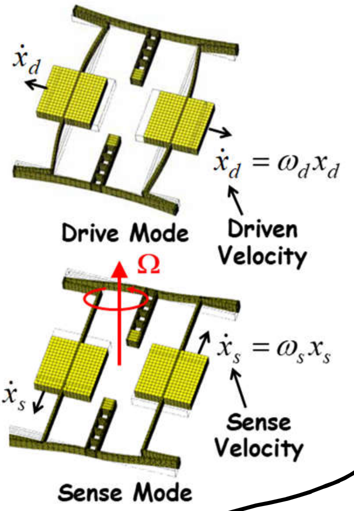
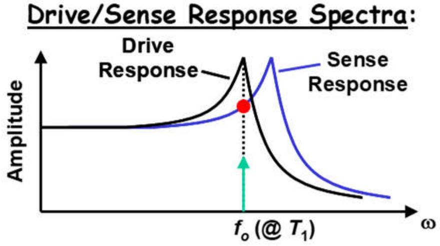
## Lecture 26: Gyro Minimum Detectable Signal

- Announcements:
- HW#6 online and due Friday, 5/1, at 8 a.m.
- Project Out-Briefs Sign-Up (google) sheet shared with you; out-briefs will be on Thursday & Friday of RRR week
- Final Exam info sheet today
    - Will also be online with some sample exams
- Course wrap up
- Course evaluations
- ----------------------------------------
- Reading: Senturia Chpt. 16
- Lecture Topics:
    - Minimum Detectable Signal
    - Noise
        - Circuit Noise Calculations
        - Noise Sources
        - Equivalent Input-Referred Noise
    - Gyro MDS
        - Example ARW Determination
- ----------------------------------------
- Last Time:
- Started example angle random walk (ARW) determination
- Now continue with this …

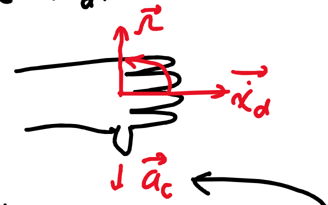
## Gyroscope Drive-to-Sense Xfer Fcn





Rotation-Induced Coriolis Force:

$\vec{a}_c = 2\vec{\dot{x}}_d \times \vec{\Omega}$



$\vec{a}_c = 2\omega_d \vec{x}_d \times \vec{\Omega} \rightarrow$ act in the sense mode direction

$a_s = a_c = 2\omega_d x_d \Omega \sin 90°$

$$\boxed{a_s = 2\omega_d x_d \Omega}$$

rotation rate (what we want to measure)

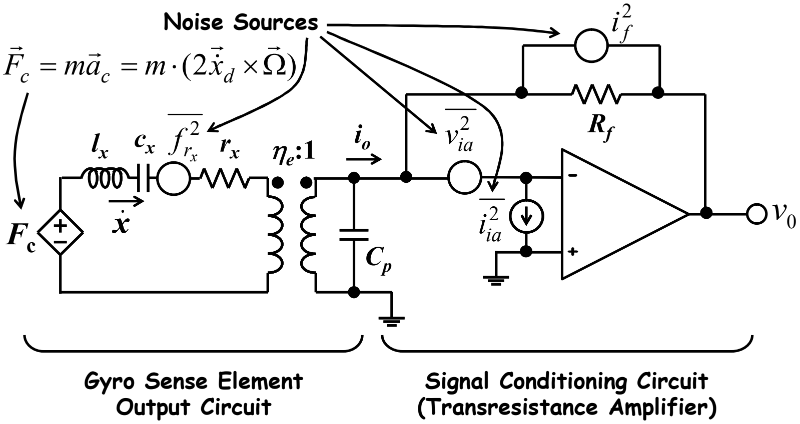
drive radian frequency

drive displacement amplitude

$$\boxed{F_c = m a_s = m a_c}$$

Coriolis Force

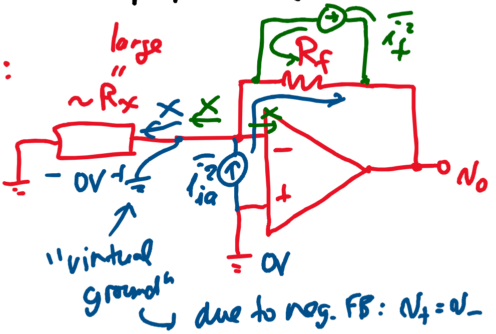
## Determine MDS



① Determine $\overline{v_{on}^2}$:

