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EE C247B – ME C218
Introduction to MEMS Design
Spring 2020

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Lecture Module 11: Equivalent Circuits I

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Lecture Outline

- Reading: Senturia, Chpt. 5
- Lecture Topics:
  - Lumped Mass
  - Lumped Stiffness
  - Lumped Damping
  - Lumped Mechanical Equivalent Circuits
  - Electromechanical Analogies

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Lumped Parameter Mechanical Equivalent Circuit

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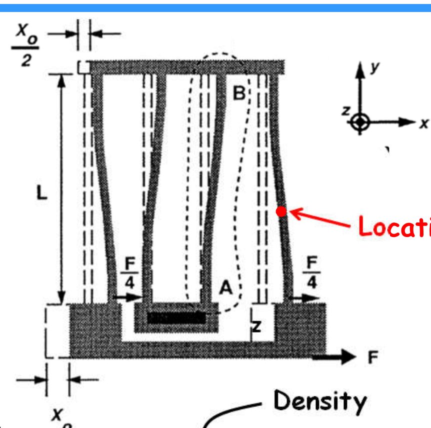
Equivalent Dynamic Mass

- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location x using knowledge of kinetic energy and velocity



Maximum Kinetic Energy

Density

$$\text{Equivalent Mass} = M_{eq\,x} = \frac{K.E.}{\frac{1}{2}V_x^2} = \frac{\frac{1}{2}\rho A \int_0^{\ell} V^2(x)\,dx}{\frac{1}{2}V_x^2}$$

Maximum Velocity @ location x

Maximum Velocity Function

Equivalent Dynamic Mass

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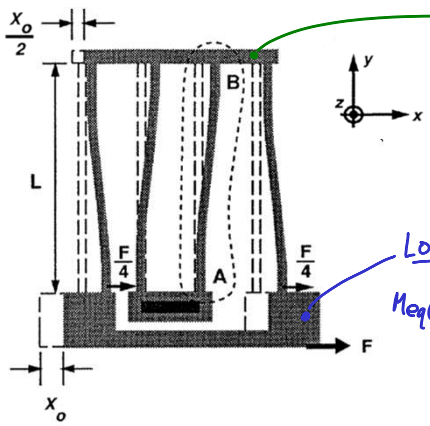
- For the folded-beam structure, we've already determined the maximum kinetic energy
- And in our resonance frequency analysis, we've already determined expressions for velocity



Location on the Truss:

$$M_{eq(truss)} = \frac{KE_{max}}{\frac{1}{2}V_{truss}^2} = \frac{\omega_o^2 X_o^2 (\frac{1}{2})[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b]}{\frac{1}{2}(\frac{1}{4})\omega_o^2 X_o^2}$$

$$\therefore M_{eq(truss)} = 4\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]$$

Location on the Shuttle:

$$M_{eq(shuttle)} = \frac{KE_{max}}{\frac{1}{2}V_{shuttle}^2} = \frac{\omega_o^2 X_o^2 (\frac{1}{2})[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b]}{\frac{1}{2}\omega_o^2 X_o^2}$$

$$\therefore M_{eq(shuttle)} = M_s + \frac{1}{4}M_t + \frac{12}{35}M_b$$

Equivalent Dynamic Stiffness & Damping

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- Stiffness then follows directly from knowledge of mass and resonance frequency

$$\omega_o = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \longrightarrow \boxed{K_{eq}(x) = \omega_o^2 M_{eq}(x)}$$

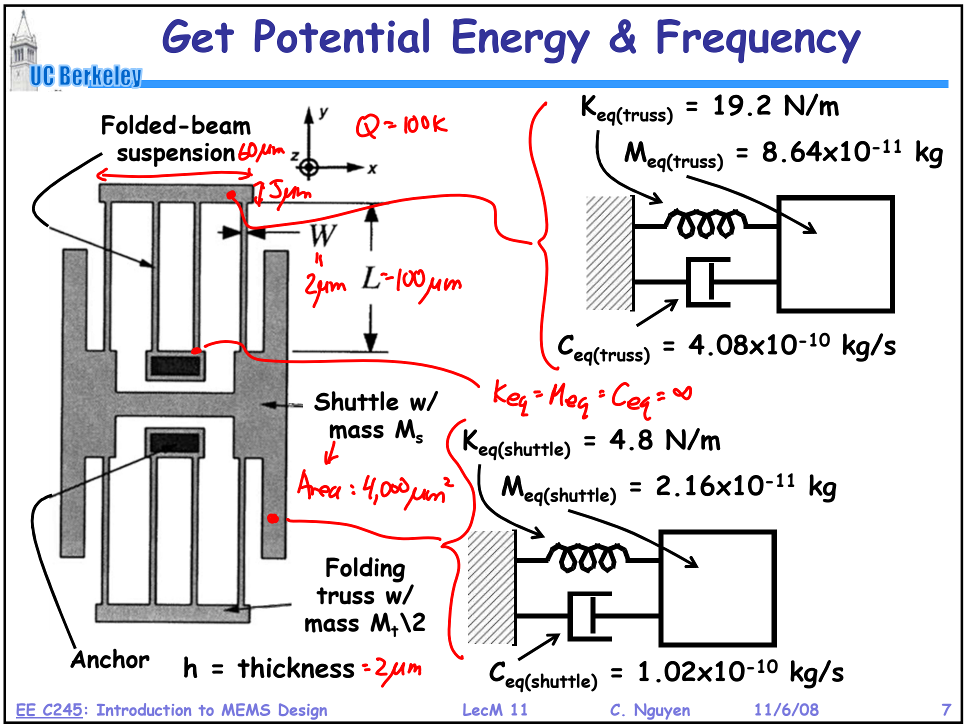
⇒ large equiv. mass & large stiffness go hand-in-hand

