Gyrosopes are inertial sensors that measure rotation rate, which is an extremely important variable to know when navigating. One must know rotation rate (as well as other parameters, e.g., time, linear acceleration, etc.) in order to determine position accurately (without the aid of GPS). Among the applications that use gyroscopes are airplanes (for navigation), boats (again, for navigation), automobiles (for skid control, among other applications), GPS receivers (to allow position determination during periods when the GPS signal cannot be received), cell phones, and game controllers (e.g., the Wii). Of these applications, the last four already use MEMS-based gyroscopes, and the first two are presently targeted by MEMS realizations.

Gyrosopes operate by taking advantage of the conservation of momentum, where an object moving in a given direction with a certain momentum will tend to continue moving in that direction even if its frame of reference rotates about an axis. This is perhaps best explained via example.

This problem involves the MEMS-based micro-gyroscope shown in Fig. PS10.1. This device generates momentum by driving a proof mass into resonance vibration using capacitive comb fingers along the x-axis. When the device rotates about the z-axis (indicated in Fig. PS10.1), the vibrating mass attempts to preserve its momentum in the original x-direction, which will then make the mass appear to deflect in the y-direction. This y-directed motion is then sensed by parallel-plate capacitors to determine rotation rate. In quantitative terms, the angular velocity $\Omega$ about the z-axis generates a Coriolis Force $F = 2m_s\dot{x}_d \times \Omega$ where $m_s$ is the equivalent mass in the sense mode, and $\dot{x}_d$ is the structure velocity in the drive mode) along the y-direction that drives the proof mass into y-directed vibration. The amplitude of the vibration is then sensed by the varying gap capacitances.

The suspended movable structure of the gyroscope is symmetric along both x-axis and y-axis. Figs. PS10.1-10.3 identify different parts of the structure, indicate which portions are freely suspended and which are anchored, and provide key dimensions. Fig. PS10.4 presents the drive and sense circuits of the gyroscope. The structures are all 20-μm thick and the suspension beams are all 2-μm wide. The entire device uses polysilicon structural material with a density $\rho = 2300 \text{ kg/m}^3$ and Young’s Modulus $E = 150 \text{ GPa}$. The movable structure is DC-biased relative to all electrodes at $V_P = 20$V.

1. Calculate the x- and y-directed resonance frequencies of the gyroscope structure when all the ports are grounded. Ignore the suspension beam masses in these calculations.

2. Determine the capacitance and change in capacitance per unit displacement for (i) one of the drive electrodes and (ii) the positive sense electrodes (+).

3. Calculate the y-directed resonance frequency of the gyroscope structure when the structure is biased at 20V (i.e., $V_P = 20$V).

4. Suppose the measured quality factors of the structure the in x- and y-directions are 150 and 100, respectively. Draw and specify (numerically) all element values in the equivalent circuits (transformer + LCR) modeling the (i) drive mode and (ii) sense mode when $V_P = 20$V.
Figure PS10.1 – Perspective view of the gyroscope
Figure PS10.2. – Top view of the gyroscope

Figure PS10.3 – Partial zoom-in of the gyroscope
Figure PS10.4 – Perspective view of the gyroscope showing the actuating and sensing circuits