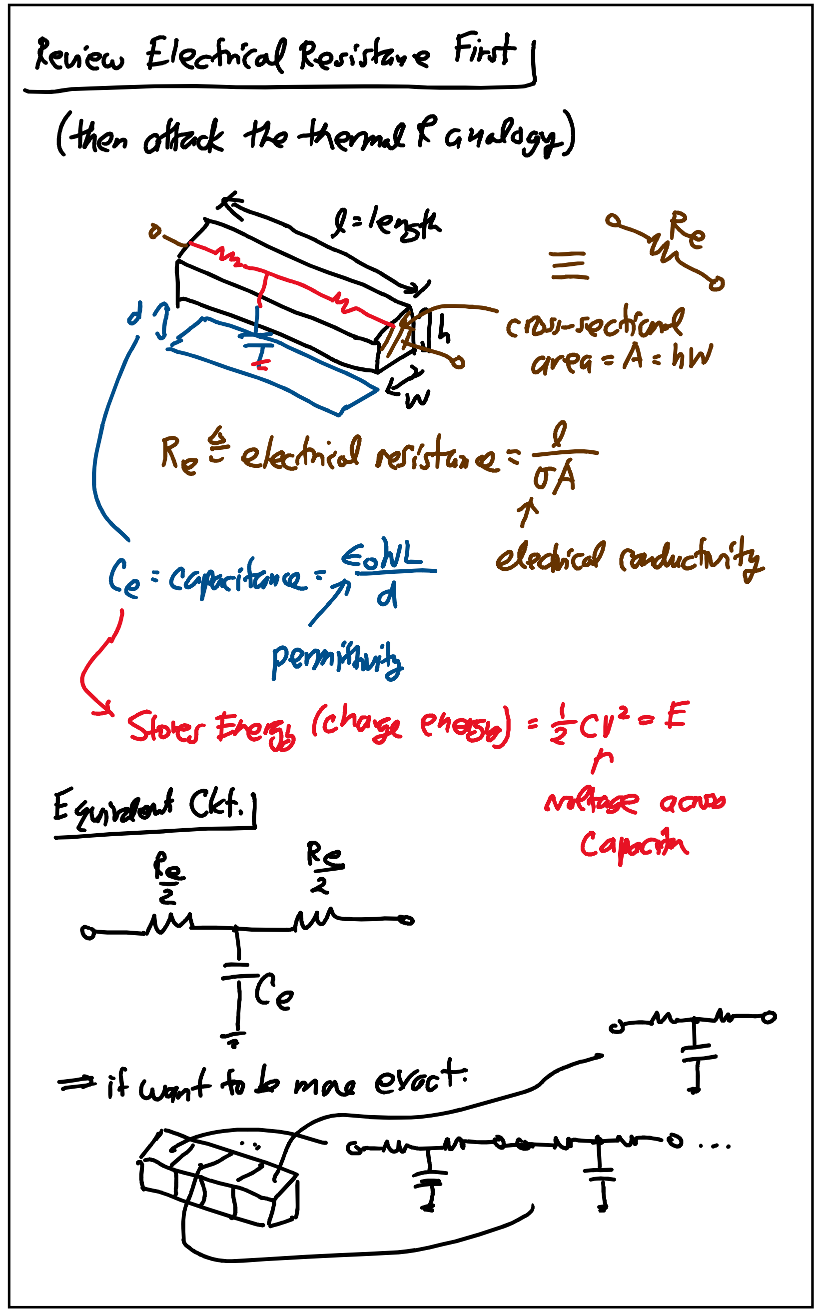
Lecture 4: Benefits of Scaling III

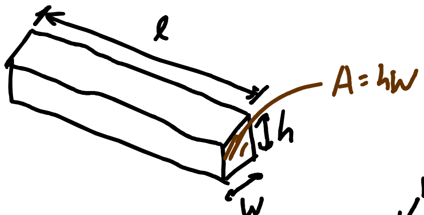
- Announcements:
- Modules 1 & 2 are online
- HW#1 online and due Feb. 11 at 8 a.m.
- Will go longer today by about 20 minutes or so to make up a bit for the lecture lost on first day
- Will do this a few more times until we're caught up
- --
- Today:
- Reading: Senturia, Chapter 1
- Lecture Topics:
  - Benefits of Miniaturization
  - Examples
    - GHz micromechanical resonators
    - Chip-scale atomic clock
    - Micro gas chromatograph
- --
- Last Time:
- Going through Module 2, looking at Chip-Scale Atomic Clock
- Continue with this now

1



2

Thermal Ckt.



