

EE C247B - ME C218
Introduction to MEMS Design
Spring 2020


Prof. Clark T.-C. Nguyen

Dept. of Electrical Engineering & Computer Sciences
University of California at Berkeley
Berkeley, CA 94720

Lecture Module 8: Microstructural Elements

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 1

1



Outline

- Reading: Senturia, Chpt. 9
- Lecture Topics:
 - ↔ Bending of beams
 - ↔ Cantilever beam under small deflections
 - ↔ Combining cantilevers in series and parallel
 - ↔ Folded suspensions
 - ↔ Design implications of residual stress and stress gradients

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 2

2

UC Berkeley

Bending of Beams

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 3

3

UC Berkeley

Beams: The Springs of Most MEMS

- Springs and suspensions very common in MEMS
 - ↳ Coils are popular in the macro-world; but not easy to make in the micro-world
 - ↳ Beams: simpler to fabricate and analyze; become "stronger" on the micro-scale → use beams for MEMS

Comb-Driven Folded Beam Actuator

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 4

4

Bending a Cantilever Beam

Clamped end condition:
 At $x=0$:
 $y=0$
 $dy/dx = 0$

Free end condition

Objective: Find relation between tip deflection $y(x=L_c)$ and applied load F

Assumptions:

1. Tip deflection is small compared with beam length
2. Plane sections (normal to beam's axis) remain plane and normal during bending, i.e., "pure bending"
3. Shear stresses are negligible

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 5

5

Reaction Forces and Moments

Reaction Moment $M_R = M_1$

Reaction Force $F_R = F$

Point Load F

Moment due to F, here:
 $M_1 = FL$
 $M_2 = F(L-x)$

split

Reactions (Senturia gives expressions)

For equilibrium:
 $M_{x,r} = M_3 = F(L-x)$
 $V_{x,r} = F$

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 6

6

Sign Conventions for Moments & Shear Forces

UC Berkeley

Positive **Negative**

Moment

(+) moment leads to deformation with a (+) radius of curvature (i.e., upwards)

(-) moment leads to deformation with a (-) radius of curvature (i.e., downwards)

Shear

(+) shear forces produce clockwise rotation

(-) shear forces produce counter-clockwise rotation

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 7

7

Beam Segment in Pure Bending

UC Berkeley

Portions above the neutral axis go into Tension

Portions below the neutral axis go into compression

Neutral Axis → Length unchanged by bending

Applied Moment

Note: (+) direction of z is downward

Consider a segment bounded by the dashed lines defined by $d\theta$:

At $z=0$: (i.e., at the neutral axis): segment length = $dx = R d\theta$ (1)

At any z : segment length = $dL = (R-z)d\theta$ (2)

Combining (1) & (2): $dL = dx - z d\theta = dx - \frac{z}{R} dx$

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 8

8

Beam Segment in Pure Bending (cont.)

UC Berkeley

Thus, the axial strain @ z : $\epsilon_x = \frac{dL - dx}{dx} = -\frac{z}{R} \Rightarrow \boxed{\epsilon_x = -\frac{z}{R}}$
 [$dx = \text{Original (unstressed) segment length}$]

Thus, strain varies linearly along beam thickness, and has 0 maximum value
 $\epsilon_{x,max} = \frac{h/2}{R}$

Of course, there is a corresponding axial stress:
 $\sigma_x = \epsilon_x E = \boxed{-\frac{zE}{R} = \sigma_x}$

This gradient in stress then generates a bending moment... *in response to the applied moment!*

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 9

9

Internal Bending Moment

UC Berkeley

Moment around this point

Small section of a beam bent in response to a transverse load

To get the bending moment:
 \Rightarrow integrate stress through the thickness of the beam

$$M = \int_{-h/2}^{h/2} [(Wdz)\sigma_x] \cdot z = - \int_{-h/2}^{h/2} \frac{EWz^2}{R} dz \Rightarrow \boxed{M = -\left(\frac{1}{12}Wh^3\right) \frac{E}{R}}$$

force $\left[\sigma_x = -\frac{zE}{R}\right]$ $\frac{1}{12}Wh^3 = I \hat{=} \text{Moment of Inertia}$

$\frac{1}{R} = -\frac{M}{EI}$ Note: (+) radius of curvature
 \rightarrow (-) internal bending moment!

