Lecture 16b: Energy Methods

Announcements:
• HW#6 online & due Wednesday, 3/17, at 12 noon
• Module 9 on “Energy Methods” online
• Midterm Exam next Thursday, 3/18
  ✉️ Can we do 5:00 p.m. - 7:30 p.m.
  ✉️ We will go through the Midterm Info Sheet
• This part of lecture happening during Friday discussion section (just finishing up Energy Methods)

Today:
• Reading: Senturia, Chpt. 10
• Lecture Topics:
  ✐ Energy Methods
  ✐ Virtual Work
  ✐ Energy Formulations
  ✐ Tapered Beam Example

Last Time:
• Looked at bending energy density
• Now, continue with energy due to an axial load ...

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Energy Method

= take some problem as before: apply a force to the tip of a cantilever

\[ \begin{align*}
\text{①} & \quad \text{Apply force.} \\
\text{②} & \quad \text{Beam responds by bending.} \\
\text{③} & \quad \text{This force has done work:} \\
& \quad W = F \cdot y(c) \\
\text{④} & \quad \text{Strain generated ( & stress) } \\
& \quad \text{So the beam has received an influx of stored energy} \\
& \quad \text{magnitude of } \frac{d}{dx} \text{ determines the slope} \\
\text{⑤} & \quad \text{Then:} \\
& \quad U = \text{Stored Energy} - \text{Work Done} \to 0 \\
& \quad \text{(Choose the right shape!)}
\end{align*} \]

Transfer function:

\[ y(x) = f(x, F) \] 

This is how we got the beam’s response to F.
Fundamentals of Energy Density

**General Definition of Work:**

\[ W(q_i) = \int_0^{q_i} e(q) \, dq \]

where \( e = e_{	ext{final}} - e_{	ext{initial}} \)

for EE: \( W(Q_i) = \int_0^Q \frac{1}{C} \, dQ \)

**Strain Energy Density:**

\[ \sigma = \sum \sigma_x \, e_x \]

value of strain at position \((x, y, z)\)

\[ \sigma_x = E \varepsilon_x \]

\[ \sigma = \int_0^x E \varepsilon_x \, dx \]

**Total Strain Energy:** \([J]\)

\[ W = \iiint \left( \frac{1}{2} E (e_x^2 + e_y^2 + e_z^2) + \frac{1}{2} G (\gamma_{xx}^2 + \gamma_{yy}^2 + \gamma_{zz}^2) \right) \, dV \]

\[ W_{\text{total}} = \frac{1}{2} EI_z \int_0^L \left( \frac{d^2 y}{dx^2} \right)^2 \, dx \]
Energy due to Axial Load

\[ ds = \left[ (dx)^2 + (dy)^2 \right]^{1/2} = dx \left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{1/2} \]

_by biarmpial theorem_

\[ ds = dx \left[ 1 + \frac{1}{2} \left( \frac{dy}{dx} \right)^2 \right] \]

\[ \varepsilon_x = \frac{ds}{dx} \cdot \frac{dx}{dx} = \frac{1}{2} \left( \frac{dy}{dx} \right)^2 \]

\[ \left[ dW_{\text{axial}} = 5 \varepsilon_x \cdot dx = \frac{1}{2} \cdot 5 \cdot \left( \frac{dy}{dx} \right)^2 \cdot dx \right] \]

\[ W_{\text{axial}} = \frac{1}{2} \cdot 5 \int_0^1 \left( \frac{dy}{dx} \right)^2 \cdot dx \]

\[ \Rightarrow \text{Axial Strain Energy} \]

\[ \Rightarrow \text{Look at shear strain energy in your module.} \]