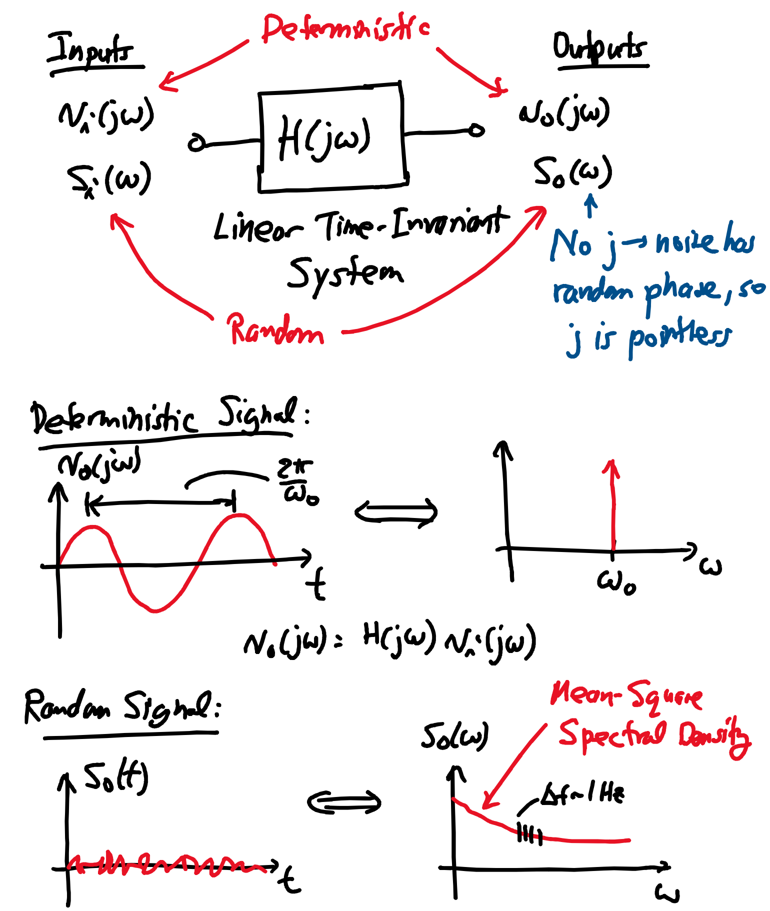
Lecture 24: Noise I

- **Announcements:**
- Project Slide Set #3 due Friday, April 24
- HW#6 online and due Friday, 5/1, at 8 a.m.
- Module 17 is online (on Noise and MDS)
- ------------------------------------
- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Referred Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination
- ------------------------------------
- **Last Time:**
- Finished MEMS/transistor integration
- Now move on to noise, beginning in Module 17 …

1

Circuit Noise Calculations



2

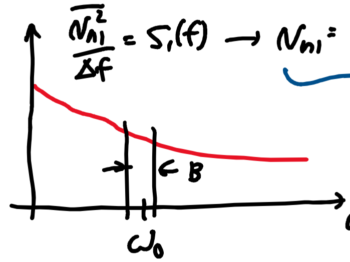
$$S_o(\omega) = \left[H(j\omega)H^*(j\omega)\right] S_i(\omega) = |H(j\omega)|^2 S_i(\omega)$$

$$\underbrace{\sqrt{S_o(\omega)}}_{} = |H(j\omega)| \underbrace{\sqrt{S_i(\omega)}}_{}$$ → How is it we can do this?

root mean-square values

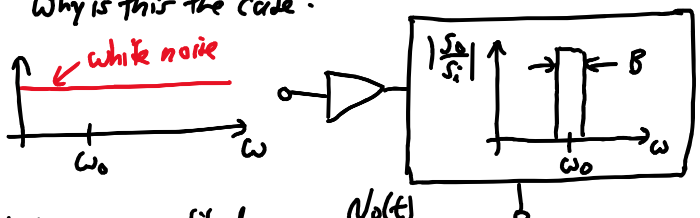
Handling Noise Quasi-Deterministically

$$\frac{\overline{v_{n1}^2}}{\Delta f} = S_i(f) \rightarrow v_{n1} = \sqrt{S_i(f)B}$$ ← bandwidth

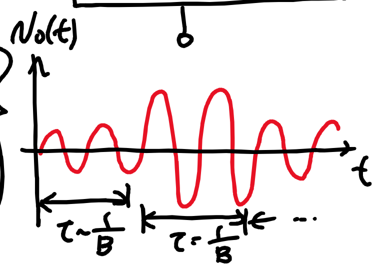


Can approximate this by a sinusoidal voltage generator (esp. when B is small, e.g., 1Hz)

Why is this the case?

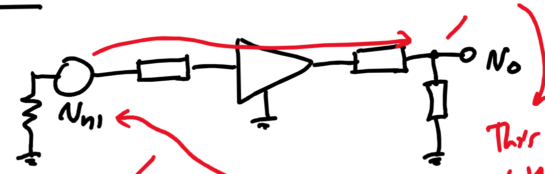


Neither the amplitude nor the phase of a signal can change appreciably within a time period $\frac{1}{B}$!



