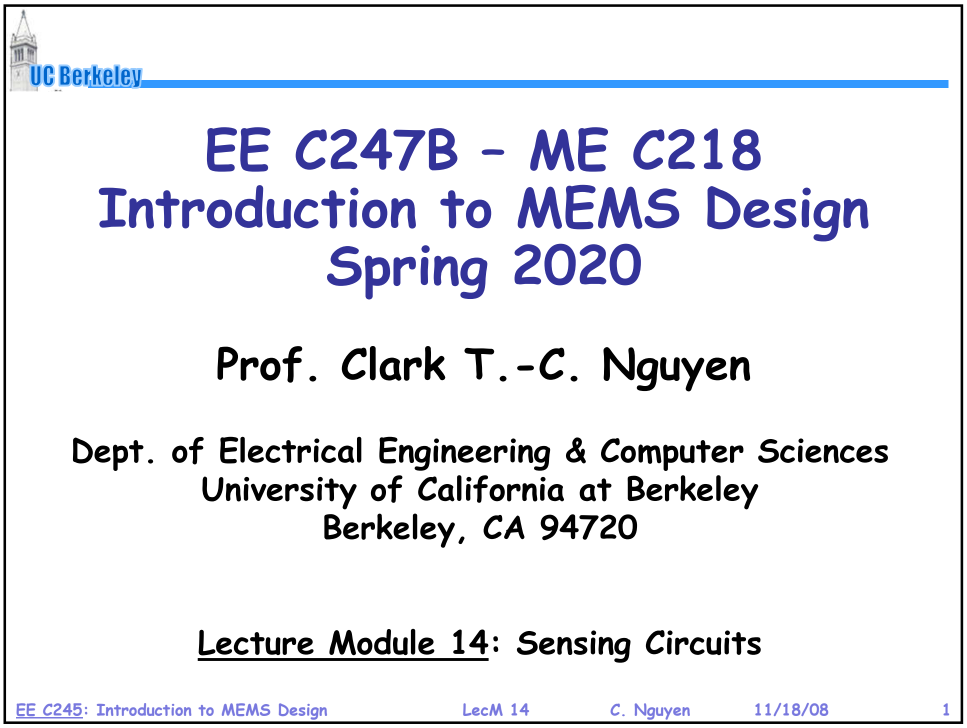

EE C247B – ME C218
Introduction to MEMS Design
Spring 2020

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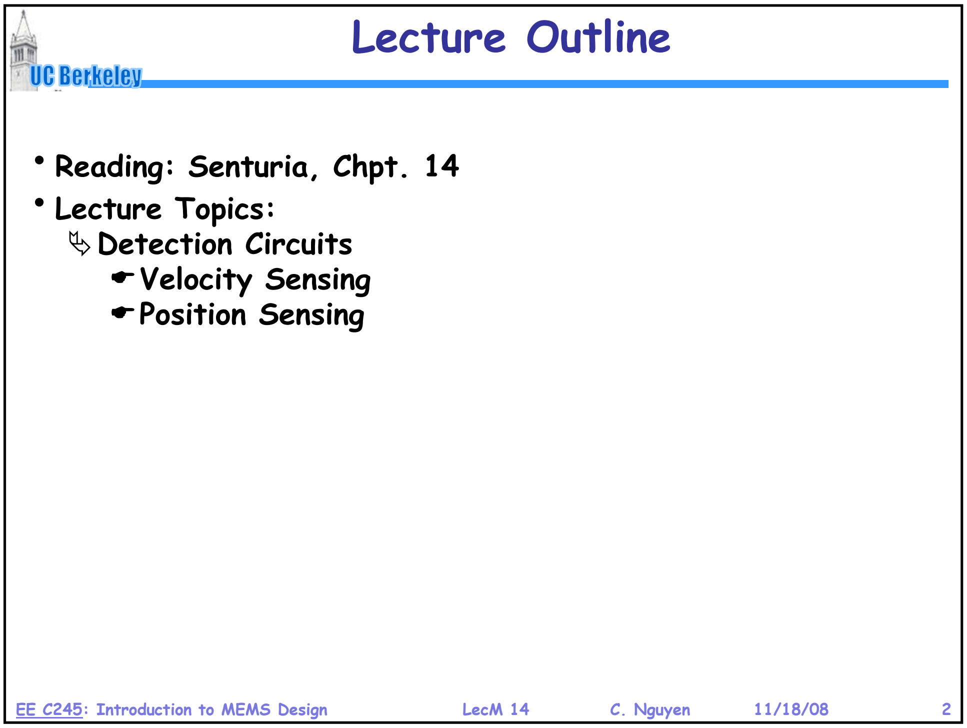
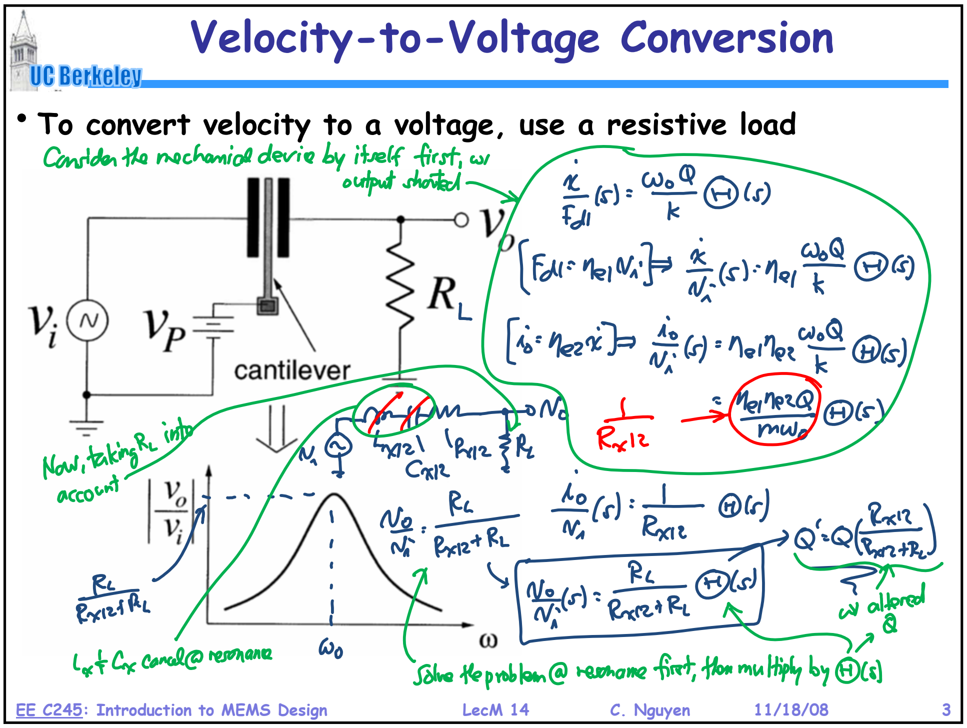
Berkeley, CA 94720