- And damping also follows readily from knowledge of Q or other loss measurands

$$Q = \frac{\omega_o M_{eq}(x)}{C_{eq}(x)} \longrightarrow \boxed{C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x) M_{eq}(x)}}{Q}}$$

($C_{eq}(x)$ ← damping)

- With mass, stiffness, and damping ⇨ lumped parameter equivalent circuit


Get Potential Energy & Frequency
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Folded-beam suspension
60μm
Q = 100K
3μm
W = 2μm
L = 100μm
Shuttle w/ mass M_s
Area: 4,000μm²
Folding truss w/ mass M_t\2
Anchor
h = thickness = 2μm
K_eq(truss) = 19.2 N/m
M_eq(truss) = 8.64x10^-11 kg
C_eq(truss) = 4.08x10^-10 kg/s
K_eq = M_eq = C_eq = ∞
K_eq(shuttle) = 4.8 N/m
M_eq(shuttle) = 2.16x10^-11 kg
C_eq(shuttle) = 1.02x10^-10 kg/s
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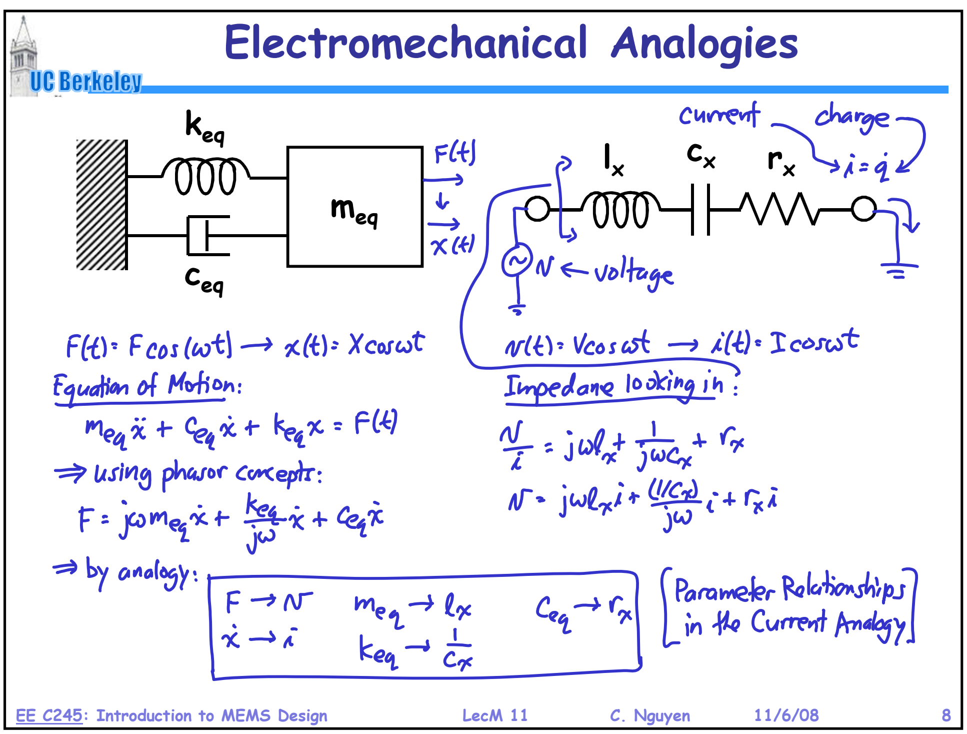
Electromechanical Analogies
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k_eq
m_eq
c_eq
F(t)
x(t)
current
charge
i = q̇
l_x
c_x
r_x
v ← voltage
F(t) = F cos(ωt) → x(t) = X cos ωt
Equation of Motion:
m_eq ẍ + c_eq ẋ + k_eq x = F(t)
⇒ using phasor concepts:
F = jω m_eq ẋ + (k_eq / jω) ẋ + c_eq ẋ
⇒ by analogy:
F → v   ẋ → i   m_eq → l_x   k_eq → 1/C_x   c_eq → r_x
v(t) = V cos ωt → i(t) = I cos ωt
Impedance looking in:
v/i = jωl_x + 1/(jωC_x) + r_x
v = jωl_x i + ((1/C_x)/jω) i + r_x i
[Parameter Relationships in the Current Analogy]
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Electromechanical Analogies (cont)

