1. Thermoelastic damping (TED) can occur in any material subject to periodic stress. It is pronounced in flexural mode resonators when heat moves from compressed parts to tensioned parts during vibration. For example, as shown in Fig. PS6.1-1, when a clamped-clamped beam (CC-Beam) resonator bends the tensile and compressive parts in the structure will generate temperature gradient. Thermal conductivity in the material will allow these hot and cold regions to equilibrate, which causes heat flux and energy loss, thereby limiting the $Q$. For low frequency CC-Beam resonators, TED is often the main loss mechanism and dominates the $Q$. The equations in Module 7 of Lecture 12 govern TED in a clamped-clamped beam.

Suppose you want to design a 100-MHz clamped-clamped beam resonator with very high $Q$ using the following guidelines/constraints:

- Lithographic resolution limits the CC-beam resonator length ($L$) and width ($W$) to no smaller than 1 μm.
- The beam thickness $h$ should be less than 3 μm.
- To avoid mode shape distortion, the length/thickness ratio $L/h$ should be no smaller than 5, and the beam width $W$ should be exactly 5 times smaller than the beam length $L$.
- You have the freedom to choose either quartz or single crystalline silicon as the structure material, the properties of which are shown in Table PS6.1-2.

Design a 100-MHz clamped-clamped beam resonator that attains the highest $Q$ if TED is the dominant loss. Clearly state the material you pick, the beam thickness $h$ you choose, the key dimensions of the beam ($L$, $W$) and the theoretical quality factor $Q$ it can reach. (Assume the resonator is operating in vacuum at room temperature 300 K.)

![Fig. PS6.1-1](image)

**Table PS6.1-2**

<table>
<thead>
<tr>
<th>Property</th>
<th>Silicon</th>
<th>Quartz</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal expansion coefficient</td>
<td>2.60</td>
<td>13.7</td>
<td>ppm/K</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>170</td>
<td>78</td>
<td>GPa</td>
</tr>
<tr>
<td>Material density</td>
<td>2330</td>
<td>2600</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Heat capacity</td>
<td>0.7</td>
<td>0.75</td>
<td>J/(g·K)</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>150</td>
<td>10</td>
<td>W/(m·K)</td>
</tr>
</tbody>
</table>
2. Fig. PS6.2 below presents top views of a 2μm-thick micromechanical structure with increasing suspension complexity from Design A to Design C. For each structure, everything is suspended 2 μm above the substrate except for the anchoring locations indicated as the darkly shaded regions. Key dimensions for the beams and data on the structural material used in this problem are given in the box below the figure. Also, assume that all folding trusses and shuttles are rigid in all directions, including the vertical (i.e., z) direction. As indicated in the box, assume that all suspension and coupling beam widths are 2 μm.

(a) Write an expression for the static spring constant in the x-direction at location A in Design A and calculate its numerical value (with units).

(b) Write an expression for the static spring constant in the x-direction at location B in Design B and calculate its numerical value (with units).

(c) Write an expression for the static spring constant in the x-direction at location C in the top half of Design C (i.e., assume the bottom half has been cut away, anchors and all) and calculate its numerical value (with units).

![Diagram of micromechanical structures](image)

**Beam Dimensions:**

$L_a = 100 \text{ μm}, L_b = 150 \text{ μm}, L_c = 50 \text{ μm}, W = 2 \text{ μm}$

**Structural Material Properties:**

Young’s Modulus, $E = 150 \text{ GPa}$; Density, $\rho = 2,300 \text{ kg/m}^3$; Poisson ratio, $\nu = 0.226$

Fig. PS6.2