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 10

10

Differential Beam Bending Equation

Write out geometric relationships: Small Angle Approx.

$$\cos\theta = \frac{dx}{ds} \rightarrow ds = \frac{dx}{\cos\theta} \rightarrow ds \approx dx$$

$$\tan\theta = \frac{dw}{dx} = \text{slope of beam @ any point} \rightarrow \theta \approx \frac{dw}{dx} \quad (1)$$

$$ds = R d\theta \rightarrow \frac{1}{R} = \frac{d\theta}{ds} \rightarrow \frac{1}{R} = \frac{d^2w}{dx^2} \quad (2)$$

Inserting (1) in (2): $\frac{1}{R} = \frac{d^2w}{dx^2} = -\frac{M}{EI}$ Differential Equation for Small Angle Bending of Beams

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 11

11

Example: Cantilever Beam w/ a Concentrated Load

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 12

12

Cantilever Beam w/ a Concentrated Load
 UC Berkeley

Clamped end condition:
 At $x=0$:
 $w=0$
 $dw/dx = 0$

Free end condition

Internal Moment @ position x : $M = -F(L-x)$

Thus: $\frac{d^2w}{dx^2} = \frac{F}{EI}(L-x)$, w/ $\begin{cases} \text{Clamped End B.C.'s: } w(x=0)=0, \frac{dw}{dx}(x=0)=0 \\ \text{Free End B.C.'s: none} \end{cases}$

Solve to get expression for w :
 \Rightarrow use Laplace; or use trial solution $w = A + Bx + Cx^2 + Dx^3$, then apply B.C.'s

$$w = \frac{FL}{2EI} x^2 \left(1 - \frac{x}{3L}\right) \quad \left[\begin{array}{l} \text{Deflection @ } x \text{ due to a point load} \\ \text{F applied at } x=L \end{array} \right]$$

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 13

13

Cantilever Beam w/ a Concentrated Load
 UC Berkeley

Clamped end condition:
 At $x=0$:
 $w=0$
 $dw/dx = 0$

Free end condition

Maximum deflection @ $x=L$:
 $w_{max} = \left(\frac{L^3}{3EI}\right)F \rightarrow F = \left(\frac{3EI}{L^3}\right)w(x=L) = k_c w(x=L)$

Where $k_c = \frac{3EI}{L^3} \hat{=} \text{stiffness @ location } x=L$

Note that in general, stiffness is a function of location x .

Ex. $L=100\mu\text{m}$, $w=2\mu\text{m}$, $h=2\mu\text{m}$
 polysilicon $\rightarrow E=150\text{GPa}$
 $k_c = \frac{1}{4}(150\text{G})(2\mu)\left(\frac{2\mu}{100\mu}\right)^3 = 0.6\text{ N/m}$

$I = \frac{1}{12}Wh^3 \Rightarrow k_c = \frac{1}{4}EW\frac{h^3}{L^3}$

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 14

14


UC Berkeley

Maximum Stress in a Bent Cantilever

From before, the radius of curvature is given by

$$\frac{1}{R} = \frac{d^2w}{dx^2} = \frac{F}{EI}(L-x) \Rightarrow \text{maximized where } R \rightarrow 0$$

↳ occurs at the support, where $x=0$:

$$\text{at } [x=0] \Rightarrow \frac{1}{R} = \frac{d^2w}{dx^2} = \frac{FL}{EI}$$


Strain is maximized:

- ① At top surface → tensile
- ② At bottom surface → compressive

$$\epsilon_{max} = -\frac{z}{R} = \frac{h}{2} \frac{1}{R} = \frac{h}{2} \frac{FL}{EI}$$

$$[I = \frac{1}{12}Wh^3] \Rightarrow \epsilon_{max} = \frac{K}{2} \frac{FL}{E} \frac{12}{Wh^3} = \frac{6L}{EWh^2} F$$

$\therefore \sigma_{max} = \epsilon_{max} E = \frac{6L}{Wh^2} F$

(Maximum Stress in a Bent Cantilever)

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 15

15

UC Berkeley

Stress Gradients in Cantilevers

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 16

16

Vertical Stress Gradients

UC Berkeley

- Variation of residual stress in the direction of film growth
- Can warp released structures in z-direction

200µM 20KV 00 017 S

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 17

17

Stress Gradients in Cantilevers

UC Berkeley

- **Below:** surface micromachined cantilever deposited at a high temperature then cooled → assume compressive stress

Before release After release, but before bending After bending

Average stress

σ_0

σ_x

Compression

Tension

Compression

Tension

Compression

After which, stress is relieved

Stress before release

Stress after release, but before bending

After bending

But stress gradient remains → induces moment that bends beam

Once released, beam length increases slightly to relieve average stress

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 18

18

Stress Gradients in Cantilevers (cont)

UC Berkeley

Find the radius of curvature.