⇒ thermal capacitance: $C_{th} = \rho V c_p$

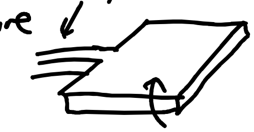
- V: volume
- c_p: specific heat
- ρ: density
- stores thermal energy

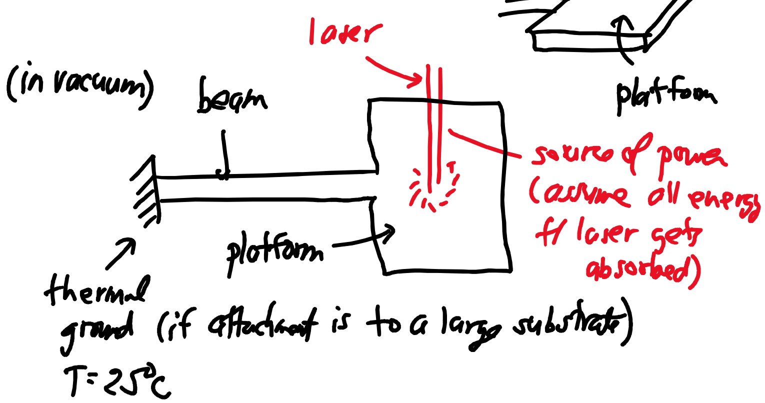
⇒ thermal resistance:

$$R_{th} = \frac{\ell}{kA}$$

- ℓ: length
- A: cross-sectional area
- k: thermal conductivity

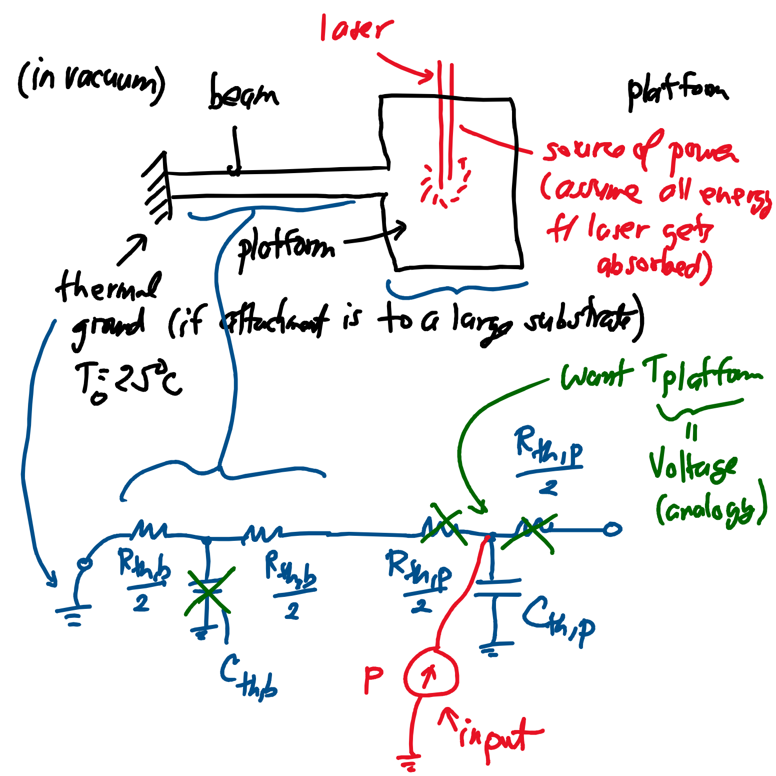
Example MEMS Type Structure





3

What is the temperature on the platform?