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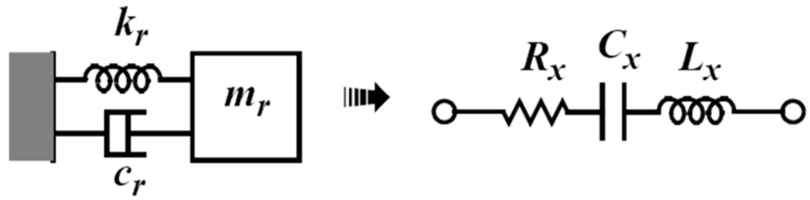

- Mechanical-to-electrical correspondence in the current analogy:

Mechanical Variable	Electrical Variable
Damping, c	Resistance, R
Stiffness^{-1}, k^{-1}	Capacitance, C
Mass, m	Inductance, L
Force, f	Voltage, V
Velocity, v	Current, I

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Bandpass Biquad Transfer Function

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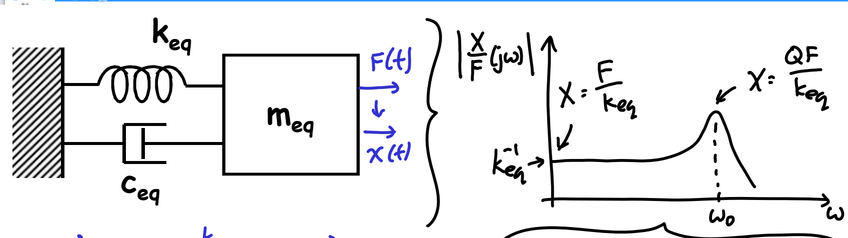

$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

⇒ converting to full phasor form:

$$F = (j\omega)(j\omega X)m_{eq} + \frac{k_{eq}}{j\omega}(j\omega X) + c_{eq}(j\omega X)$$

$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\omega^2\frac{m_{eq}}{k_{eq}} + 1 + j\frac{c_{eq}\omega}{k_{eq}}\right]^{-1} = \frac{1}{k_{eq}}\left[-\left(\frac{\omega}{\omega_0}\right)^2 + 1 + j\frac{\omega}{Q\omega_0}\right]^{-1}$$

$$\left[\frac{k_{eq}}{m_{eq}} = \omega_0^2,\ Q = \frac{m_{eq}\omega_0}{c_{eq}} = \frac{k_{eq}}{\omega_0 c_{eq}} \rightarrow \frac{k_{eq}}{c_{eq}} = Q\omega_0\right]$$

$$\boxed{\frac{X}{F}(j\omega) = \frac{k_{eq}^{-1}}{1 - \left(\frac{\omega}{\omega_0}\right)^2 + j\frac{\omega}{Q\omega_0}}}$$

