**Parallel-Plate Capacitive Nonlinearity**

- Example: clamped-clamped laterally driven beam with balanced electrodes
- Nomenclature:
  - $V_a$ or $v_a$ vs $V_d$ = $|v_a| \cos \omega t$
  - Total Value
    - DC Component (upper case variable; upper case subscript)
    - AC or Signal Component (lower case variable; lower case subscript)

**Voltage-Controllable Center Frequency**

- Quadrature force + voltage-controllable electrical stiffness:
  - $k_e = \frac{\varepsilon_0 A_0 V^2}{d^3}$
  - Frequency vs DC-Bias
  - $f_0 = \frac{1}{2\pi} \sqrt{\frac{k_e}{m}}$
  - $d = 1000\,\text{Å}$

**Microresonator Thermal Stability**

- Thermal stability of poly-Si micromechanical resonator is 10X worse than the worst case of AT-cut quartz crystal

**Geometric-Stress Compensation**

- Use a temperature dependent mechanical stiffness to null frequency shifts due to Young's modulus thermal dep.
- Problems:
  - stress relaxation
  - compromised design flexibility

* Problems: [Hsu et al, IEDM 2000]
Voltage-Controllable Center Frequency

- Quadrature force = voltage-controllable electrical stiffness:
  \[ k_e = \frac{1}{d^3} v \frac{v}{P} \]

- Frequency:
  \[ f_o \approx \left( \frac{k_m - k_e}{2 \pi} \right)^{0.5} \]

- Counteracts reduction in frequency due to Young's modulus temp. dependence

Excellent Temperature Stability

- Uncompensated resonator:
  \[ -1.7 \text{ ppm/}^\circ C \]
  \[ -0.24 \text{ ppm/}^\circ C \]

Can One Cancel \( k_e \) w/ Two Electrodes?

- What if we don't like the dependence of frequency on \( V_P \)?
- Can we cancel \( k_e \) via a differential input electrode configuration?
- If we do a similar analysis for \( F_{d2} \) at Electrode 2:

\[
F_{d2} = -V_P^2 \frac{C_{d2}}{d_2} v_2 \cos \omega t + V_P^2 \frac{C_{d2}}{d_2} v_1 \sin \omega t
\]

- Subtracts the \( F_{d1} \) term, as expected
- Adds to the quadrature term \( k_e \)’s add, no matter the electrode configuration!
Problems With Parallel-Plate C Drive

- Nonlinear voltage-to-force transfer function
  - Resonance frequency becomes dependent on parameters (e.g., bias voltage \( V_P \))
  - Output current will also take on nonlinear characteristics as amplitude grows (i.e., as \( x \) approaches \( d_o \))
  - Noise can alias due to nonlinearity
- Range of motion is small
  - For larger motion, need larger gap ... but larger gap weakens the electrostatic force
  - Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)

Electrostatic Comb Drive

- Use of comb-capacitive transducers brings many benefits
  - Linearizes voltage-generated input forces
  - (Ideally) eliminates dependence of frequency on dc-bias
  - Allows a large range of motion