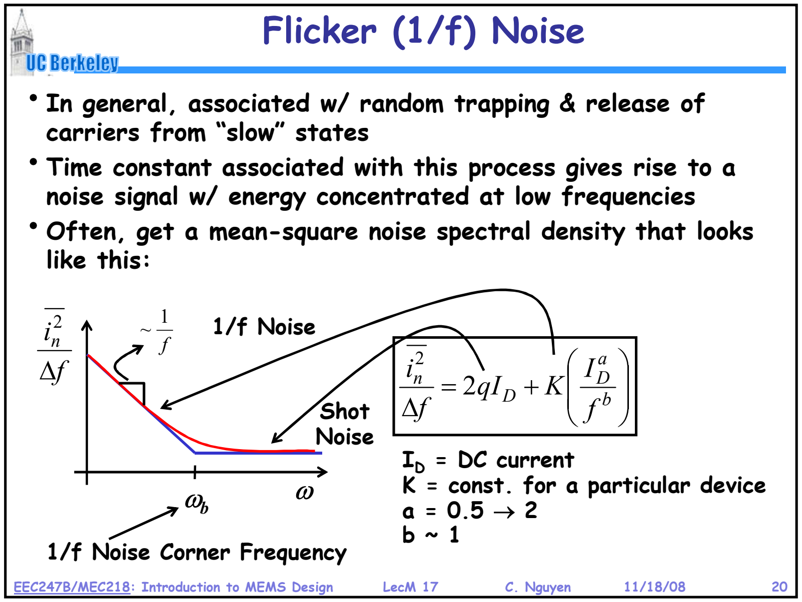
Shot Noise

- Associated with direct current flow in diodes and bipolar junction transistors
- Arises from the random nature by which e^-'s and h^+'s surmount the potential barrier at a pn junction
- The DC current in a forward-biased diode is composed of h^+'s from the p-region and e^-'s from the n-region that have sufficient energy to overcome the potential barrier at the junction → noise process should be proportional to DC current
- **Attributes**:
  - Related to DC current over a barrier
  - Independent of temperature
  - White (i.e., const. w/ frequency)
  - Noise power ~ I_D & bandwidth

$$\boxed{\frac{\overline{i_n^2}}{\Delta f} = 2qI_D}$$

Charge on an e^- (=1.6×10^{-19}C)

DC Current



Flicker (1/f) Noise

- In general, associated w/ random trapping & release of carriers from "slow" states
- Time constant associated with this process gives rise to a noise signal w/ energy concentrated at low frequencies
- Often, get a mean-square noise spectral density that looks like this:

$$\boxed{\frac{\overline{i_n^2}}{\Delta f} = 2qI_D + K\left(\frac{I_D^a}{f^b}\right)}$$

I_D = DC current
K = const. for a particular device
a = 0.5 → 2
b ~ 1