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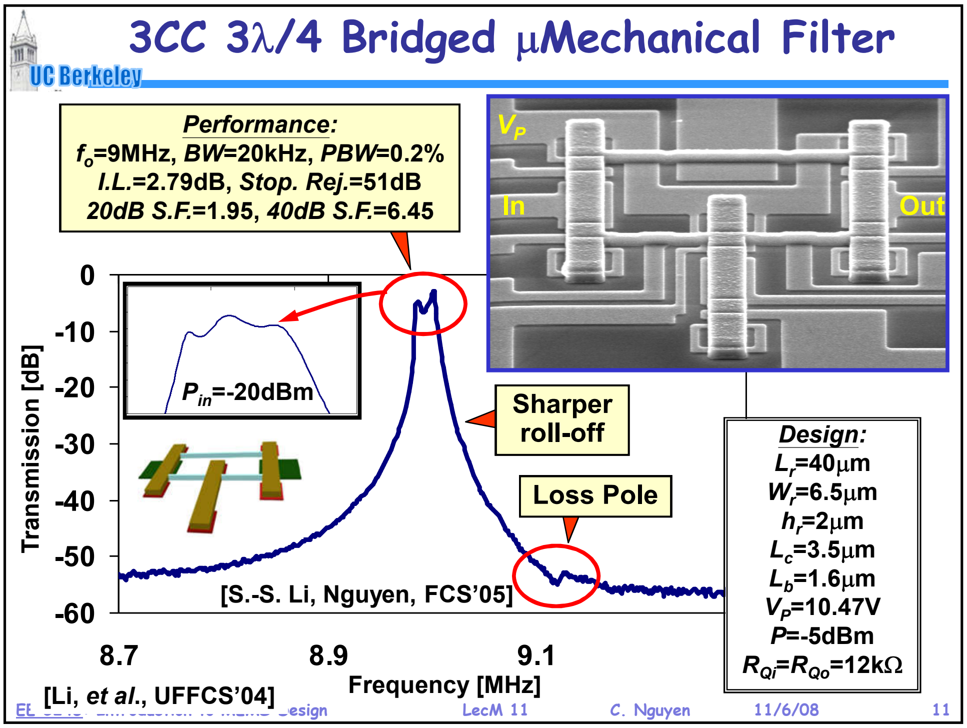
3CC 3λ/4 Bridged μMechanical Filter
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Performance:
fo=9MHz, BW=20kHz, PBW=0.2%
I.L.=2.79dB, Stop. Rej.=51dB
20dB S.F.=1.95, 40dB S.F.=6.45
VP
In
Out
Pin=-20dBm
Sharper roll-off
Loss Pole
Design:
Lr=40μm
Wr=6.5μm
hr=2μm
Lc=3.5μm
Lb=1.6μm
VP=10.47V
P=-5dBm
RQi=RQo=12kΩ
Transmission [dB]
0
-10
-20
-30
-40
-50
-60
8.7
8.9
9.1
Frequency [MHz]
[S.-S. Li, Nguyen, FCS'05]
[Li, et al., UFFCS'04]
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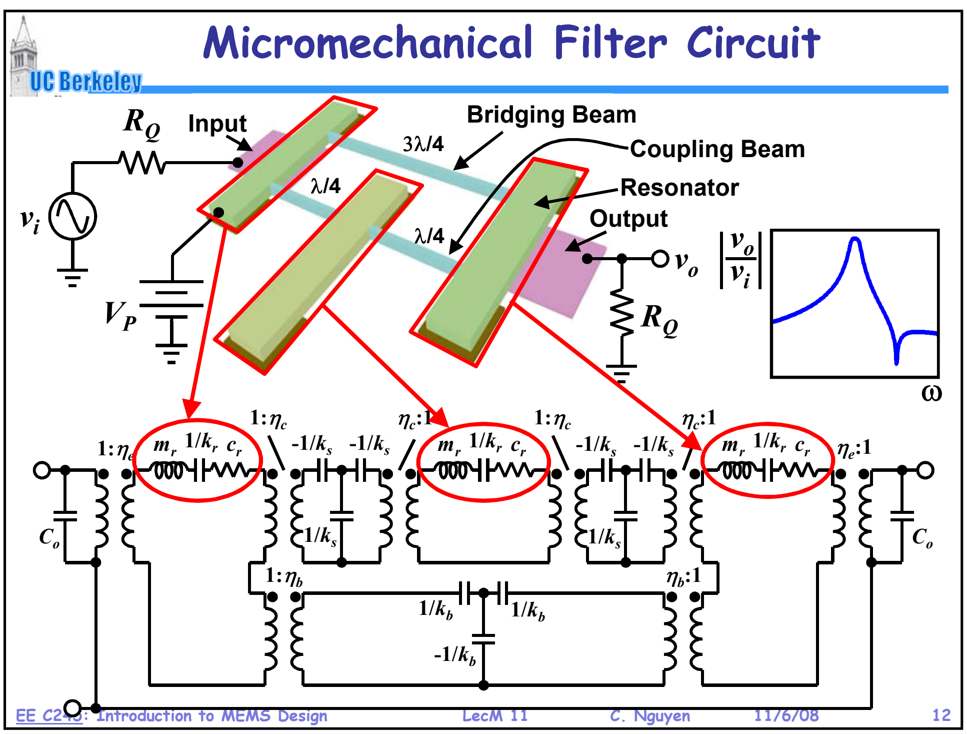
Micromechanical Filter Circuit
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RQ
Input
Bridging Beam
3λ/4
Coupling Beam
λ/4
Resonator
Output
λ/4
vi
VP
vo
|vo/vi|
RQ
ω
1:ηe
1:ηc
ηc:1
1:ηc
ηc:1
ηe:1
mr 1/kr cr
-1/ks -1/ks
-1/ks -1/ks
1/ks
1/ks
Co
Co
1:ηb
ηb:1
1/kb
1/kb
-1/kb
