Lecture 2: Benefits of Scaling I

- Announcements:
- The notes from last time are online in the Lecture link table; video also already up
- Modules 1 & 2 are also online (also, in the Lecture link table)
- HW#1 online and due Feb. 11 at 8 a.m.
- Get your computer accounts by following the instructions at the end of the Course Info Sheet
- You all have received invites to join the class Piazza group
- -------------------------------------
- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - —GHz micromechanical resonators
    - —Chip-scale atomic clock
    - —Micro gas chromatograph
- -------------------------------------
- Last Time: Going through Module 1
- Finish Module 1, then start going through Module 2

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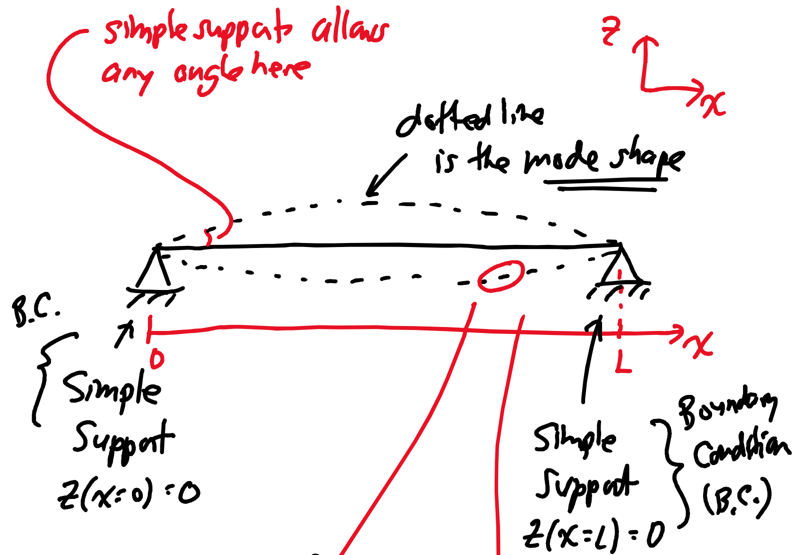
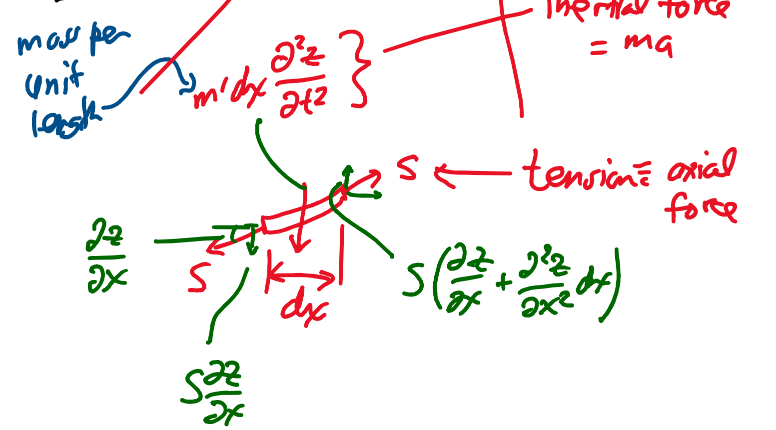
Scaling of Guitar Strings

guitar string ≡ transversely vibrating stretched wire



Find an equation for resonance frequency.

Free Body Diagram



2

⇒ condition for dynamic equilibrium:

$$S\left(\frac{\partial z}{\partial x} + \frac{\partial^2 z}{\partial x^2}dx\right) - S\frac{\partial z}{\partial x} - m'dx\frac{\partial^2 z}{\partial x^2} = 0$$

