Lecture 23: Circuit Problems & Gyros

- **Announcements:**
  - HW#9 due Wednesday, 4/21, at 12 noon
  - Project Slide Set #2 due Friday, April 16
  - Module 13 on “Equivalent Circuits II” online
  - Module 15 on “Gyros, Noise, & MDS” online
  - Discussion tomorrow

- **Today:**
  - Reading: Senturia, Chpt. 6, Chpt. 14
  - Lecture Topics:
    - Current Modeling
      - Output Current Into Ground
      - Input Current
      - Complete Electrical-Port Equiv. Ckt.
    - Impedance & Transfer Functions

- Reading: Senturia, Chpt. 14, Chpt. 16, Chpt. 21
  - Lecture Topics:
    - Gyros

- Reading: Senturia, Chpt. 14
  - Lecture Topics:
    - Detection Circuits
      - Velocity Sensing
      - Position Sensing

- **Last Time:**
  - Developing equivalent circuit for MEMS device
  - Now, finish this and do some circuit problems ...

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Get \( I_s(\omega) \):

\[
i_i(t) = C_1 \frac{dV_1(t)}{dt} + V_1(t) \frac{dC_1}{dX} \frac{dX}{dt}
\]

\[
[V_1(t) = V_L - V_p] \Rightarrow \int i_s(t) dt = C_1 \frac{dV_1(t)}{dt} + (V_L - V_p) \frac{dX}{dt}
\]

\[
I_s(\omega) = j\omega C_1 V_1 + j\omega V_p \frac{\partial X}{\partial X} - j\omega V_p \frac{\partial C_1}{\partial X}
\]

Feed-through Current

Motional Current (due to motion)

@ Drive Frequency

V1 < Vp  \( X \) is small \( \neq 0 \)
\[ \frac{\text{off resonance}}{k} = \frac{F_{0 \text{ff}}}{k} = -\frac{1}{k} V_p \left( \frac{\partial x}{\partial \theta} \right) \]

\[ \frac{\text{on resonance}}{j \omega} = X = \frac{Q_{\text{eff}}}{j \omega} = -\frac{Q}{j \omega} V_p \frac{\partial x}{\partial \theta} V_1 = X \]

\[ \omega_0 = \text{constant} \]

\[ I_c(j \omega) = j \omega_0 C V_1 + j \frac{Q}{j \omega_0} V_1 \]

\[ = j \omega_0 C V_1 + \frac{Q}{\omega_0} \eta^2 V_1 \]

\[ \text{90° phase shift from } \eta_1 \]

\[ \text{This is an effective resistance seen looking into electrode 1} \]

\[ \text{static capacitance between Electrode 1 to the structure} \]

\[ \text{Look at some examples} \]

\[ R_x = \frac{V_1}{I_1} = \frac{k}{\omega_0} \frac{Q}{\eta_1} \]

\[ \text{"motion"} \]

\[ R_x \text{ is important in practical applications, so our equivalent ckt. needs to get this right!} \]
\[ [e_2] = \begin{bmatrix} \eta & 0 \\ 0 & -\frac{1}{\eta} \end{bmatrix} [f_1] \Rightarrow e_2 = \eta e_1 \rightarrow e_1 = \frac{e_2}{\eta} \]

\[ f_2 = \frac{1}{\eta} f_1 \rightarrow f_1 = -\eta f_2 \]

\[ e_{\text{res}} = \frac{e_2}{\eta_2} \frac{1}{\eta_1} \left( j \omega x + \frac{1}{j \omega c_x} + r_x \right) \]

\[ = j \omega \left( \frac{l_x}{n_{e1}^2} \right) + \frac{1}{j \omega \left( n_{e1} c_x \right)} + \frac{r_x}{n_{e1}^2} \]

\[ L_{\text{res}} = \frac{1}{n_{e1}^2} \]

\[ C_{\text{res}} = -\frac{1}{n_{e1}^2 \omega} \]

\[ R_{\text{res}} = \frac{r_x}{n_{e1}^2} \]

\[ \text{motional resistance} \]

\[ \text{Input Impedance into Port 2} \]

\[ Z_2' = N_{i1}^2 \]

\[ Z_2 = Z_2' \]
\[ V_2 = \frac{I_2}{N_2} \]

\[ I_1 = \eta_{el} I_x \]

\[ I_0 = \eta_{e2} I_x \]

\[ \dot{I}_2 = \frac{N_2}{\eta_{el}} \frac{n_{el} I_x}{j \omega L_x + j \omega C_x + R_{x12}} \]

\[ \dot{I}_2 = \frac{n_{el} n_{e2}}{N_2} \left( \frac{1}{j \omega C_x + R_{x12}} + \frac{1}{j \omega L_x + j \omega C_x + R_{x12}} \right)^{-1} \]