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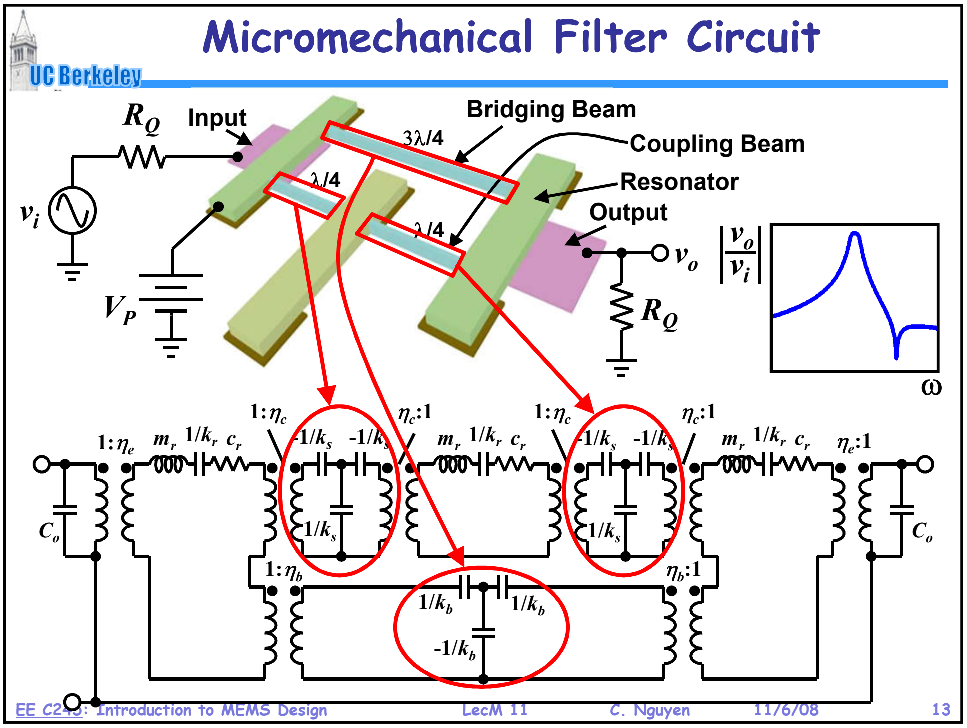
Micromechanical Filter Circuit
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R_Q
Input
Bridging Beam
3λ/4
Coupling Beam
Resonator
λ/4
λ/4
Output
v_i
V_P
v_o
R_Q
$\left|\frac{v_o}{v_i}\right|$
ω
$1:\eta_e$
m_r $1/k_r$ c_r
$1:\eta_c$
$-1/k_s$ $-1/k_s$
$\eta_c:1$
$1/k_s$
C_o
$1:\eta_b$
$\eta_b:1$
$1/k_b$ $1/k_b$
$-1/k_b$
$\eta_e:1$
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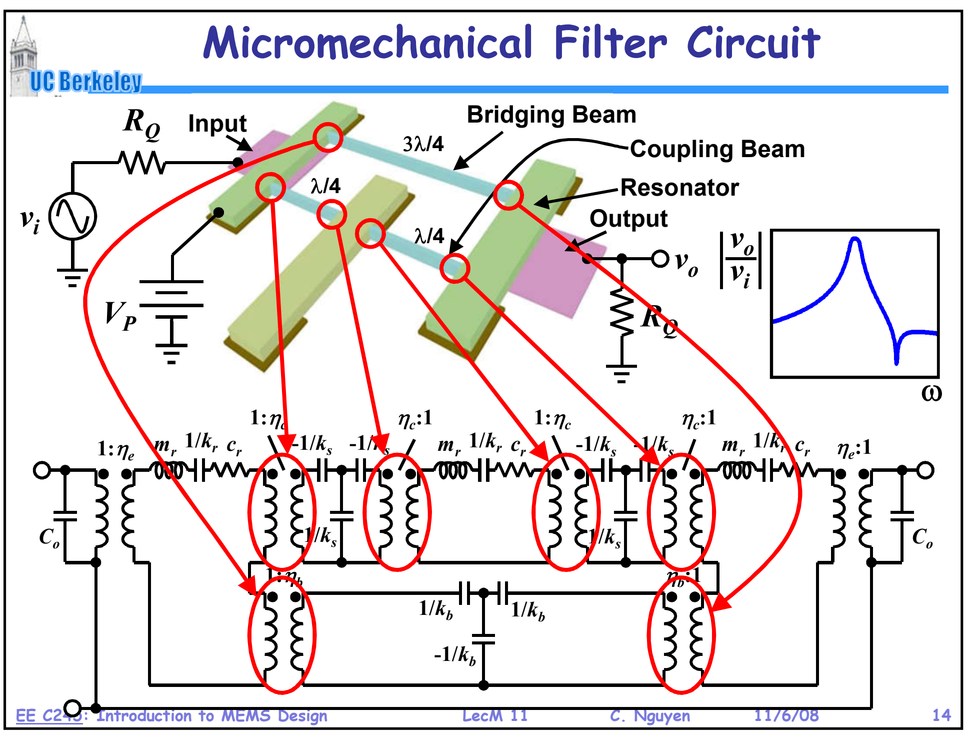
Micromechanical Filter Circuit
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R_Q
Input
Bridging Beam
3λ/4
Coupling Beam
Resonator
λ/4
λ/4
Output
v_i
V_P
v_o
R_Q
$\left|\frac{v_o}{v_i}\right|$
ω
$1:\eta_e$
m_r $1/k_r$ c_r
$1:\eta_c$
$-1/k_s$ $-1/k_s$
$\eta_c:1$
$1/k_s$
C_o
$1/k_b$ $1/k_b$
$-1/k_b$
$\eta_e:1$
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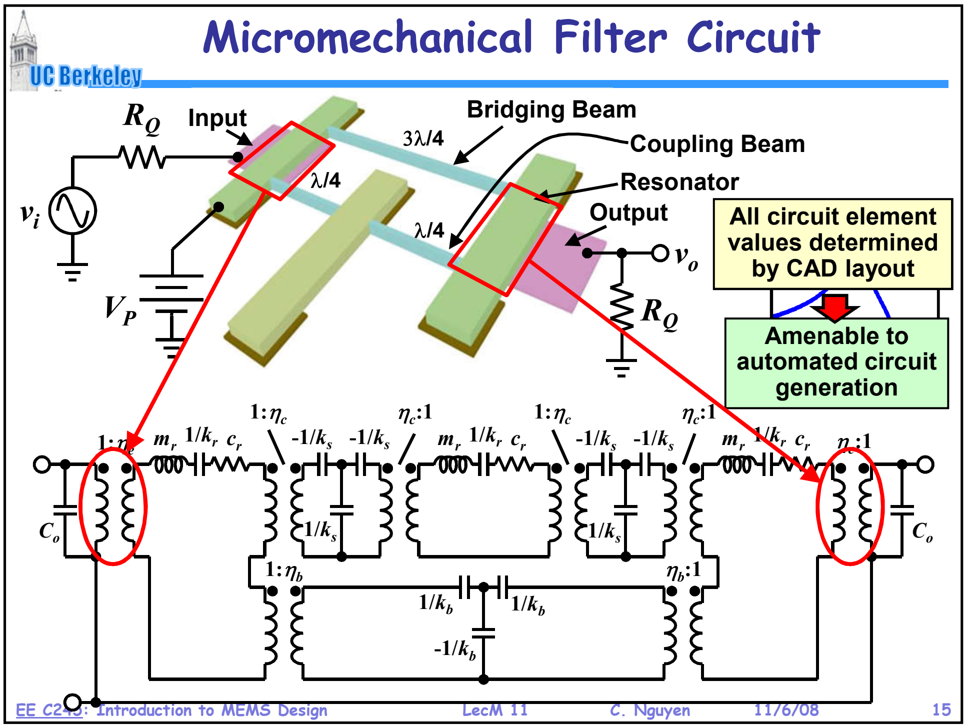