⇒ use superposition (handle one source at a time)

$\overline{i_{ia}^2}$:



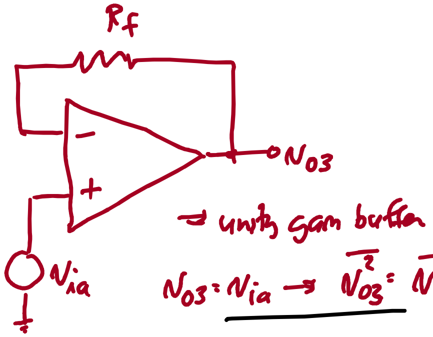
due to neg. FB: $v_+ = v_-$

$v_{o1} = i_{ia} R_f \rightarrow \overline{v_{o1}^2} = \overline{i_{ia}^2} R_f^2$

(ignore the "−" sign, since it doesn't matter for noise)

$\overline{i_f^2}$: $v_{o2} = i_f R_f \rightarrow \overline{v_{o2}^2} = \overline{i_f^2} R_f^2 = \frac{4kT}{R_f} R_f^2 = 4kTR_f \Delta f$

$\overline{v_{ia}^2}$:



$v_{o3} = v_{ia} \rightarrow \overline{v_{o3}^2} = \overline{v_{ia}^2}$

$\overline{f_{r_x}^2}$: $v_{o4} = \dot{x}\eta_e R_f = \frac{f_{r_x}}{r_x}\Theta(j\omega_d)\eta_e R_f$

$\left[\dot{x} = \frac{f_{r_x}}{r_x}\Theta(j\omega_d)\right]$

↑ drive freq.

$$\overline{v_{o4}^2} = \frac{\overline{f_{r_x}^2}}{r_x^2}|\Theta(j\omega_d)|^2 \eta_e^2 R_f^2$$

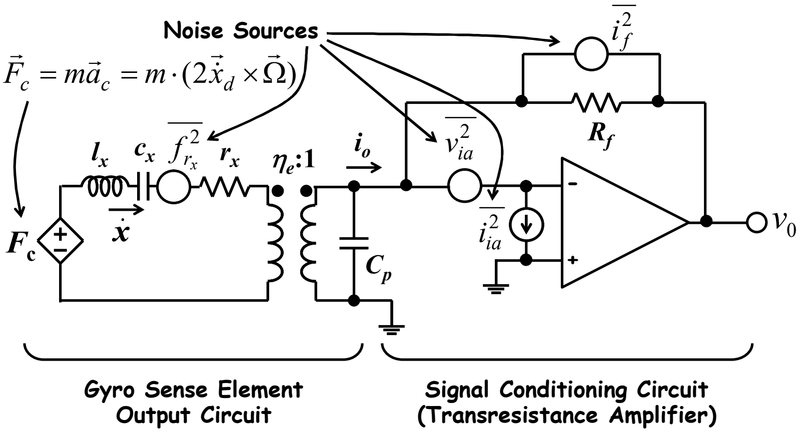
$\left[\overline{f_{r_x}^2} = 4kTr_x\Delta f\right] \Rightarrow \overline{v_{o4}^2} = 4kT|\Theta(j\omega_d)|^2\eta_e^2\frac{R_f^2}{r_x}$

$$\therefore \boxed{\overline{v_{on}^2} = \overline{i_{ia}^2}R_f^2 + \overline{v_{ia}^2} + 4kTR_f\left(1 + \eta_e^2|\Theta(j\omega_d)|^2\frac{R_f}{r_x}\right)\Delta f}$$

Get from the op amp data sheet.