solve

$$\boxed{f_i = \frac{i}{2L}\sqrt{\frac{S}{m'}}}$$

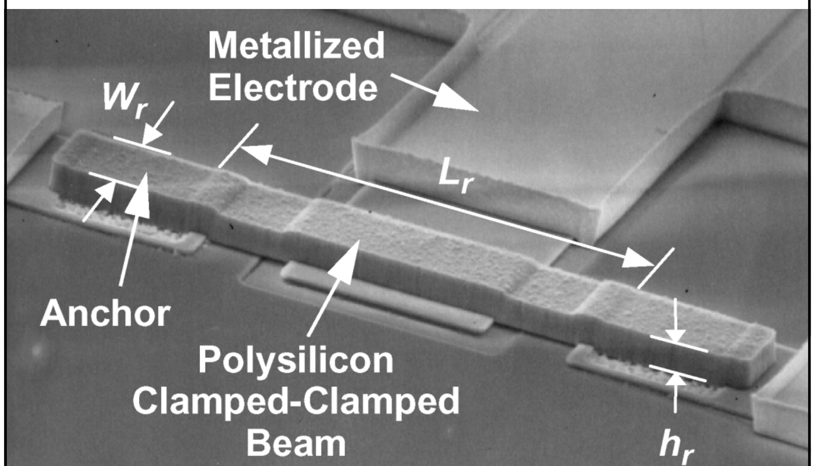
frequency

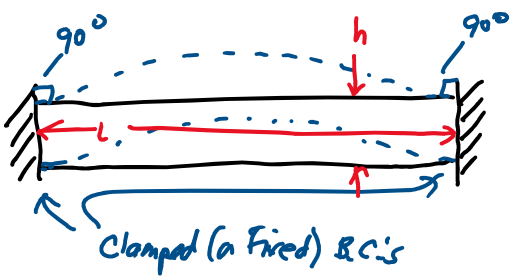
if $L\downarrow \rightarrow f_i\uparrow$

i = mode: 1, 2, 3, …

!!!

Clamped-Clamped (or Fixed-Fixed) Beam





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→ Eqn. for Resonance

Euler-Bernoulli Eq.

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L^2} \quad (1)$$

where $E \triangleq$ Young's modulus of elasticity [GPa]

$\rho \triangleq$ density [kg/m³]

$h \triangleq$ thickness [m]

$L \triangleq$ length [m]

Example: $L = 40\mu m$, $h = 2\mu m$

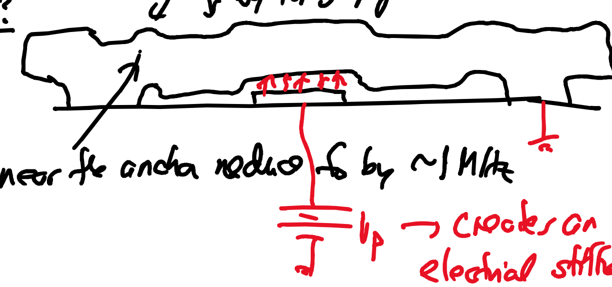
polysi → $E = 150\,GPa$, $\rho = 2300\,kg/m^3$

$$\therefore f_o = (1.03)\sqrt{\frac{150G}{2300}}\frac{2\mu}{(40\mu)^2} \Rightarrow \boxed{f_o = 16.4MHz}$$

Not quite the same as the measurement!?

Why?

kinks caused by topography



near the anchor reduce f_o by ~1MHz

V_p → creates an electrical stiffness $k_e \rightarrow \Delta f_o$

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Scaling:

2x, $\frac{1}{2}$x

① Scale all dimensions equally by a factor S

$$f_o \sim \frac{S}{S^2} = \frac{1}{S}$$

② If scale L only: $f_o \sim \frac{1}{S^2}$ → even faster rise in f_o!

(But problems...)

Example:

acoustic velocity → $\sqrt{\frac{E}{\rho}}$ m/s

$$L = 4\mu m \rightarrow f_o = (1.03)(8076)\frac{2\mu}{(4\mu)^2} = 1.04\,GHz!$$

really get ~800MHz

Q↓

- Remarks:
- Eq. (1) not accurate when L ≈ h
- Anchor loss when L ≈ h ! → beam becomes too stiff → lowers Q

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