Lecture Module 14: Sensing Circuits



Lecture Outline

- Reading: Senturia, Chpt. 14
- Lecture Topics:
  - ➩ Detection Circuits
    - ☛ Velocity Sensing
    - ☛ Position Sensing



Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load

Consider the mechanical device by itself first, w/ output shorted

$$\frac{\dot{x}}{F_{d1}}(s) = \frac{\omega_0 Q}{k}\textcircled{H}(s)$$

$$\left[F_{d1} = \eta_{e1} v_i\right] \Rightarrow \frac{\dot{x}}{v_i}(s) = \eta_{e1}\frac{\omega_0 Q}{k}\textcircled{H}(s)$$

$$\left[i_o = \eta_{e2}\dot{x}\right] \Rightarrow \frac{i_o}{v_i}(s) = \eta_{e1}\eta_{e2}\frac{\omega_0 Q}{k}\textcircled{H}(s)$$

$$= \frac{\eta_{e1}\eta_{e2} Q}{m\omega_0}\textcircled{H}(s)$$

$$\frac{1}{R_{x12}}$$

Now, taking R_L into account

$$\frac{i_o}{v_i}(s) = \frac{1}{R_{x12}}\textcircled{H}(s)$$

$$\frac{v_o}{v_i} = \frac{R_L}{R_{x12}+R_L}$$

$$\boxed{\frac{v_o}{v_i}(s) = \frac{R_L}{R_{x12}+R_L}\textcircled{H}(s)}$$

$$Q' = Q\left(\frac{R_{x12}}{R_{x12}+R_L}\right)$$

w/ altered Q

$$\frac{R_L}{R_{x12}+R_L}$$

L_x & C_x cancel @ resonance

Solve the problem @ resonance first, then multiply by $\textcircled{H}(s)$

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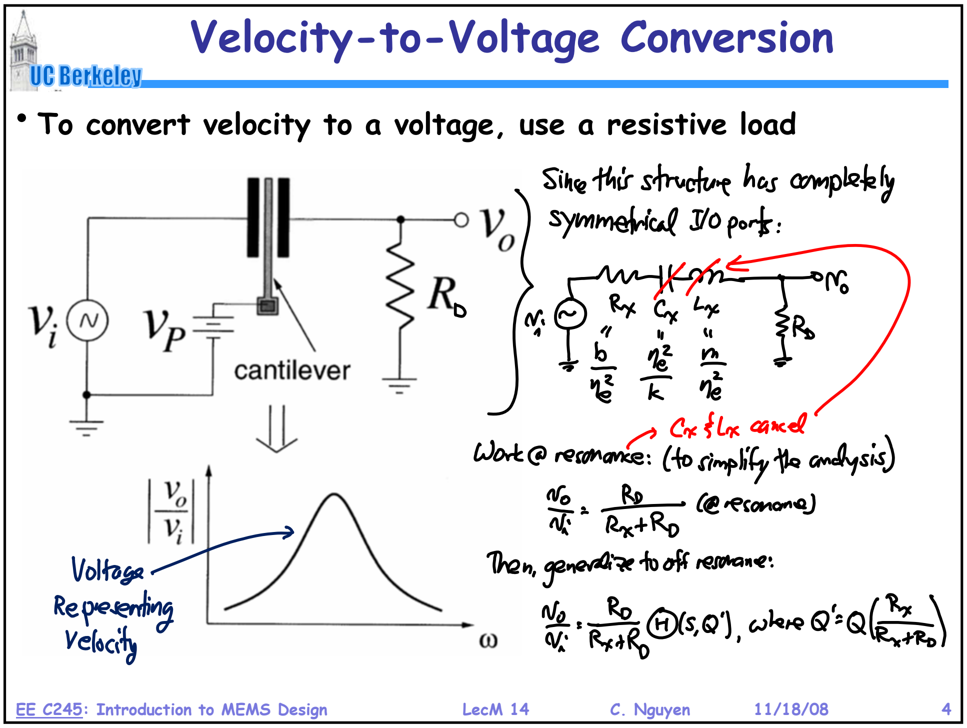
3



Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load

Since this structure has completely symmetrical I/O ports:

$$R_x = \frac{b}{\eta_e^2} \quad C_x = \frac{\eta_e^2}{k} \quad L_x = \frac{m}{\eta_e^2}$$

C_x & L_x cancel

Work @ resonance: (to simplify the analysis)

$$\frac{v_o}{v_i} = \frac{R_D}{R_x+R_D} \text{ (@ resonance)}$$

Then, generalize to off resonance:

$$\frac{v_o}{v_i} = \frac{R_D}{R_x+R_D}\textcircled{H}(s,Q'), \text{ where } Q' = Q\left(\frac{R_x}{R_x+R_D}\right)$$