Micromechanical Filter Circuit
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R_Q
Input
Bridging Beam
3λ/4
Coupling Beam
λ/4
Resonator
Output
λ/4
v_i
v_o
V_P
R_Q
All circuit element values determined by CAD layout
Amenable to automated circuit generation
1:η_e
m_r 1/k_r c_r
1:η_c
-1/k_s -1/k_s
η_c:1
1:η_c
η_c:1
η_e:1
C_o
1/k_s
1:η_b
η_b:1
1/k_b
1/k_b
-1/k_b
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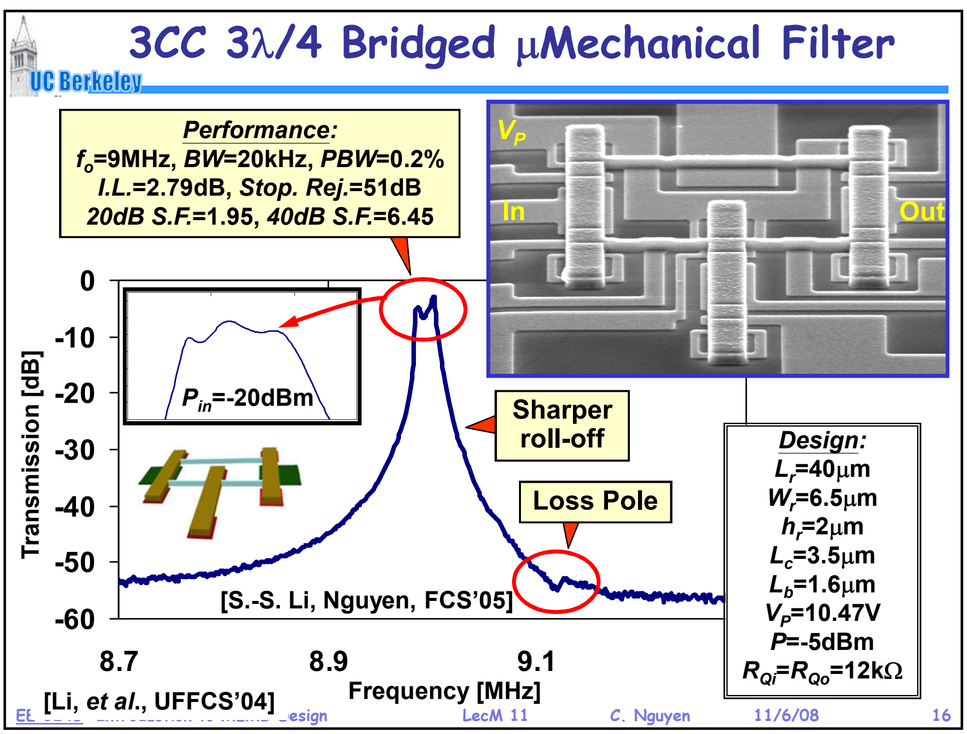
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In
Out
P_in=-20dBm
Sharper roll-off
Loss Pole
Design:
L_r=40μm
W_r=6.5μm
h_r=2μm
L_c=3.5μm
L_b=1.6μm
V_P=10.47V
P=-5dBm
R_Qi=R_Qo=12kΩ
Transmission [dB]
0
-10
-20
-30
-40
-50
-60
8.7
8.9
9.1
Frequency [MHz]
[S.-S. Li, Nguyen, FCS'05]
[Li, et al., UFFCS'04]
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Beam Resonator Equivalent Circuits (Pretty Much the Same Stuff)