Prior to release, axial stress is: $\sigma = \sigma_0 - \frac{\sigma_1}{(H/2)} z$

The internal moment:

$$M_x = \int_{-H/2}^{H/2} [(W dz) \sigma] \cdot z = W \int_{-H/2}^{H/2} \left(z \sigma_0 - \frac{\sigma_1 z^2}{(H/2)} \right) dz = W \left(\frac{1}{2} \sigma_0 z^2 - \frac{2 \sigma_1 z^3}{3H} \right) \Big|_{-H/2}^{H/2}$$

$$= W \left(\frac{1}{2} \sigma_0 \frac{H^2}{4} - \frac{2}{3} \sigma_1 \frac{H^2}{8} - \frac{1}{2} \sigma_0 \frac{H^2}{4} + \frac{2 \sigma_1 H^2}{3(8)} \right) = -\frac{1}{6} \sigma_1 W H^2 = M_x$$

Thus, the radius curvature is

$$\frac{1}{R} = -\frac{M_x}{EI} \rightarrow R = -\frac{EI}{M_x} = -\frac{E' \left(\frac{1}{12} W H^3 \right)}{-\frac{1}{6} \sigma_1 W H^2} = \frac{1}{2} \frac{E H}{\sigma_1}$$

↑
Biaxial Stress Gradient $[I = \frac{1}{12} W h^3]$

$R = \frac{1}{2} \frac{E}{(1-\nu)} \frac{H}{\sigma_1}$

[Radius of Curvature for a Cantilever w/ Stress Gradient]

$\sigma_1 = \frac{1}{2} \frac{E}{(1-\nu)} \frac{H}{R}$

⇒ R can be used to determine stress gradient

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 19

19

Measurement of Stress Gradient

UC Berkeley

- Use cantilever beams
 - ↪ Strain gradient (Γ = slope of strain-thickness curve) causes beams to deflect up or down
 - ↪ Assuming linear strain gradient Γ , $z = \Gamma L^2 / 2$

compressive

 tensile

[P. Krulevitch Ph.D.]

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 20

20

UC Berkeley

Folded-Flexure Suspensions

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 21

21

UC Berkeley

Folded-Beam Suspension

- Use of folded-beam suspension brings many benefits
 - ↳ Stress relief: folding truss is free to move in y-direction, so beams can expand and contract more readily to relieve stress
 - ↳ High y-axis to x-axis stiffness ratio

Comb-Driven Folded Beam Actuator

Folding Truss

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 22

22

Beam End Conditions

UC Berkeley

TABLE 4.1
Types of commonly used support conditions for beams and frames

Type of support	Displacement boundary conditions	Force boundary conditions
 FREE	None	All, as specified
 PINNED	$u = 0$ $w = 0$	Moment is specified
 ROLLER (vertical)	$u = 0$	Transverse force and moment are specified
 ROLLER (horizontal)	$w = 0$	Horizontal force and bending moment are specified
 FIXED or CLAMPED	$u = 0$ $w = 0$ $dw/dx = 0$	None specified

[From Reddy, Finite Element Method]

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 23

23

Common Loading & Boundary Conditions

UC Berkeley

- Displacement equations derived for various beams with concentrated load F or distributed load f
- Gary Fedder Ph.D. Thesis, EECS, UC Berkeley, 1994

cantilever	guided-end	fixed-fixed
$x = \frac{F_x L}{Ehw}$	$x = \frac{F_x L}{Ehw}$	$x = \frac{F_x L}{4Ehw}$
$y = 4 \frac{F_y L^3}{Eh w^3}$	$y = \frac{F_y L^3}{Eh w^3}$	$y = \frac{1}{16} \frac{F_y L^3}{Eh w^3}$
$z = 4 \frac{F_z L^3}{Ew h^3}$	$z = \frac{F_z L^3}{Ew h^3}$	$z = \frac{1}{16} \frac{F_z L^3}{Ew h^3}$

(a) Concentrated load.

cantilever	guided-end	fixed-fixed
$x = \frac{f_x L}{E}$	$x = \frac{f_x L}{E}$	$x = \frac{f_x L}{4E}$
$y = \frac{3}{2} \frac{f_y L^4}{Eh w^3}$	$y = \frac{1}{2} \frac{f_y L^4}{Eh w^3}$	$y = \frac{1}{32} \frac{f_y L^4}{Eh w^3}$
$z = \frac{3}{2} \frac{f_z L^4}{Ew h^3}$	$z = \frac{1}{2} \frac{f_z L^4}{Ew h^3}$	$z = \frac{1}{32} \frac{f_z L^4}{Ew h^3}$

(b) Distributed load.