Voltage Representing Velocity

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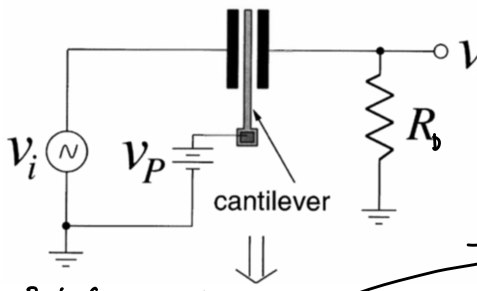
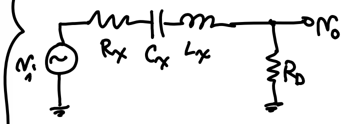
4

Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load



Since this structure has completely symmetrical I/O ports:



$$\left[Q = \frac{\omega_o L_x}{R_x} \rightarrow Q' = \frac{\omega_o L_x}{R_x + R_D} \rightarrow \frac{R_x + R_D}{L_x} = \frac{\omega_o}{Q'}\right]$$

$$\left[\frac{1}{L_x C_x} = \omega_o^2\right]$$

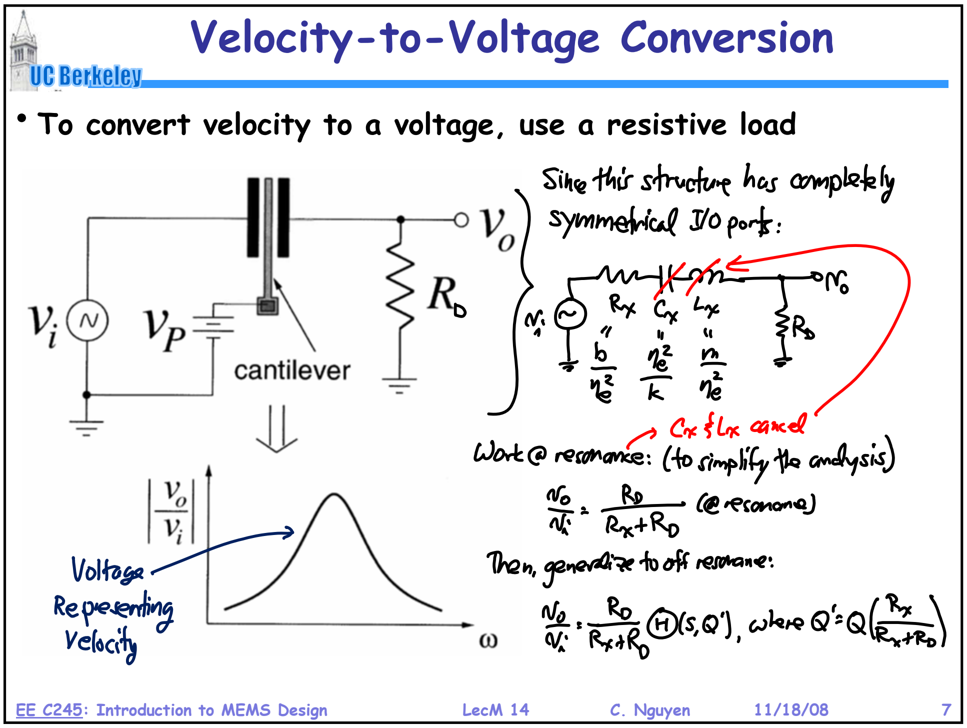
Brute force approach:

$$\frac{v_o}{v_i}(s) = \frac{R_D}{R_x + \frac{1}{sC_x} + sL_x + R_D} = \frac{sR_D C_x}{sR_x C_x + 1 + s^2 L_x C_x + sR_D C_x} = \frac{s\frac{R_D}{L_x}}{s^2 + s\frac{R_x + R_D}{L_x} + \frac{1}{L_x C_x}}$$

$$= \frac{R_D}{R_x + R_D} \frac{s\left(\frac{R_x + R_D}{L_x}\right)}{s^2 + s\left(\frac{R_x + R_D}{L_x}\right) + \frac{1}{L_x C_x}} \Rightarrow \boxed{\frac{v_o}{v_i}(s) = \frac{R_D}{R_x + R_D} \frac{s(\omega_o/Q')}{s^2 + s(\omega_o/Q') + \omega_o^2} = \frac{R_D}{R_x + R_D} Ⓗ(s, Q') \quad Q' = Q\left(\frac{R_x}{R_x + R_D}\right)}$$

✓

Velocity Sensing Circuits



Velocity-to-Voltage Conversion

- To convert velocity to a voltage, use a resistive load

Voltage Representing Velocity

Since this structure has completely symmetrical I/O ports:

C_x & L_x cancel

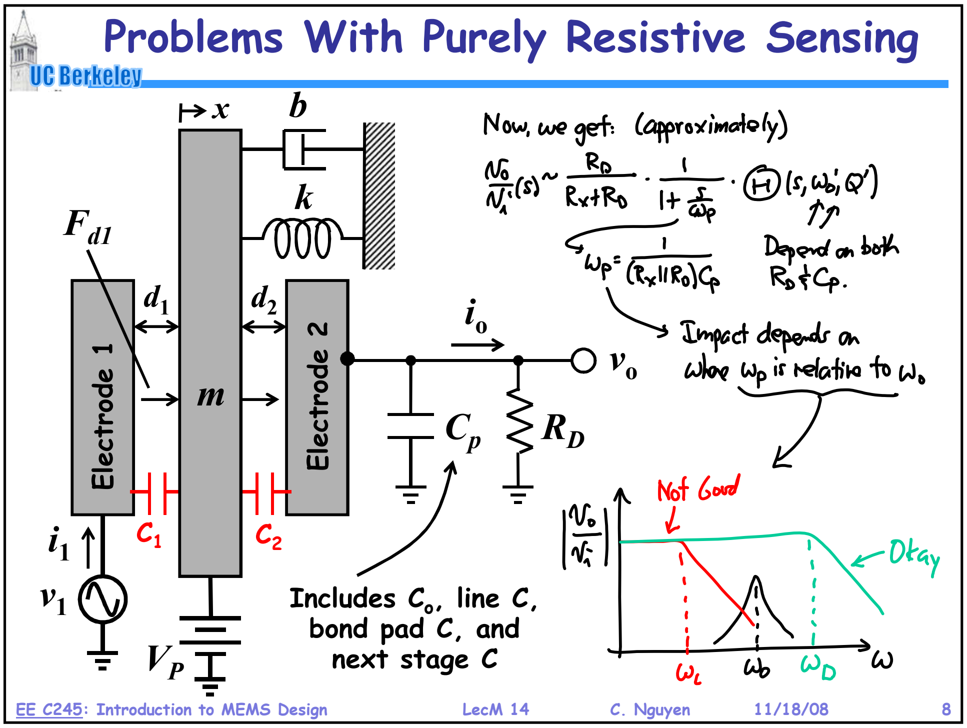
Work @ resonance: (to simplify the analysis)

