Lecture 3: Benefits of Scaling II

- Announcements:
- Modules 1 & 2 are online
- HW#1 online and due Feb. 11 at 8 a.m.
- ------------------------------------
- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Micro gas chromatograph
- ------------------------------------
- Last Time:
- Going through Module 2, looking at how scaling vibrating RF MEMS provides both benefits and problems that one must circumvent
- Continue with this now

1

→ Eqn. for Resonance

→ Euler-Bernoulli Eq.

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.03\sqrt{\frac{E}{\rho}}\frac{h}{L^2} \quad (1)$$

where $E \triangleq$ Young's modulus of elasticity [GPa]

$\rho \triangleq$ density [kg/m^3]

$h \triangleq$ thickness [m]

$L \triangleq$ length [m]

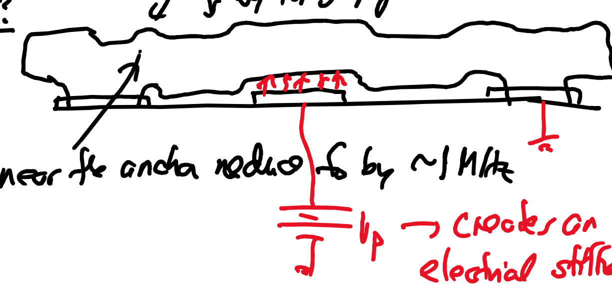
Example: $L = 40\mu m$, $h = 2\mu m$

polySi → $E = 150\,GPa$, $\rho = 2300\,kg/m^3$

$$\therefore f_o = (1.03)\sqrt{\frac{150G}{2300}}\frac{2\mu}{(40\mu)^2} \Rightarrow f_o = 16.4\,MHz$$

Not quite the same as the measurement!?

Why?



2

Scaling:

① Scale all dimensions equally by a factor S (2x, ½x)

$$f_o \sim \frac{S}{S^2} = \frac{1}{S}$$

② If scale L only: $f_o \sim \frac{1}{S^2}$ → even faster rise in f_o! (But problems...)

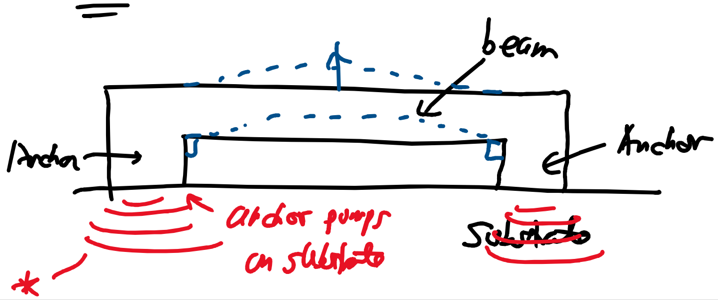
Example: acoustic velocity $\sqrt{\frac{E}{\rho}}$ → m/s

$$L = 4\mu m \rightarrow f_o = (1.03)(8076)\frac{2\mu}{(4\mu)^2} = 1.04\,GHz!$$

really get ~800MHz

- Remarks:
- Eq. (1) not accurate when L ≈ h (to fix: Timoshenko theory)
- Anchor loss when L ≈ h ! → beam becomes too stiff → lowers Q (Q↓)

Problem: Anchor Loss



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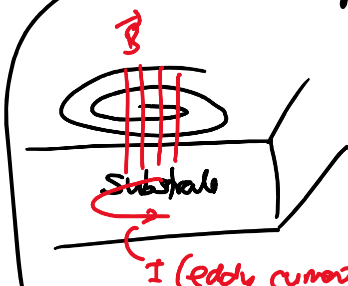
* ↳ energy radiates into substrate } anchor loss

$$Q = \frac{\text{Energy per Cycle}}{\text{Energy lost per Cycle}} \downarrow$$

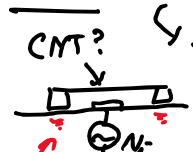
↳ Example of a scaling con



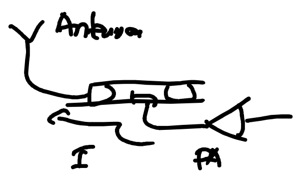
} Equivalent Spiral Inductor Problem

↳ goes thru R → Q↓ (1-5)