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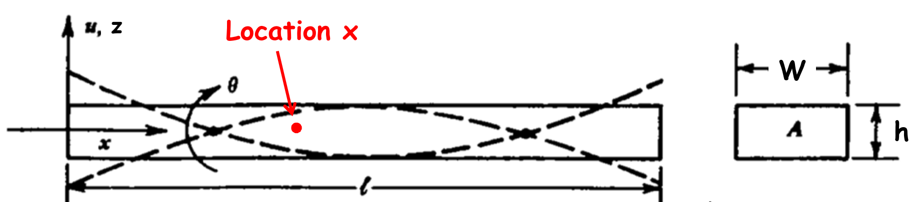
Equivalent Dynamic Mass

- Once the mode shape is known, the lumped parameter equivalent circuit can then be specified
- Determine the equivalent mass at a specific location x using knowledge of kinetic energy and velocity



Maximum Kinetic Energy → K.E.; Density → ρ

$$\text{Equivalent Mass} = M_{eq\,x} = \frac{K.E.}{\frac{1}{2}V_x^2} = \frac{\frac{1}{2}\rho A \int_0^\ell V^2(x)\,dx}{\frac{1}{2}V_x^2}$$

Maximum Velocity @ location x → V_x; Maximum Velocity Function → $V(x)$

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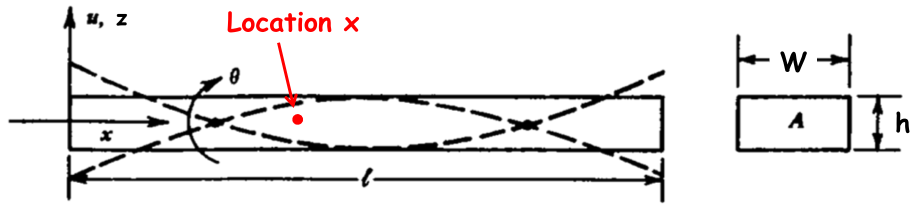
Equivalent Dynamic Mass



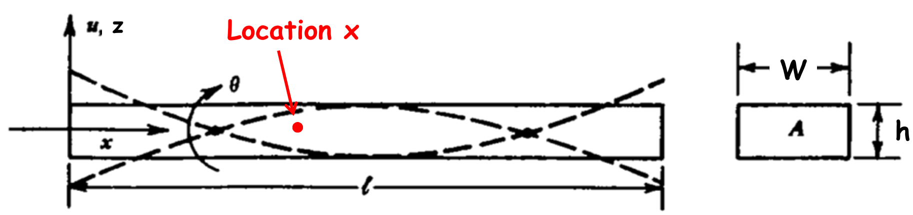
- We know the mode shape, so we can write expressions for displacement and velocity at resonance