$$\frac{v_o}{v_i} = \frac{R_D}{R_x+R_D} \quad (@\text{ resonance})$$

Then, generalize to off resonance:

$$\frac{v_o}{v_i} = \frac{R_D}{R_x+R_D}\,\text{Ⓗ}(s,Q'),\ \text{where } Q' = Q\left(\frac{R_x}{R_x+R_D}\right)$$

7



Problems With Purely Resistive Sensing

Includes C_o, line C, bond pad C, and next stage C

Now, we get: (approximately)

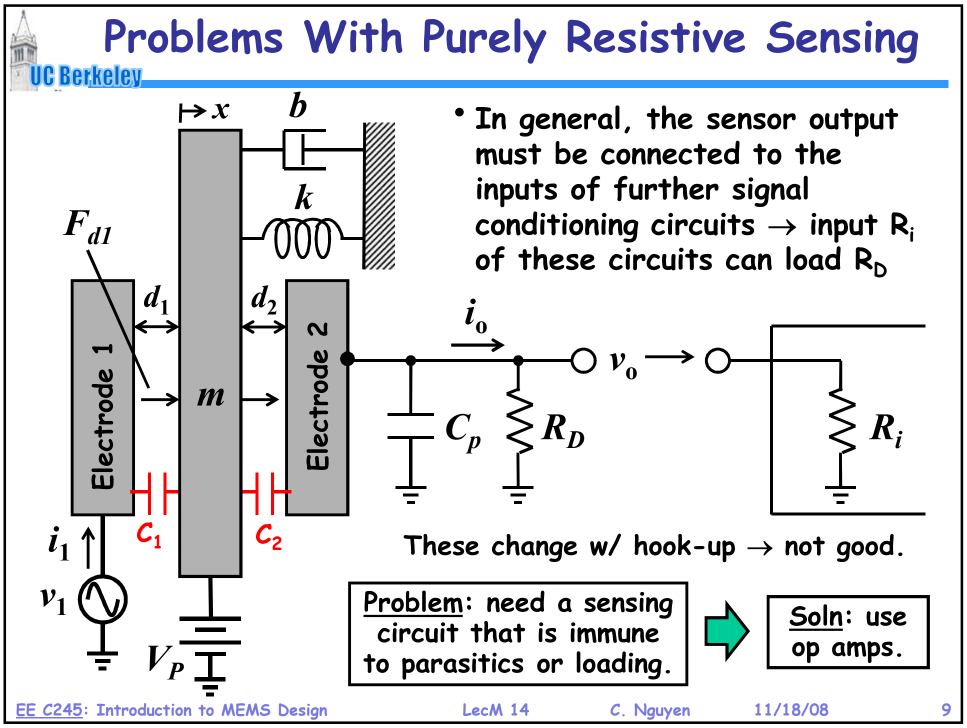
$$\frac{v_o}{v_i}(s) \sim \frac{R_D}{R_x+R_D}\cdot\frac{1}{1+\frac{s}{\omega_p}}\cdot\text{Ⓗ}(s,\omega_o',Q')$$

Depend on both R_D & C_p.

$$\omega_p = \frac{1}{(R_x \| R_D)C_p}$$

Impact depends on where ω_p is relative to ω_o

8



Problems With Purely Resistive Sensing

- In general, the sensor output must be connected to the inputs of further signal conditioning circuits → input R_i of these circuits can load R_D

These change w/ hook-up → not good.

Problem: need a sensing circuit that is immune to parasitics or loading.

Soln: use op amps.

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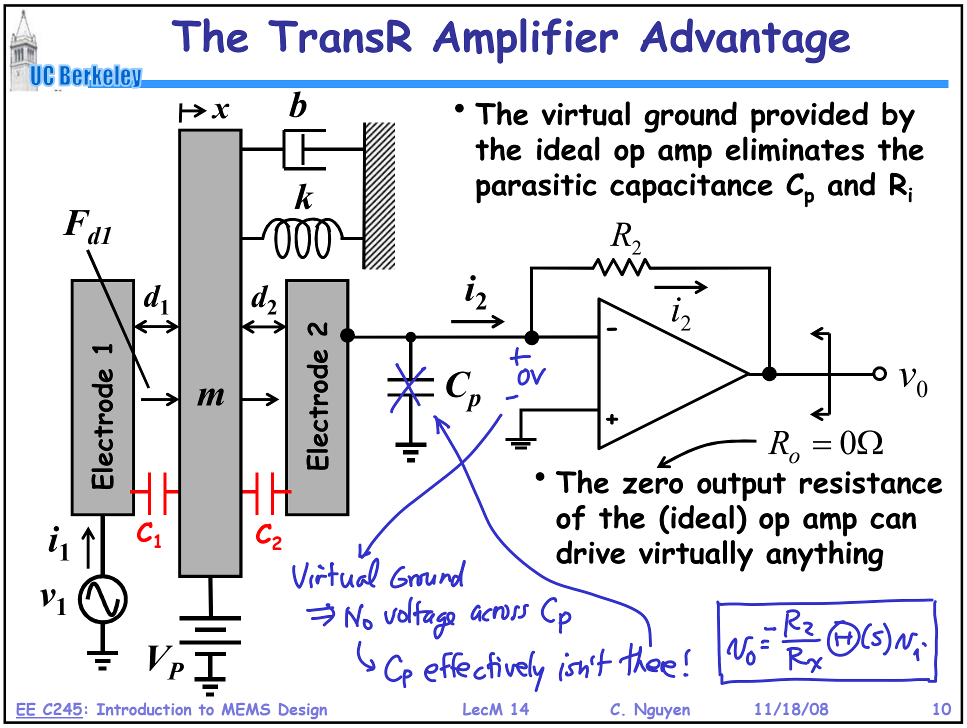