Total Output Mean-Square Voltage Noise

② Get the scale factor, i.e., find $v_o$ in terms of rotation rate $\Omega$:



⇒ Find the rotation-to-$i_o$ transfer function:

$$\dot{x}_s = F_c\left(\frac{1}{r_x}\,Ⓗ_s(j\omega_d)\right) = \frac{\omega_s Q_s}{k_s}\cdot 2\omega_d x_d \Omega m \cdot Ⓗ_s(j\omega_d)$$

$\left[F_c = m a_c = 2\omega_d x_d \Omega m\right]$ $\left[\frac{1}{r_x} = \frac{\omega_s Q_s}{k_s}\right]$

$= \frac{1}{\omega_s^2}$

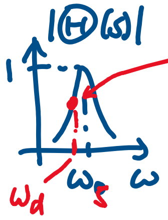
$$\dot{x}_s = 2\frac{\omega_d}{\omega_s} Q_s x_d\, Ⓗ_s(j\omega_d)\cdot\Omega$$

$$Ⓗ(s) = \frac{s(\omega_o/Q)}{s^2 + s(\omega_o/Q) + \omega_o^2}$$

$s=0$: $Ⓗ(0) = 0$

$s=j\omega_o$: $Ⓗ(j\omega_o) = 1$

$s=\infty$: $Ⓗ(\infty) = 0$



$$i_0 = \eta_e \dot{x}_s = 2\frac{\omega_d}{\omega_s} Q_s x_d \eta_e Ⓗ_s(j\omega_d)\cdot\Omega$$

rotation rate

$$\therefore |v_o| = i_o R_f = \underbrace{2R_f\frac{\omega_d}{\omega_s} Q_s x_d \eta_e Ⓗ_s(j\omega_d)}_{A \,\hat{=}\, \text{scale factor} \,\rightarrow\, |v_o| = A\Omega}\cdot\Omega$$

③ $\Omega = \Omega_{min}$ when $|v_o| = \sqrt{\overline{v_{on}^2}}$ ← rms noise voltage

↑ minimum detectable rotation rate (MDS)

$$2R_f\frac{\omega_d}{\omega_s} Q_s x_d \eta_e Ⓗ_s(j\omega_d)\cdot\Omega_{min} =$$

$$\sqrt{\overline{i_{ia}^2} R_f^2 + \overline{v_{ia}^2} + 4kTR_f\left(1 + \eta_e^2 |Ⓗ(j\omega_d)|^2 \frac{R_f}{r_x}\right)\Delta f}$$

Solve for $\Omega_{min}$:

$$\boxed{\Omega_{min} = \frac{\sqrt{\overline{i_{ia}^2} R_f^2 + \overline{v_{ia}^2} + 4kTR_f\left(1 + \eta_e^2 |Ⓗ(j\omega_d)|^2 \frac{R_f}{r_x}\right)\Delta f}}{2R_f\frac{\omega_d}{\omega_s} Q_s x_d \eta_e Ⓗ_s(j\omega_d)}}$$

Often most interested in spectral density:

$$\boxed{\sqrt{\frac{\overline{\Omega_{min}^2}}{\Delta f}} = \Omega_{min} \times \left(\frac{1}{\sqrt{\Delta f}}\right)\left(\frac{3600\,s}{hr}\right)\left(\frac{180^\circ}{\pi}\right) \rightarrow \left[(°/hr)/\sqrt{Hz}\right]}$$

↓

★

*Angle Random Walk (ARW) $= \frac{1}{60}\Omega_{min}$ [$°/\sqrt{hr}$]

Easier to determine directional error as a function of elapsed time.

- Go through Module 17, slides 36-49.