Displacement: $u(x) = B\left[\varsigma(\cosh kx + \cos kx) + (\sinh kx + \sin kx)\right]$, $\varsigma = \frac{A}{B}$

$$[V(x) = \omega_0 u(x)] \Rightarrow M_{eq}(x) = \frac{KE_{max}}{\frac{1}{2}[V(x)]^2} = \frac{\frac{1}{2}\rho A \int_0^\ell \omega_0^2 [u(x')]^2 dx'}{\frac{1}{2}\omega_0^2 [u(x)]^2}$$

$$\boxed{M_{eq}(x) = \frac{\rho A \int_0^\ell B^2\left[\varsigma(\cosh kx' + \cos kx') + (\sinh kx' + \sin kx')\right]^2 dx'}{B^2\left[\varsigma(\cosh kx + \cos kx) + (\sinh kx + \sin kx)\right]^2}}$$

Equivalent Dynamic Stiffness & Damping



- Stiffness then follows directly from knowledge of mass and resonance frequency

$$\omega_o = \sqrt{\frac{K_{eq}(x)}{M_{eq}(x)}} \longrightarrow \boxed{K_{eq}(x) = \omega_o^2 M_{eq}(x)}$$

- And damping also follows readily

$$Q = \frac{\omega_o M_{eq}(x)}{c_{eq}(x)} \longrightarrow \boxed{c_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q} = \frac{\sqrt{K_{eq}(x) M_{eq}(x)}}{Q}}$$

($c_{eq}(x)$ ← damping)

Equivalent Lumped Mechanical Circuit

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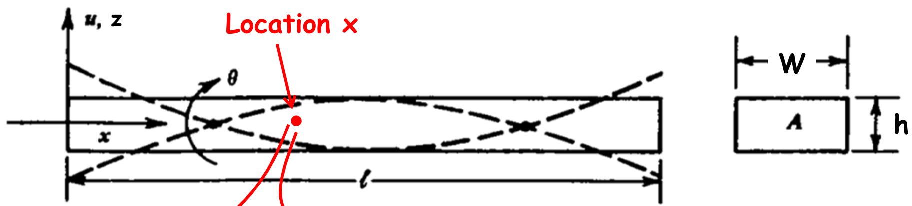

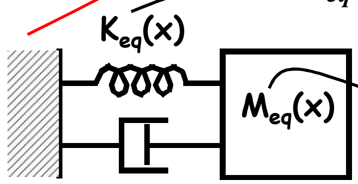

$$K_{eq}(x) = \omega_o^2 M_{eq}(x)$$

$$M_{eq}(x) = \frac{\rho A \int_o^l [u(x')]^2 dx'}{[u(x)]^2}$$

$$C_{eq}(x) = \frac{\omega_o M_{eq}(x)}{Q}$$

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Equivalent Lumped Mechanical Circuit

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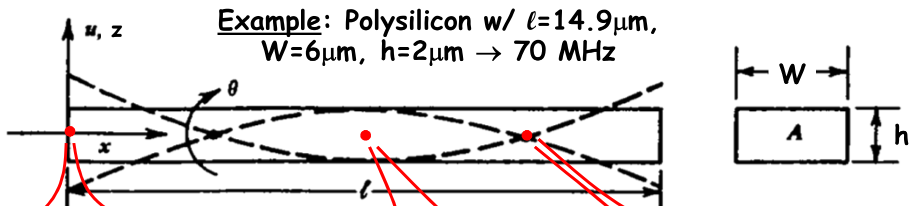
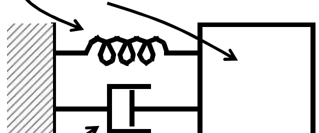
Example: Polysilicon w/ ℓ=14.9μm, W=6μm, h=2μm → 70 MHz



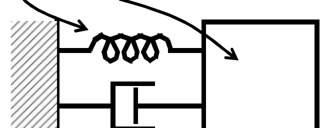
K_{eq}(node) = ∞
M_{eq}(node) = ∞
C_{eq}(node) = ∞

$K_{eq}(0)$ = 19,927 N/m
$M_{eq}(0)$ = 1.03x10^{-13} kg



$C_{eq}(0)$ = 5.66x10^{-9} kg/s

$K_{eq}(\ell/2)$ = 53,938 N/m
$M_{eq}(\ell/2)$ = 2.78x10^{-13} kg



$C_{eq}(\ell/2)$ = 1.53x10^{-8} kg/s

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