The TransR Amplifier Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p and R_i

$R_o = 0\Omega$

- The zero output resistance of the (ideal) op amp can drive virtually anything

Virtual Ground
⇒ No voltage across Cp
↳ Cp effectively isn't there!

$\boxed{v_0 = -\frac{R_2}{R_x} Ⓗ(s) v_i}$

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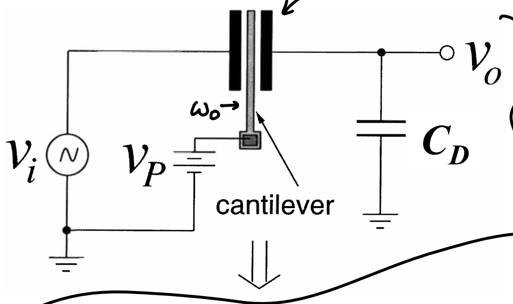
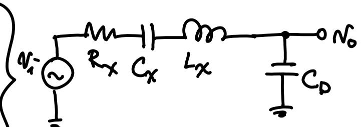
Position Sensing Circuits

Position-to-Voltage Conversion

- To sense position (i.e., displacement), use a capacitive load

Again, have port-to-port I/O symmetry:





Brute force approach:

$$\frac{v_o}{v_i}(s) = \frac{\frac{1}{sC_D}}{R_x + \frac{1}{sC_x} + sL_x + \frac{1}{sC_D}}$$

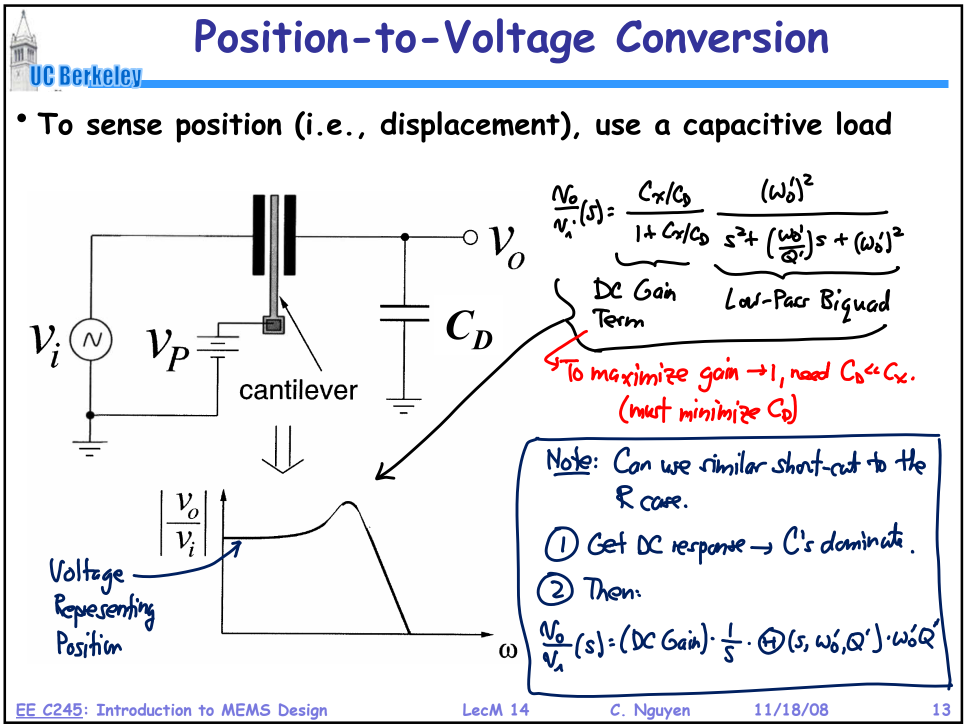
$$\frac{v_o}{v_i}(s) = \frac{\frac{\cancel{s}C_x}{\cancel{s}C_D}}{sR_xC_x + 1 + s^2L_xC_x + \frac{\cancel{s}C_x}{\cancel{s}C_D}} = \frac{C_x/C_D}{1+C_x/C_D}\,\frac{1}{1 + \frac{sR_xC_x}{1+C_x/C_D} + s^2\frac{L_xC_x}{1+C_x/C_D}}$$

$$= \frac{C_x/C_D}{1+C_x/C_D}\,\frac{\frac{1+C_x/C_D}{L_xC_x}}{s^2 + s\left(\frac{R_x}{L_x}\right) + \frac{(1+C_x/C_D)}{L_xC_x}}$$

$$\left[\omega_o^2 = \frac{1}{L_xC_x} \rightarrow (\omega_o')^2 = \omega_o^2(1+C_x/C_D)\right]$$

$$\left[Q' = \frac{\omega_o' L_x}{R_x} \rightarrow \frac{R_x}{L_x} = \frac{\omega_o'}{Q'},\ Q' = Q\sqrt{1+C_x/C_D}\right]$$

over



Position-to-Voltage Conversion

- To sense position (i.e., displacement), use a capacitive load

$$\frac{v_o}{v_i}(s) = \underbrace{\frac{C_x/C_D}{1+C_x/C_D}}_{\text{DC Gain Term}} \underbrace{\frac{(\omega_o')^2}{s^2+\left(\frac{\omega_o'}{Q'}\right)s+(\omega_o')^2}}_{\text{Low-Pass Biquad}}$$