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 24

24

Series Combinations of Springs

UC Berkeley

- For springs in series w/ one load
 - ↳ Deflections add
 - ↳ Spring constants combine like "resistors in parallel"

$$y(L) = F/k = 2 y(L_c) = 2 (F/k_c) = F(1/k_c + 1/k_c)$$

Compliances effectively add:

$1/k = 1/k_c + 1/k_c \rightarrow k = k_c || k_c$

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 25

25

Parallel Combinations of Springs

UC Berkeley

- For springs in parallel w/ one load
 - ↳ Load is shared between the two springs
 - ↳ Spring constant is the sum of the individual spring constants

$$y(L) = F/k = F_a/k_a = F_b/k_b = (F/2) (1/k_a)$$

$k = 2 k_a$

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 26

26

Folded-Flexure Suspension Variants

UC Berkeley

- Below: just a subset of the different versions
- All can be analyzed in a similar fashion

(a) Inner fold, continuous truss

(b) Inner fold, discontinuous truss

(c) Outer fold, continuous truss

(d) Outer fold, discontinuous truss

[From Michael Judy, Ph.D. Thesis, EECS, UC Berkeley, 1994]

EE C245: Introduction to MEMS Design
LecM 8
C. Nguyen
9/28/07
27

27

Deflection of Folded Flexures

UC Berkeley

This equivalent to two cantilevers of length $L_c = L/2$
 Composite cantilever free ends attach here
 $L_c = \frac{L}{2}$
 $\frac{F}{4} = F_c$
 Half of F absorbed in other half (symmetrical)
 4 sets of these pairs, each of which gets $\frac{1}{4}$ of the total force F

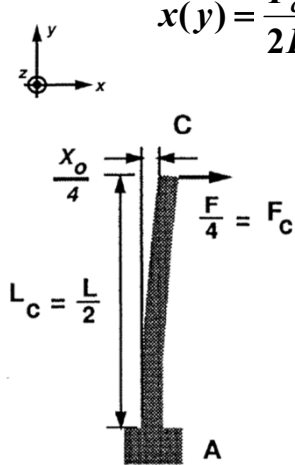
EE C245: Introduction to MEMS Design
LecM 8
C. Nguyen
9/28/07
28

28

Constituent Cantilever Spring Constant

UC Berkeley

- From our previous analysis:

$$x(y) = \frac{F_c L_c}{2EI_z} y^2 \left(1 - \frac{y}{3L_c} \right) = \frac{F_c y^2}{6EI_z} (3L_c - y)$$


- From which the spring constant is:

$$k_c = \frac{F_c}{x(L_c)} = \frac{3EI_z}{L_c^3}$$

- Inserting $L_c = L/2$

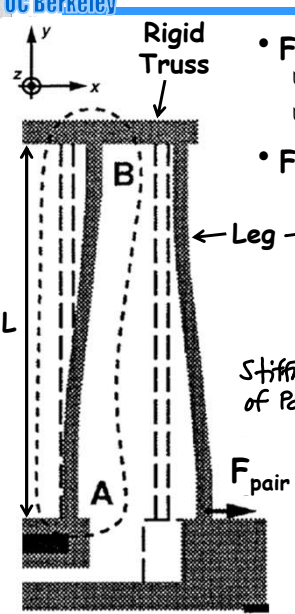
$$k_c = \frac{3EI_z}{(L/2)^3} = \frac{24EI_z}{L^3}$$

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 29

29

Overall Spring Constant

UC Berkeley



- Four pairs of clamped-guided beams
 - In each pair, beams bend in series
 - (Assume trusses are inflexible)
- Force is shared by each pair $\rightarrow F_{\text{pair}} = F/4$

total force on shuttle

Leg \rightarrow Displacement of two legs add
 \hookrightarrow thus, springs are in series:

$$x = \frac{F_{\text{pair}}}{k_{\text{pair}}} = \frac{F_{\text{pair}}}{(k_{\text{leg}} \parallel k_{\text{leg}})} = \left(\frac{F}{4} \right) \left(\frac{1}{k_{\text{leg}}} + \frac{1}{k_{\text{leg}}} \right)$$