Analogies

(temperature) $T \rightarrow V$ (voltage)

(power) $P \rightarrow I$ (current)

thermal ↵ ↳ electrical

4

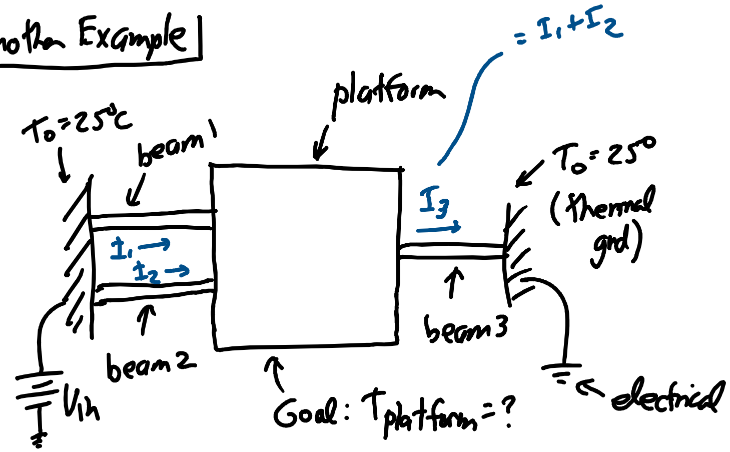
To Analyze:

① Remove small element to simplify the ckt.

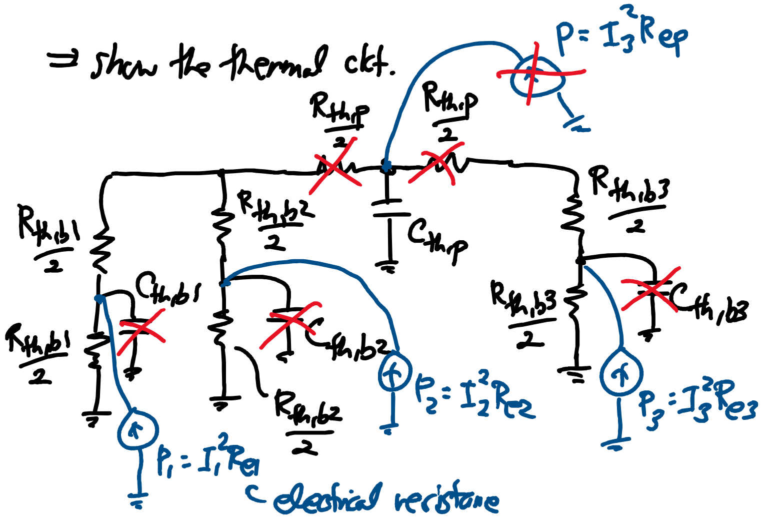
② $(T_{platform} - T_0) = PR_{th,b}$ (in steady-state)

(just like $V = IR$ in an electrical system)

Another Example



⇒ show the thermal ckt.



5

To Analyze:

① $I_3 = I_1 + I_2 = \frac{V_{in}}{R_{e,tot}}$ (electrical analysis)

② Get P_i's. (powers)

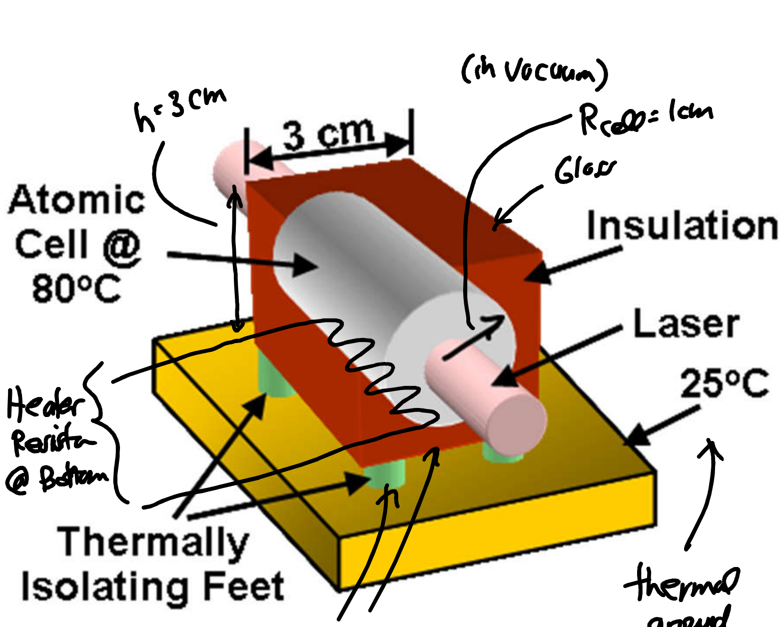
③ Use superposition to solve the thermal ckt.

handle one power source at a time & sum the temperatures (i.e., thermal voltages) to get the total temperature at any point (or node)