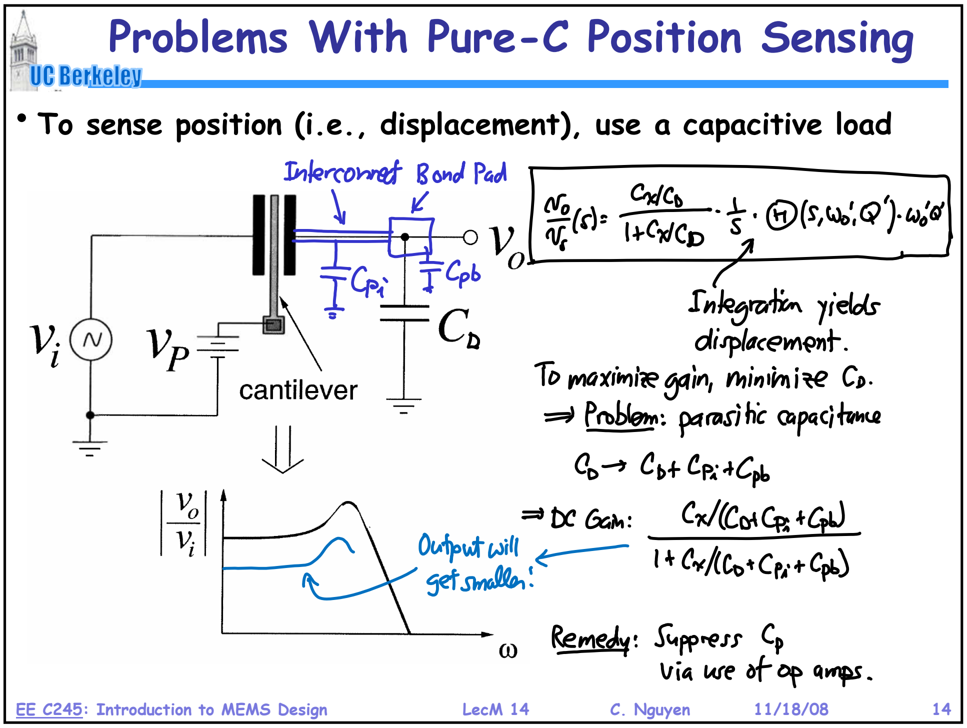
To maximize gain → 1, need $C_D \ll C_x$. (must minimize C_D)

Note: Can use similar short-cut to the R case.

① Get DC response → C's dominate.

② Then:

$$\frac{v_o}{v_i}(s) = (\text{DC Gain}) \cdot \frac{1}{s} \cdot Ⓗ(s, \omega_o', Q') \cdot \omega_o' Q'$$



Problems With Pure-C Position Sensing

- To sense position (i.e., displacement), use a capacitive load

$$\boxed{\frac{v_o}{v_s}(s) = \frac{C_x/C_D}{1+C_x/C_D} \cdot \frac{1}{s} \cdot Ⓗ(s, \omega_o', Q') \cdot \omega_o' Q'}$$

Integration yields displacement.

To maximize gain, minimize C_D.

⇒ Problem: parasitic capacitance

$$C_D \rightarrow C_D + C_{Pi} + C_{pb}$$

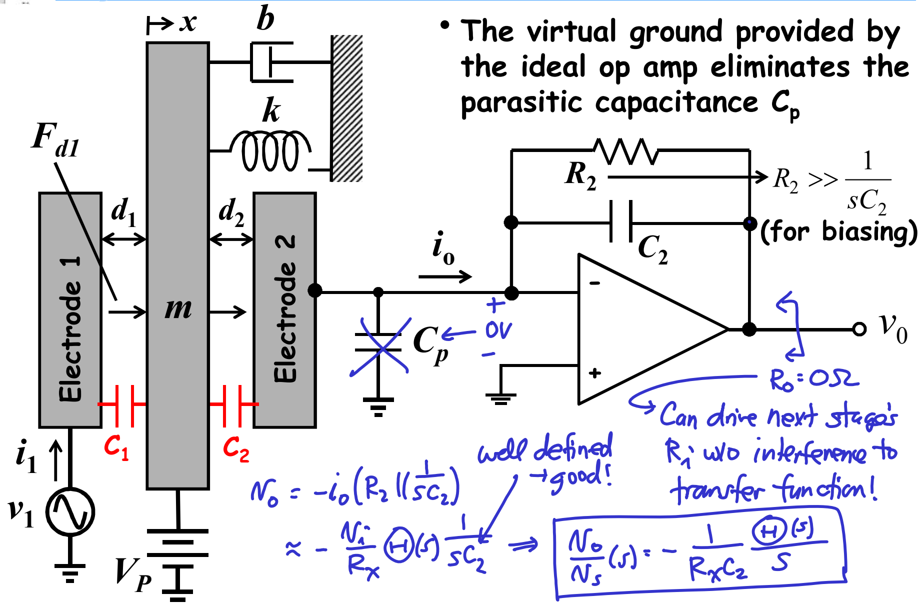
$$\Rightarrow \text{DC Gain: } \frac{C_x/(C_D+C_{Pi}+C_{pb})}{1+C_x/(C_D+C_{Pi}+C_{pb})}$$

Output will get smaller!

Remedy: Suppress C_p via use of op amps.