Stiffness of Pair \rightarrow From before: $k_{\text{leg}} = k_c \parallel k_c = \frac{k_c}{2}$

Thus:

$$x = \left(\frac{F}{4} \right) \left(\frac{2}{k_c} + \frac{2}{k_c} \right) = \frac{F}{k_c} = \frac{F}{k_{\text{tot}}}$$

$$\Rightarrow k_{\text{tot}} = k_c = \frac{24EI_z}{L^3}$$

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 30

30

Folded-Beam Stiffness Ratios

UC Berkeley

- In the x-direction:

$$k_x = \frac{24EI_z}{L^3}$$
- In the z-direction:
 - ↳ Same flexure and boundary conditions
$$k_z = \frac{24EI_x}{L^3}$$
- In the y-direction:
 - [See Senturia, §9.2] $k_y = \frac{8EWh}{L}$
- Thus:

$\frac{k_y}{k_x} = 4 \left(\frac{L}{W} \right)^2$

 Much stiffer in y-direction!

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 31

31

Folded-Beam Suspensions Permeate MEMS


UC Berkeley

Accelerometer [ADXL-05, Analog Devices] Gyroscope [Draper Labs.]

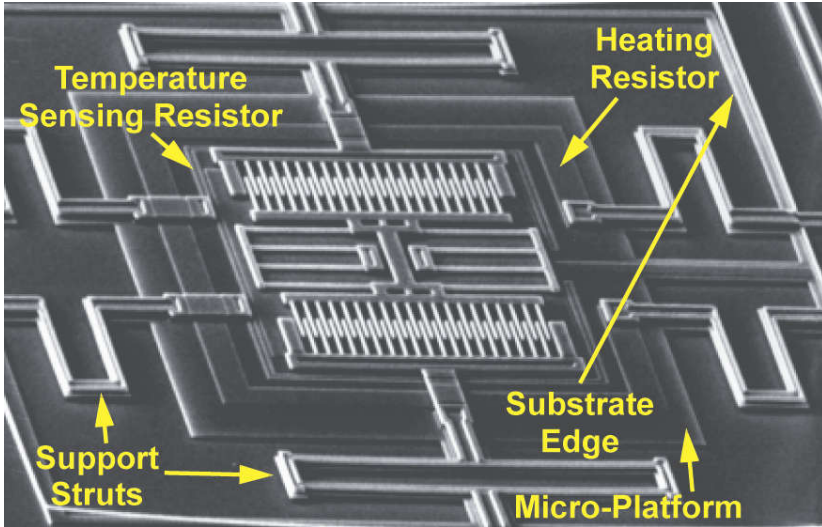
Micromechanical Filter [K. Wang, Univ. of Michigan]

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 32

32

 **Folded-Beam Suspensions Permeate MEMS**

- Below: Micro-Oven Controlled Folded-Beam Resonator



EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 33

33

 **Stressed Folded-Flexures**

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 34

34

Clamped-Guided Beam Under Axial Load
 UC Berkeley

- Important case for MEMS suspensions, since the thin films comprising them are often under residual stress
- Consider small deflection case: $y(x) \ll L$

Governing differential equation: (Euler Beam Equation)

$$EI_z \frac{d^4 y}{dx^4} - S \frac{d^2 y}{dx^2} = F \delta(x-L)$$

Axial Load
Unit impulse @ $x=L$

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 35

35

The Euler Beam Equation
 UC Berkeley

- Axial stresses produce no net horizontal force; but as soon as the beam is bent, there is a net downward force
 - ↳ For equilibrium, must postulate some kind of upward load on the beam to counteract the axial stress-derived force
 - ↳ For ease of analysis, assume the beam is bent to angle π

Downward Vertical Force = $2\sigma_0 W H$
 Upward Force due to P_0 :

$$F_u = \int_0^\pi (P_0 \sin \theta) W (R d\theta) = -P_0 W R \cos \theta \Big|_0^\pi = 2R W P_0$$

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 36

36

The Euler Beam Equation

UC Berkeley

[Equilibrium] $\Rightarrow 2RWp_0 = 2\sigma_0 WH \rightarrow p_0 = \frac{\sigma_0 H}{R}$