Example Thermal Ckt. for the CSAC Physics Package

⇒ determine the power needed to get the atomic cell to 80°C (from room temperature) & how fast

6



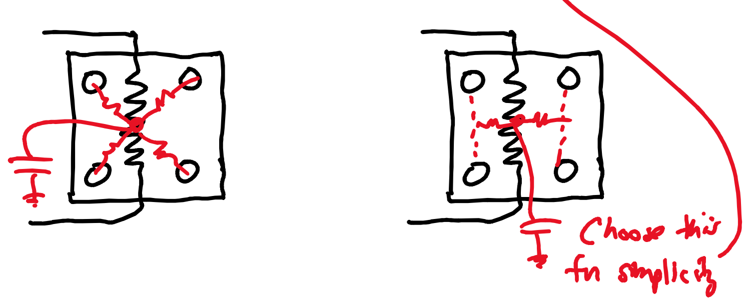
all glass:

$\rho_{glass} = 2500\ kg/m^3$

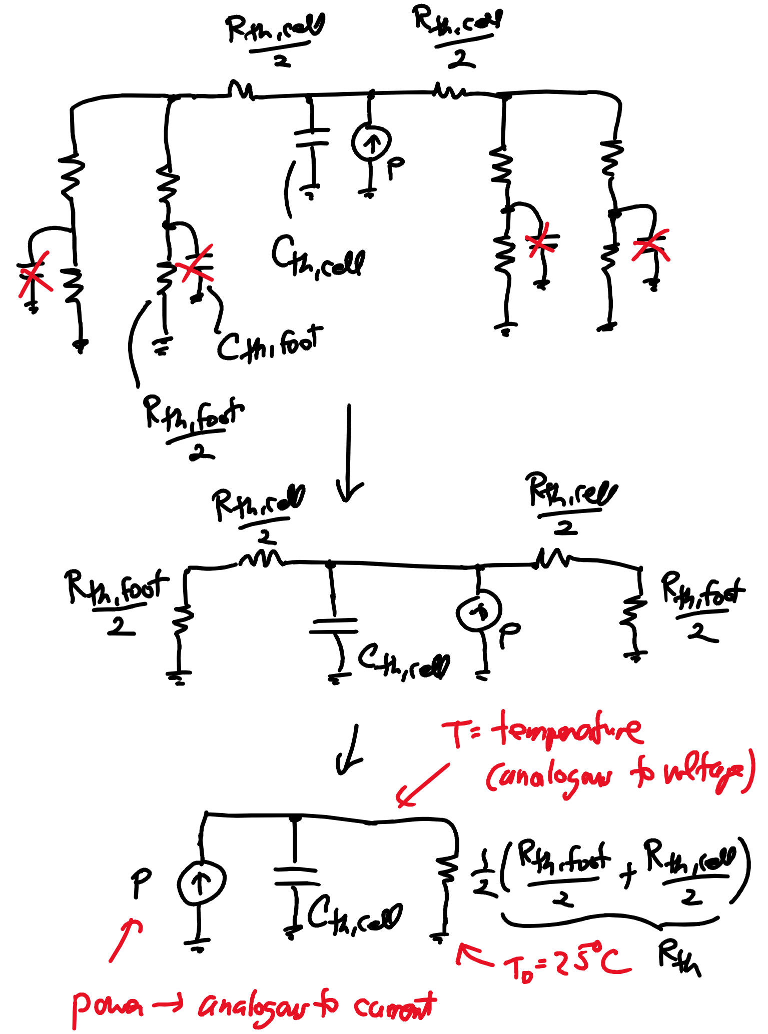
$C_{p,glass} = 0.5\ J/(g \cdot K)$

$K_{glass} = 1.05\ \frac{W}{m \cdot K}$

Top Views:

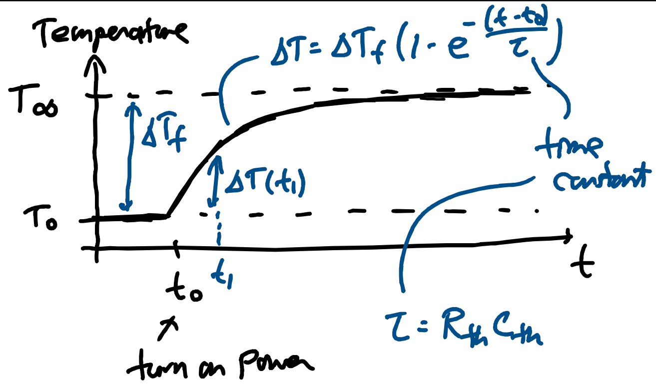


7



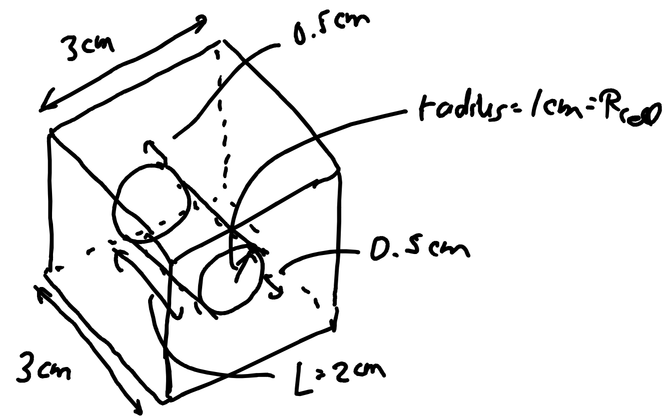
$\Delta T = T - T_0 = P R_{th}$ (in steady-state)

8



Find $C_{th,cell}$:

⇒ first, find the cell volume



$V_{cell} = hWL - \pi R_{cell}^2 \cdot L$

$= (3cm)(3cm)(3cm) - \pi(1cm)^2(2cm) = 20.7 cm^3$

9

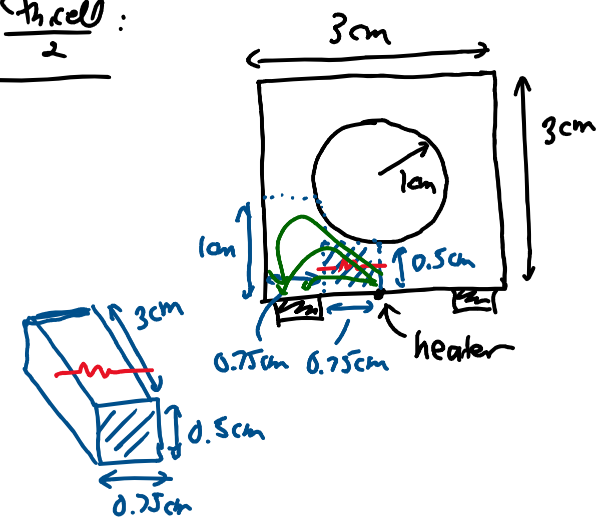
$C_{th,cell} = \rho_{glass} V_{cell} c_{p,glass}$

$= \left(2500 \frac{kg}{m^3}\right)\left(1000 \frac{g}{kg}\right)\left(\frac{1}{100^3} \frac{m^3}{cm^3}\right)$

$\times (20.7 cm^3)\left(0.5 \frac{J}{g \cdot K}\right)$

$\Rightarrow C_{th,cell} = 25.9 J/K$

Find $\frac{R_{th,cell}}{2}$:



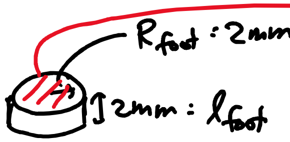
$\frac{R_{th,cell}}{2} = \frac{\frac{3}{4}}{K(3)(\frac{1}{2})} + \frac{\frac{3}{4}}{K(3)(1)} = \frac{1}{K}\left(\frac{1}{8} + \frac{1}{4}\right) = \frac{3}{8}\frac{1}{K}$

$\left[R_{th} = \frac{l}{KA}\right]$ $\quad \therefore \frac{R_{th,cell}}{2} = \frac{3}{8} \frac{1}{1.05} \times \left(100 \frac{cm}{m}\right)$

$\Rightarrow \frac{R_{th,cell}}{2} = 35.7 K/W$

10

Find $R_{th,foot}$:



$A_{foot} = \pi R_{foot}^2$

$$\therefore R_{th,foot} = \frac{l_{foot}}{kA_{foot}} = \frac{2mm}{(1.05\frac{W}{m\cdot K})\pi(2mm)^2} = 151.6\frac{K}{W}$$

