Lecture 15: Beam Combos II

- Announcements:
  - HW#5 online & due Wednesday, 3/10, at 12 noon
  - Midterm Exam about 1.5 weeks away
    - Will provide information next lecture
    - Let me know if you are in a drastically different time zone than Pacific Time

- Today:
  - Reading: Senturia, Chpt. 9
  - Lecture Topics:
    - Bending of beams
    - Cantilever beam under small deflections
    - Strain Gradients
    - Combining cantilevers in series and parallel
    - Folded suspensions
    - Design implications of residual stress and stress gradients

- Last Time:
  - Learned how to combine springs
  - Now, apply to common MEMS cases …

Typical Questions:
- All demand that we know $y = f(F)$
  1. How does the structure move in response to a force at a specific location?
  2. What is the frequency response to an AC force applied at a specific location?
  3. Noise?
  4. Response to environmental stimuli? (e.g., rotation)
  5. How does stress affect the behavior of the structure?
Procedure:

1. Build the def. (excite the def.) \( \rightarrow \) in the x-direction
   (for this example)

\[ k_1, k_2 \rightarrow m_1, k_3 \rightarrow m_2 \rightarrow \ldots \]
\[ k_4 \rightarrow \ldots \]
\[ k_5 \rightarrow m_3, k_6 \rightarrow \ldots \]
 Anch

\( \Rightarrow \) best way is to approach: simplify this mechanical def.
\( \Rightarrow \) need to know how springs combine

2. Analyze to get \( \kappa = f(F) \)
   displacement \( \rightarrow \) force

\[ k \rightarrow m \rightarrow F = k\kappa \rightarrow \kappa = F \cdot \left( \frac{1}{k} \right) \]

(a) Case 1: series springs
\( \kappa_1 \rightarrow \kappa_2 \rightarrow \kappa_3 \)
\( \kappa \) are across variables

(b) Case 2: parallel springs
\( F = \frac{1}{k_1 + k_2} \)

\( \kappa_1 \rightarrow \kappa_2 \rightarrow \kappa_3 \)

\( \kappa_1 \) is indicator parallel

\( \frac{1}{k_1 + k_2} \) if only need to go thru

\( F = f_1 + f_2 = k_1\kappa_1 + k_2\kappa_2 \) one of the springs to get from
anch to the far cry pt., then
the springs are in parallel.

\[ k_{tot} = k_1 + k_2 \] (for \( k_1 \) & \( k_2 \) in parallel)
**Series Combination of Springs**

1. **Clamped B.C.**
   -\[ L_c \]
2. **Free B.C.**
   -\[ y(L) \]
3. **Guided B.C.**

**Y_{tot} = Y_1 + Y_2 \quad \text{series}**

- Y must go through both springs to get from anchor to forcing pt.

**Parallel Combination of Beams**

1. **Pinned B.C.**
   -\[ k_a = k_b \]
2. **Fixed B.C.**
   -\[ k_c \]

**Y_{tot} = \frac{k_a}{2} \quad \text{parallel}**

- To go from one to forcing pt., need only go through one of the beams.

\[ k_{tot} = k_a + k_b = k_c \]
**Stiffness of Folded-Beam Suspension @ Shuttle Location**

Parallel: $k_{tot} = \frac{4}{9} k_c k_c$

**Micromechanical Filter**

1. Find the stiffness at point A.
2. Assume the shuttles and folding trusses are rigid.
3. Apply force at A: what is $\chi_A$?
4. $\chi_A = \frac{F}{k_A}$

$k_A$ is stiffness at point A
Get $k_A$:  

\[ k_A = k_{\text{comb}} + k_c = \frac{k_{c\max} L}{2} + k_c = k_{\text{comb}} + k_c \]

- Guided B.C.'s everywhere
- Sides
- Free
- Constraint $\rightarrow k_{c\max}$
- Free, B.C.
- Free, B.C.
- Free, B.C.
- Anchored
- Free
- Forced pt.
- Pinned $\rightarrow k_{c\max}$

### Beam Combos II

**Torsion Spring (Non-Ideality)**

- Important case for MEMS suspensions, since the thin films comprising them are often under residual stress
- Consider small deflection case: $y(x) \ll L$

**Governing differential equation: (Euler Beam Equation)**

\[
EI_z \frac{d^4 y}{dx^4} - S \frac{d^2 y}{dx^2} = F\delta(x-L)
\]

**Heuristic Derivation for the Euler-Beam Equation**

Consider first a straight beam under an axial stress:

\[
\sigma_a \rightarrow 0
\]

\[ \Rightarrow \text{no effect on } z\text{-directed stiffness} \]

- when the beam is straight
- but, when the beam is bent

**Thin beam**

**Axial Stress**

**$z$-directed component**

\[ k_{z\text{eff}} \text{ is affected!} \]
* Upward pressure $P_o$ to counteract the downward force from $f$ to keep everything in static equilibrium.

For ease of analysis:

Assume the beam is bent to an angle $\theta$.

Downward radial force: $2GWH$

Upward force due to $P_o$:

$F_u = \int_0^\pi (P_o \sin \theta) W (R \theta) d\theta$

$= -P_o WR \cos \theta \bigg|_0^\pi$

$= 2WRP_o$

$\textbf{[Equilibrium]} \Rightarrow 2WRP_o = 2 \sigma_o WH \rightarrow P_o = \frac{\sigma_o H}{R}$

$q_o = \text{beam load/unit length} = P_o W$, \( \frac{1}{R} = \frac{d^2 w}{dx^2} \)

beam displacement

$q_o = \delta WH \frac{d^2 w}{dx^2}$

generally to the case of small displacements