$\left[q_0 = \frac{\text{beam load}}{\text{unit length}} = p_0 W, \frac{1}{R} = \frac{d^2 w}{dx^2} \right] \Rightarrow q_0 = \sigma_0 WH \frac{d^2 w}{dx^2}$

beam displacement

Using the differential beam bending equation:

$$\frac{d^2 w}{dx^2} = -\frac{M}{EI} \rightarrow \frac{d^4 w}{dx^4} = \frac{q}{EI} \leftarrow \frac{\text{load}}{\text{unit length}}$$

external load

$$EI \frac{d^4 w}{dx^4} = q + q_0 \leftarrow \text{equiv. load accounting for the axial stress contribution to the bending stiffness}$$

$\left[q_0 = \sigma_0 WH \frac{d^2 w}{dx^2} \right] \Rightarrow EI \frac{d^4 w}{dx^4} - (\sigma_0 WH) \frac{d^2 w}{dx^2} = q$ [Euler Beam Equation]

\hookrightarrow tension in the beam = $S \leftarrow$ a force

Note: Use of the full bend angle of π to establish conditions for load balance; but this returns us to case of small displacements and small angles

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 37

37

Clamped-Guided Beam Under Axial Load

UC Berkeley

- Important case for MEMS suspensions, since the thin films comprising them are often under residual stress
- Consider small deflection case: $y(x) \ll L$

Governing differential equation: (Euler Beam Equation)

$$EI_z \frac{d^4 y}{dx^4} - S \frac{d^2 y}{dx^2} = F \delta(x-L)$$

Axial Load Unit impulse @ $x=L$

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 38

38

Solving the ODE

UC Berkeley

- Can solve the ODE using standard methods
 - ↪ Senturia, pp. 232-235: solves ODE for case of point load on a clamped-clamped beam (which defines B.C.'s)
 - ↪ For solution to the clamped-guided case: see S. Timoshenko, Strength of Materials II: Advanced Theory and Problems, McGraw-Hill, New York, 3rd Ed., 1955
- Result from Timoshenko:

$$S > 0 \text{ (tension)} \quad k^{-1} = \frac{pL - 2 \tanh(pL/2)}{p|S|} = \frac{y(x=L)}{F}$$

$$S < 0 \text{ (compression)} \quad k^{-1} = \frac{-pL + 2 \tan(pL/2)}{p|S|} = \frac{y(x=L)}{F}$$
 where $p = \sqrt{\frac{|S|}{EI}}$

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 39

39

Design Implications

UC Berkeley


- Straight flexures
 - ↪ Large tensile S means flexure behaves like a tensioned wire (for which $k^{-1} = L/S$)
 - ↪ Large compressive S can lead to buckling ($k^{-1} \rightarrow \infty$)
- Folded flexures
 - ↪ Residual stress only partially released
 - ↪ Length from truss to shuttle's centerline differs by L_s for inner and outer legs

③ Beam strain:
 $\epsilon_b = \frac{\Delta L}{2L} = \frac{\Delta L_s}{2L} = \epsilon_{r-2L}$
over

① If polysil strain is ϵ_r , then shuttle expands by $\Delta L_s = \epsilon_r L_s$
 ② This then applies a load to the beams, also $\Delta L = \Delta L_s$.

EE C245: Introduction to MEMS Design LecM 8 C. Nguyen 9/28/07 40

40



Effect on Spring Constant

- Residual compression on outer legs with same magnitude of tension on inner legs: strain in the polysi

Beam Strain: $\epsilon_b = \pm \epsilon_r \left(\frac{L_s}{2L} \right)$; Stress Force: $S = \pm E \epsilon_r \left(\frac{L_s}{2L} \right) Wh$

- Spring constant becomes: Expansion of the Shoulder = $\Delta L_s = \epsilon_r L_s$ ← This expansion applies a load on the beams

strain in the beams
 $\epsilon_b = \frac{\Delta L}{2L} = \frac{\epsilon_r L_s}{2L}$
 $\{\Delta L = \Delta L_s\}$
 $\epsilon_b = \epsilon_r \frac{L_s}{2L}$

$k = 4(k_{com}^{-1} + k_{ten}^{-1})^{-1}$

$k = 4 \left[\frac{-pL + 2 \tan(pL/2)}{p|S|} + \frac{pL - 2 \tanh(pL/2)}{p|S|} \right]^{-1}$

of the flexure

- Remedies:
 - ↪ Reduce the shoulder width L_s to minimize stress in legs
 - ↪ Compliance in the truss lowers the axial compression and tension and reduces its effect on the spring constant

EE C245: Introduction to MEMS Design
LecM 8
C. Nguyen
9/28/07
41

41