The Op Amp Integrator Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance C_p



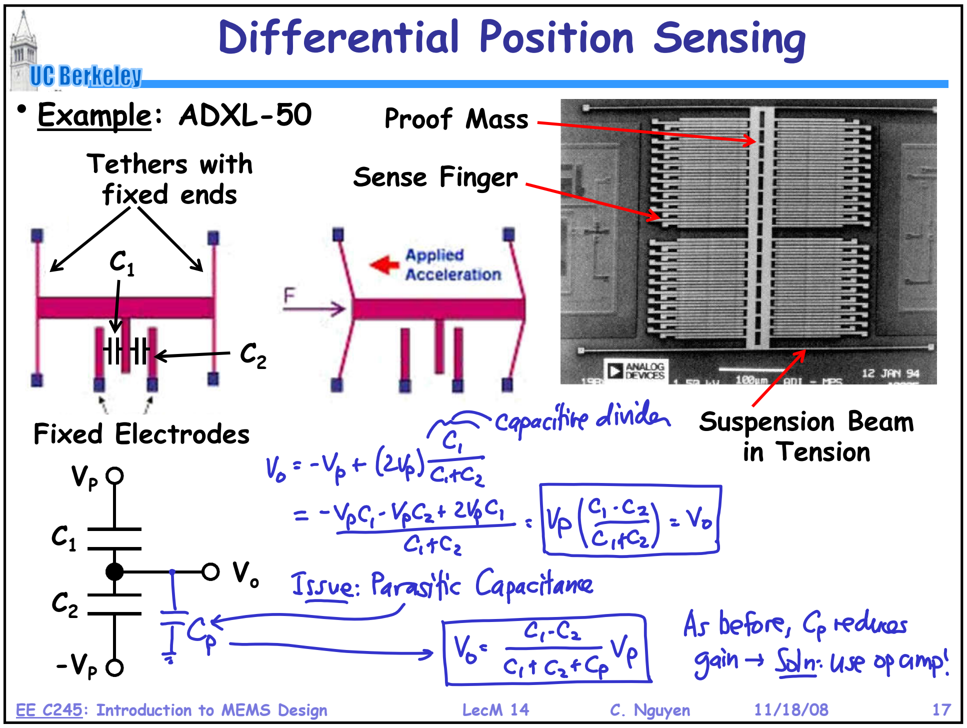
Can drive next stage's R_i w/o interference to transfer function!

$$v_o = -i_o\left(R_2 \,\|\, \frac{1}{sC_2}\right)$$

$$\approx -\frac{v_i}{R_x}\textcircled{H}(s)\frac{1}{sC_2} \Rightarrow \boxed{\frac{v_o}{v_s}(s) = -\frac{1}{R_xC_2}\frac{\textcircled{H}(s)}{s}}$$

well defined → good!

Differential Position Sensing



Differential Position Sensing

- **Example**: ADXL-50

Tethers with fixed ends

Proof Mass

Sense Finger

Fixed Electrodes

Suspension Beam in Tension

capacitive divider

$$V_o = -V_p + (2V_p)\frac{C_1}{C_1+C_2}$$

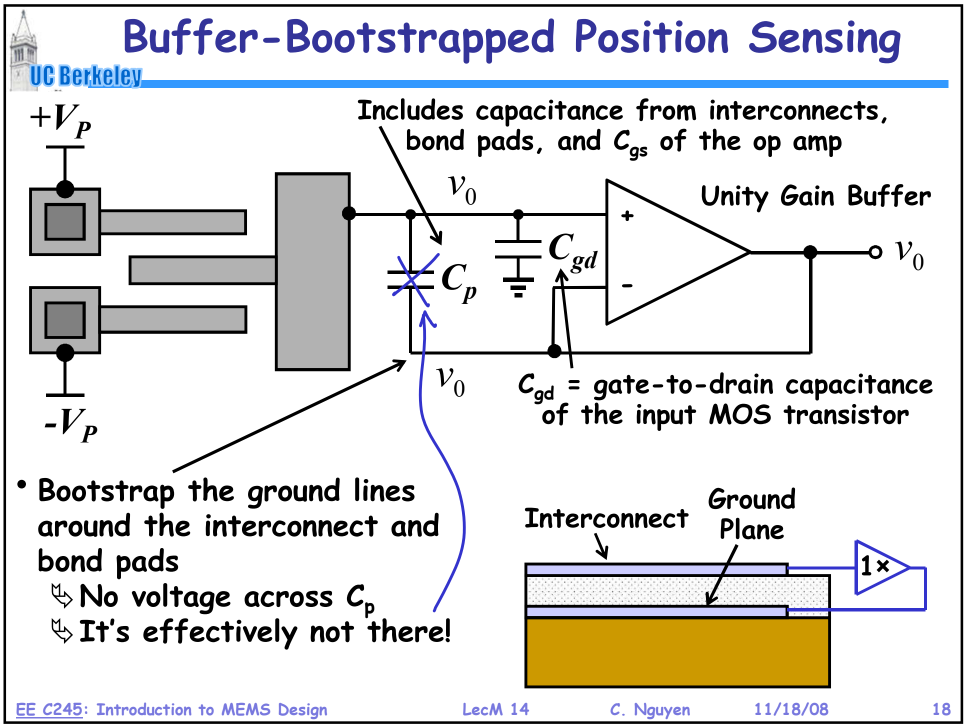
$$= \frac{-V_pC_1 - V_pC_2 + 2V_pC_1}{C_1+C_2} = \boxed{V_p\left(\frac{C_1 - C_2}{C_1+C_2}\right) = V_o}$$

Issue: Parasitic Capacitance

$$\boxed{V_o = \frac{C_1-C_2}{C_1+C_2+C_p}V_p}$$

As before, C_p reduces gain → Soln: use op amp!

