Outline

- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients

Bending of Beams

Beams: The Springs of Most MEMS

* Springs and suspensions very common in MEMS
  - Coils are popular in the macro-world; but not easy to make in the micro-world
  - Beams: simpler to fabricate and analyze; become “stronger” on the micro-scale → use beams for MEMS

Comb-Driven Folded Beam Actuator
Bending a Cantilever Beam

- **Objective**: Find relation between tip deflection \( y(x=L) \) and applied load \( F \)
- **Assumptions**:
  1. Tip deflection is small compared with beam length
  2. Plane sections (normal to beam's axis) remain plane and normal during bending, i.e., “pure bending”
  3. Shear stresses are negligible

**Clamped end condition**:
- At \( x=0 \):
  - \( y=0 \)
  - \( \frac{dy}{dx} = 0 \)

**Free end condition**:
- At \( x=L \):
  - \( y=0 \)
  - \( \frac{dy}{dx} = 0 \)

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**Reaction Forces and Moments**

- **Point Load**
  - \( M_R = M \)
  - \( M_z = F(L-x) \)
  - \( V_x = F \)
  - \( V_{x,r} = F \)

**Sign Conventions for Moments & Shear Forces**

- **Moment**:
  - \((+)\) moment leads to deformation with a \((+)\) radius of curvature (i.e., upwards)
  - \((-)\) moment leads to deformation with a \((-)\) radius of curvature (i.e., downwards)

- **Shear**:
  - \((+)\) shear forces produce clockwise rotation
  - \((-)\) shear forces produce counterclockwise rotation