Noise Source Correlation

Case ①: Single Noise Source

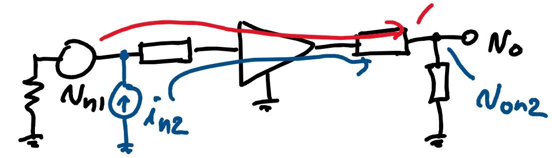


This is correlated w/ this, since it derives from it:

Thus, can write:

$$v_{on1} = H(j\omega)v_{n1}$$

Case ②: Multiple Noise Sources



Can write: $v_{on1} = H_1(j\omega)v_{n1}$

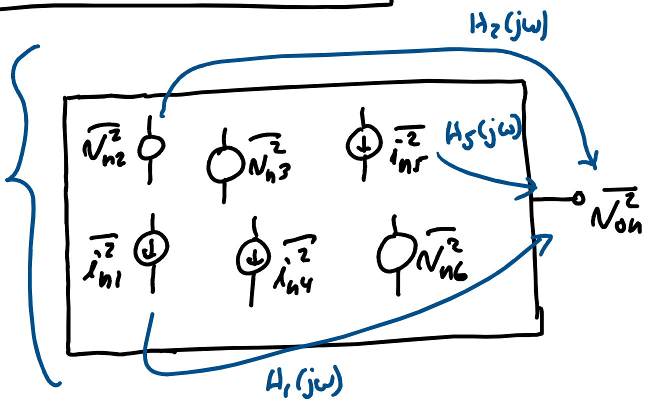
$v_{on2} = H_2(j\omega)i_{n2}$

But $v_{oT} \neq v_{on1} + v_{on2}$ → since these are not correlated

Rather: $\overline{v_{oT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2}$ (must add powers!)

Systematic Noise Calculation Procedure

General Ckt. w/ Several Noise Sources



Assume noise sources are uncorrelated.

1. For $\overline{i_{n1}^2}$, replace w/ a source i_{n1}
2. Calculate $v_{on1}(\omega) = H_1(j\omega)\, i_{n1}(\omega)$ (treating it like a deterministic signal)
3. Determine $\overline{v_{on1}^2} = |H_1(j\omega)|^2 \cdot \overline{i_{n1}^2}$
4. Repeat for each noise source: get $\overline{v_{on2}^2}, \overline{v_{on3}^2}, \overline{v_{on4}^2}, \ldots$ $\xrightarrow{}$ output ($\overline{i_{n4}^2} \xrightarrow{H_4(j\omega)} \overline{v_{on4}^2}$)
5. Add noise powers (mean-square values):

$$\overline{v_{onTOT}^2} = \overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots$$

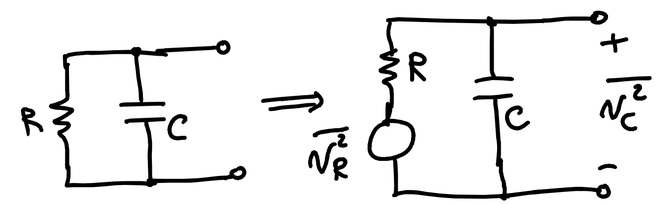
$$v_{onTOT} = \sqrt{\overline{v_{on1}^2} + \overline{v_{on2}^2} + \overline{v_{on3}^2} + \overline{v_{on4}^2} + \cdots}$$

total rms value

- Go through Module 17, slides 12-16

$\boxed{\text{Why } \frac{\overline{v_{nR}^2}}{\Delta f} = 4kTR?}$ (a heuristic argument)

Consider an RC ckt:



$$E = \frac{1}{2}kT = \frac{1}{2}C\overline{v_C^2}$$

$$\therefore \boxed{\overline{v_C^2} = \frac{kT}{C}}$$

integrated noise over all freqs. → total mean-square output voltage integrated over all freqs.

Question: What value of $\frac{\overline{v_R^2}}{\Delta f}$ gives us this (assuming white noise)

$$\overline{v_C^2} = \int_0^\infty \left| \frac{1}{1+j\omega RC} \right|^2 \frac{\overline{v_R^2}}{\Delta f}\, df$$

[noise is white] [$\omega_b = \frac{1}{RC}$] $\Rightarrow$

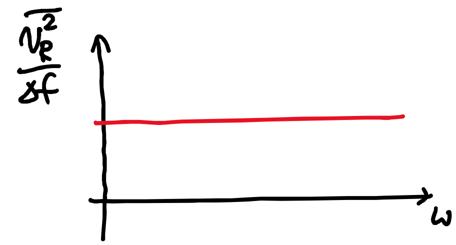
$$= \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f} \int_0^\infty \frac{\omega_b^2}{\omega_b^2+\omega^2}\, d\omega \quad *$$

$$*\left[\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]$$

$$* = \frac{1}{2\pi}\frac{\overline{v_R^2}}{\Delta f}\frac{\omega_b^{\cancel{2}}}{\cancel{\omega_b}}\tan^{-1}\left(\frac{\omega}{\omega_b}\right)\Big|_0^{\infty}$$

$$= \frac{1}{2\cancel{\pi}}\frac{\overline{v_R^2}}{\Delta f}\left(\frac{\cancel{\pi}}{2}\omega_b - 0\right) = \frac{1}{4}\omega_b\frac{\overline{v_R^2}}{\Delta f} = \frac{kT}{C}$$

$$\frac{\overline{v_R^2}}{\Delta f} = 4kT\underbrace{\left(\frac{1}{\omega_b C}\right)}_{R} \Rightarrow \boxed{\frac{\overline{v_R^2}}{\Delta f} = 4kTR} \checkmark$$



- Go through Module 17, slides 19-20

7