Then:

$$R_{th} = \frac{1}{2}\left(\frac{R_{th,foot}}{2} + \frac{R_{th,cell}}{2}\right)$$

$$= \frac{1}{2}\left(\frac{151.6}{2} + 35.7\right) \Rightarrow R_{th} = 55.8\ K/W$$

⇒ Find the power req'd to maintain $T_\infty = 80°C$ in steady-state:

$$P = \frac{T_\infty - T_o}{R_{th}} = \frac{(80-25)}{55.8} = 0.99W \sim 1W$$

⇒ Find the time constant:

$$\tau = R_{th}C_{th,cell} = \underset{K/W}{(55.8)}\underset{J/K}{(25.9)} = 24\ min.$$

It takes ~3τ to reach steady-state ∴ must wait 72 min. before using this atomic cell

How about using MEMS?

↳ (how about scaling this?)

⇒ much smaller cell volume → weight ↓

↗ Vol ↓ → C_{th} ↓

Macro:

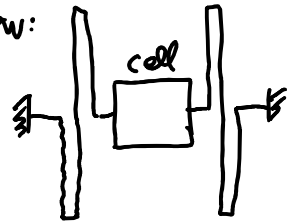
↓ shrink dimensions

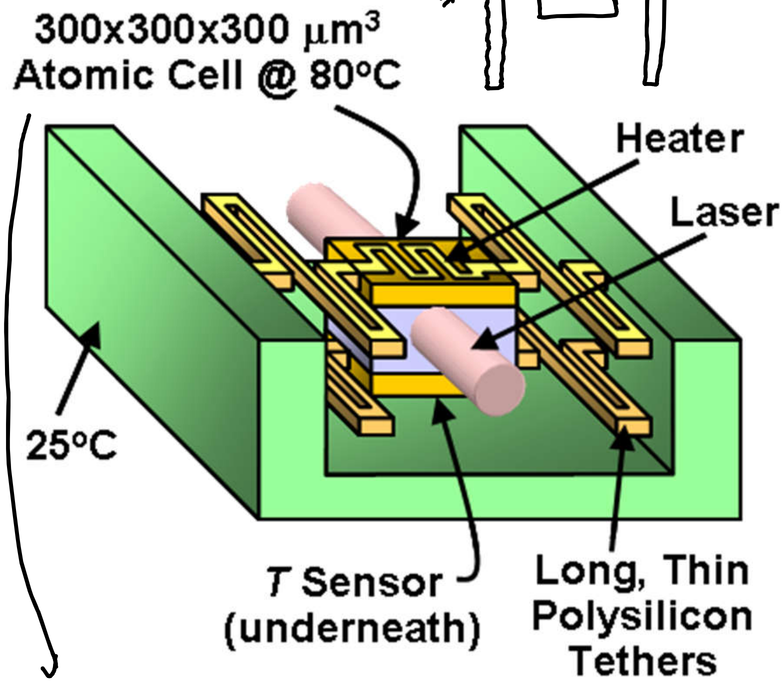
Micro:

can do this → use long, thin supports to suspend a smaller cell

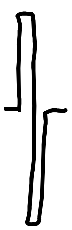
MEMS Atomic Cell

Top View:





Cell is hollow w/ 10μm-thick walls.



} 500μm-long, 10μm-thick, 20μm-wide

$V_{cell} = (300\mu m)^3 - (280\mu m)^3$
$= 5.048\times10^{-12}\,m^3$
(much smaller than macro!)

$C_{th,cell} = \rho_{glass} V_{cell} C_{p,glass}$
$= \left(2500\,\frac{kg}{m^3}\right)\left(5.048\times10^{-12}\,m^3\right)\left(500\,\frac{J}{kg\cdot K}\right)$
$= 6.31\times10^{-6}\,\frac{J}{K}$ ← 4 million x smaller than macro!

$$R_{th,supp} = \frac{l_{supp}}{k_{polySi} W_{supp} h_{supp}} = \frac{500\mu}{\left(30\frac{W}{m\cdot K}\right)(20\mu)(10\mu)}$$

$\Rightarrow R_{th,supp} = 83,333\ K/W$

↑ 1493x larger than macro!

and...

$P = \frac{(80-25)}{83,333} = 0.66\,mW$ ← 1493x smaller than macro!

$\tau = 0.53s$ ← 2727x faster!

All due to scaling!