**Module 17: Noise & MDS**

**Minimum Detectable Signal (MDS)**

- Minimum Detectable Signal (MDS): Input signal level when the signal-to-noise ratio (SNR) is equal to unity

- The sensor scale factor is governed by the sensor type
- The effect of noise is best determined via analysis of the equivalent circuit for the system

![Diagram of Sensor Signal Conditioning Circuit](image)

**Lecture Outline**

- Reading: Senturia Chpt. 16
- Lecture Topics:
  - Minimum Detectable Signal
  - Noise
    - Circuit Noise Calculations
    - Noise Sources
    - Equivalent Input-Reflected Noise
  - Gyro MDS
    - Equivalent Noise Circuit
    - Example ARW Determination
Noise

- Noise: Random fluctuation of a given parameter $I(t)$
- In addition, a noise waveform has a zero average value (e.g., could be DC current)

We can’t handle noise at instantaneous times, but we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation.

Thus, represent noise by its mean-square value:

$$\langle i^2 \rangle = \int_0^T I^2 \, dt$$

Noise Spectral Density

- We can plot the spectral density of this mean-square value:

Noise Sources

- Used in circuits
- Measured by spectrum analyzers
Thermal Noise

• Thermal Noise in Electronics: (Johnson noise, Nyquist noise)
  - Produced as a result of the thermally excited random motion of free e−'s in a conducting medium
  - Path of e−'s randomly oriented due to collisions
• Thermal Noise in Mechanics: (Brownian motion noise)
  - Thermal noise is associated with all dissipative processes that couple to the thermal domain
  - Any damping generates thermal noise, including gas damping, internal losses, etc.

• Properties:
  - Thermal noise is white (i.e., constant w/ frequency)
  - Proportional to temperature
  - Not associated with current
  - Present in any real physical resistor

Circuit Representation of Thermal Noise

- Thermal Noise can be shown to be represented by a series voltage generator \( \frac{v_R^2}{R} \) or a shunt current generator \( \frac{i_R^2}{R} \)

\[
\frac{v_R^2}{R} = 4kT \quad \text{or} \quad \frac{i_R^2}{R} = 4kTR
\]

Note: These are one-sided mean-square spectral densities! To make them 2-sided, must divide by 2.

where \( 4kT = 1.66 \times 10^{-20} \text{V}^2 \cdot \text{C} \)

and where these are spectral densities.

Noise in Capacitors and Inductors?

• Resistors generate thermal noise
• Capacitors and inductors are noiseless → why?

\[
\frac{v_C^2}{L} = \frac{1}{2} k_B T, \quad \frac{v_L^2}{C} = \frac{1}{2} k_B T
\]

Why 4kTR?

• Why is \( \frac{v_R^2}{R} = 4kT \Delta f \) (a heuristic argument)
• The Equipartition Theorem of Statistical Thermodynamics says that there is a mean energy \((1/2)kT\) associated w/ each degree of freedom in a given system
• An electronic circuit possesses two degrees of freedom:
  - Current, \( i \), and voltage, \( v \)
  - Thus, we can write:
    \[
    \frac{1}{2} L i^2 = \frac{1}{2} k_B T, \quad \frac{1}{2} C v^2 = \frac{1}{2} k_B T
    \]
• Similar expressions can be written for mechanical systems
  - For example: for displacement, \( x \)
    \[
    \frac{1}{2} k x^2 = \frac{1}{2} k_B T
    \]
**Shot Noise**

- Associated with direct current flow in diodes and bipolar junction transistors
- Arises from the random nature by which e−'s and h+’s surmount the potential barrier at a pn junction
- The DC current in a forward-biased diode is composed of h+’s from the p-region and e−’s from the n-region that have sufficient energy to overcome the potential barrier at the junction
  → noise process should be proportional to DC current
- Attributes:
  - Related to DC current over a barrier
  - Independent of temperature
  - White (i.e., const. w/ frequency)
  - Noise power ~ $I_D$ & bandwidth

\[ \frac{\Delta I}{\Delta f} = 2qI_D \]

**Flicker (1/f) Noise**

- In general, associated w/ random trapping & release of carriers from “slow” states
- Time constant associated with this process gives rise to a noise signal w/ energy concentrated at low frequencies
- Often, get a mean-square noise spectral density that looks like this:

\[ \frac{\Delta I^2}{\Delta f} = 2qI_D + K \left( \frac{I_D^a}{f^b} \right) \]

$I_D$ = DC current
$K$ = const. for a particular device
$a = 0.5 \rightarrow 2$
$b \sim 1$