① Solution: nanodimensions! ✓
↳ shrink both thickness (h) and L
↳ h~300nm, L~1μm



very little pumping → retain Q



Problem: power handling ↓ when size ↓

Soln: arrays

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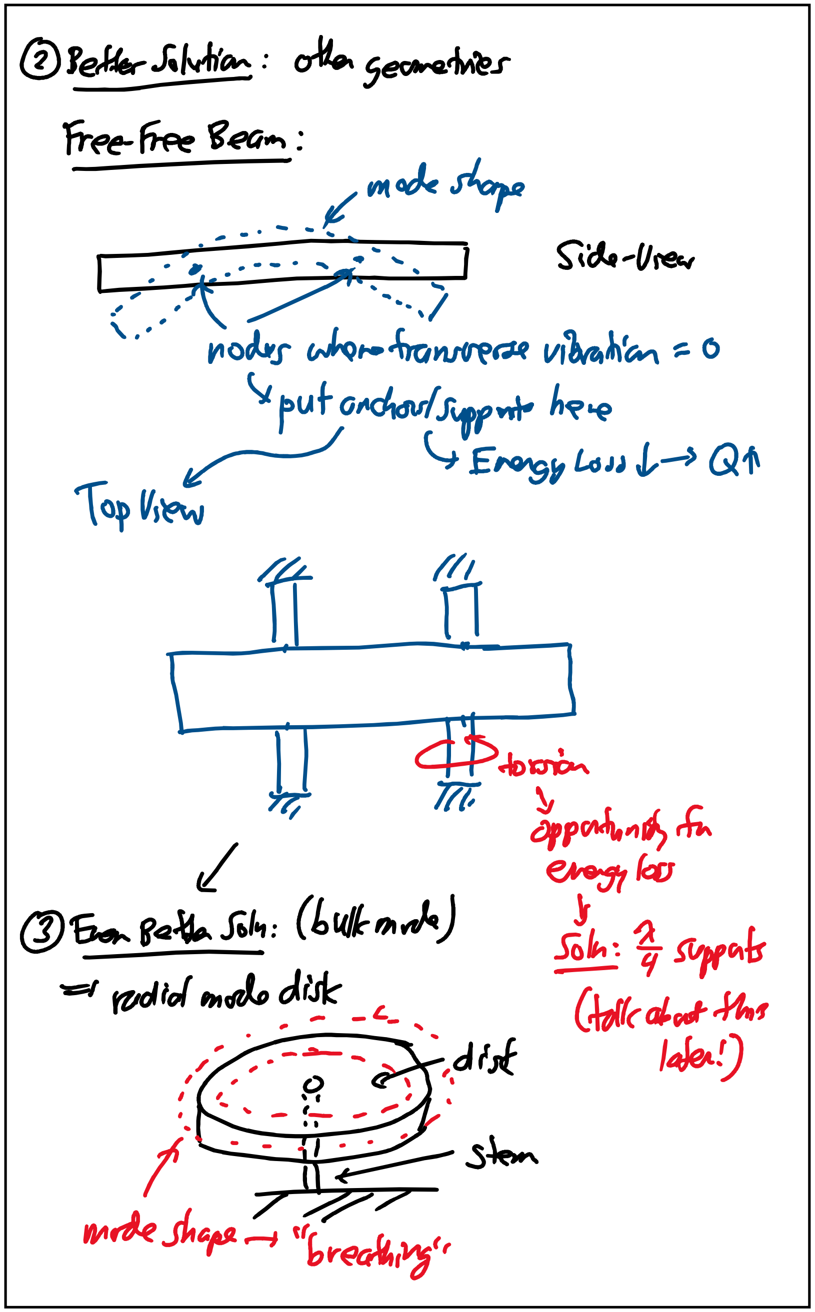
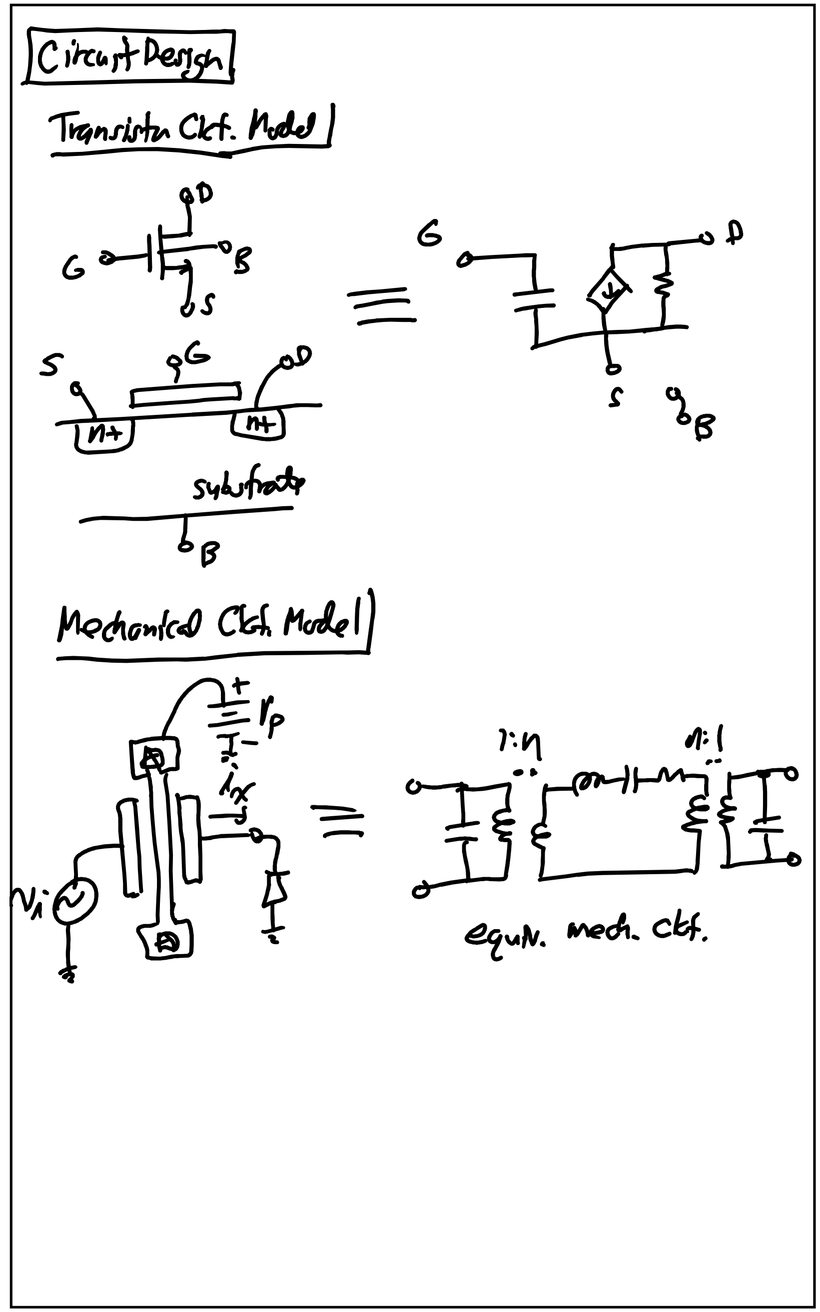

② Better Solution: other geometries
Free-Free Beam:
mode shape
Side-View
nodes where transverse vibration = 0
put anchors/supports here
Energy Loss ↓ → Q ↑
Top View
torsion
opportunity for energy loss
Soln: λ/4 supports
(talk about this later!)
③ Even Better Soln: (bulk mode)
⇒ radial mode disk
disk
stem
mode shape → "breathing"

5


Circuit Design
Transistor Ckt. Model
D
G
B
S
n+
substrate
Mechanical Ckt. Model
+
V_P
i_x
v_i
1:η
η:1
equiv. mech. ckt.

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