17



Buffer-Bootstrapped Position Sensing

Includes capacitance from interconnects, bond pads, and C_{gs} of the op amp

Unity Gain Buffer

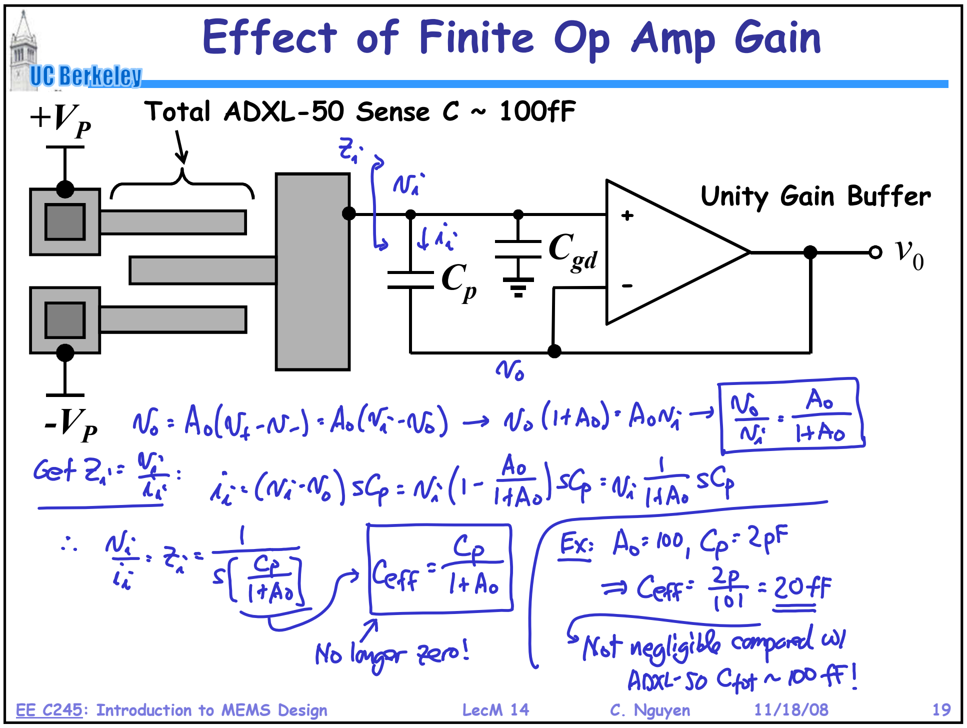
C_{gd} = gate-to-drain capacitance of the input MOS transistor

- Bootstrap the ground lines around the interconnect and bond pads
  - ↳ No voltage across C_p
  - ↳ It's effectively not there!

Interconnect

Ground Plane

18



Effect of Finite Op Amp Gain

Total ADXL-50 Sense C ~ 100fF

$v_o = A_o(v_+ - v_-) = A_o(v_i - v_o) \rightarrow v_o(1+A_o) = A_o v_i \rightarrow \boxed{\frac{v_o}{v_i} = \frac{A_o}{1+A_o}}$

Get $Z_i = \frac{v_i}{i_i}$: $i_i = (v_i - v_o)sC_p = v_i\left(1 - \frac{A_o}{1+A_o}\right)sC_p = v_i \frac{1}{1+A_o} sC_p$

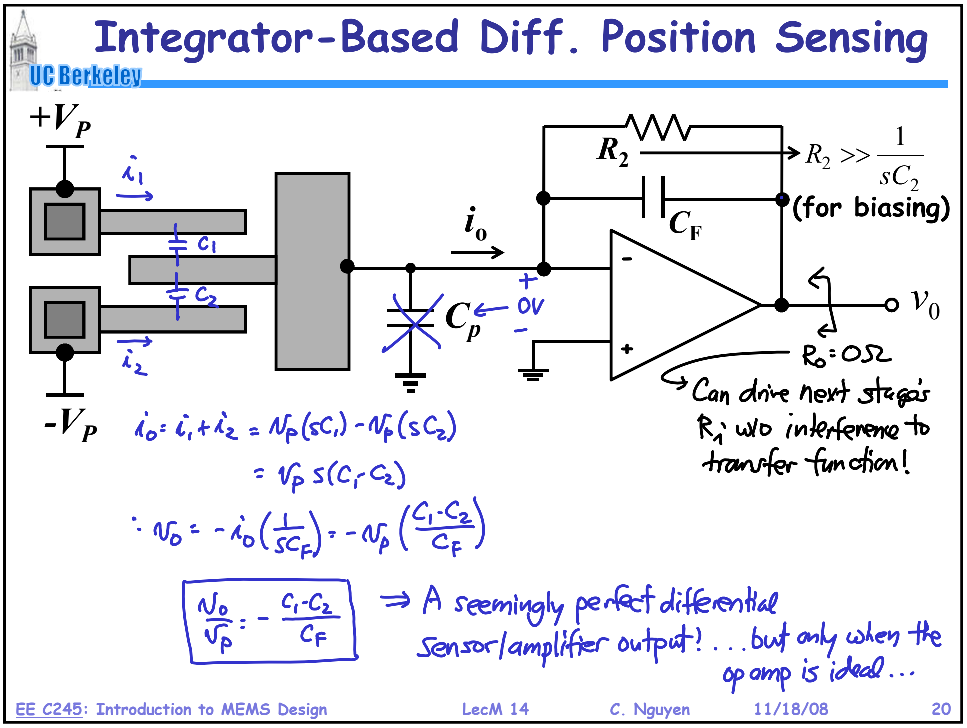
$\therefore \frac{v_i}{i_i} = Z_i = \frac{1}{s\left[\frac{C_p}{1+A_o}\right]}$ $\boxed{C_{eff} = \frac{C_p}{1+A_o}}$

No longer zero!

Ex: $A_o = 100$, $C_p = 2pF$

$\Rightarrow C_{eff} = \frac{2p}{101} = 20fF$

Not negligible compared w/ ADXL-50 $C_{tot} \sim 100fF$!



Integrator-Based Diff. Position Sensing

$R_2 >> \frac{1}{sC_2}$ (for biasing)

$R_o = 0\Omega$

Can drive next stage's R_i w/o interference to transfer function!

$i_o = i_1 + i_2 = v_p(sC_1) - v_p(sC_2)$

$= v_p s(C_1 - C_2)$

$\therefore v_o = -i_o\left(\frac{1}{sC_F}\right) = -v_p\left(\frac{C_1 - C_2}{C_F}\right)$

$\boxed{\frac{v_o}{v_p} = -\frac{C_1 - C_2}{C_F}}$ ⇒ A seemingly perfect differential sensor/amplifier output! ...but only when the op amp is ideal...