EE C247B – ME C218
Introduction to MEMS Design
Spring 2021

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Lecture Module 14: Sensing Circuits

Lecture Outline

• Reading: Senturia, Chpt. 14
• Lecture Topics:
  • Detection Circuits
    • Velocity Sensing
    • Position Sensing
Velocity-to-Voltage Conversion

To convert velocity to a voltage, use a resistive load

Consider the mechanical device by itself first. Assume free output shafted.

For velocity to voltage conversion:

\[ V_o = \frac{1}{k} \oint \omega \, d\theta \]

\[ V_o = \frac{1}{k} \oint \omega \, d\theta \]

Solve the problem at resonance first, then multiply by \( \Theta(t) \)

\[ V_o \frac{N_o}{N_o} = \frac{R_o}{R_o + R_L} \Theta(t) \]

Voltage representing velocity

Work @ resonance: (to simplify the analysis)

\[ V_o \frac{N_o}{N_o} = \frac{R_o}{R_o + R_L} \Theta(t) \]

Then, generalize to off resonance:

\[ V_o \frac{N_o}{N_o} = \frac{R_o}{R_o + R_L} \Theta(t) \]

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Velocity-to-Voltage Conversion

To convert velocity to a voltage, use a resistive load.

Brute force approach:

\[ V_o = \frac{R_0}{R_x + \frac{1}{sC_x} + \frac{sL_x}{R_0}} \]

Since this structure has completely symmetrical I/O ports:

\[ Q = \frac{\omega_x}{R_x} \rightarrow Q_x = \frac{\omega_x}{R_x + R_0} \]

\[ Q = \frac{1}{L_x} \rightarrow Q_x = \frac{1}{L_x} \]

\[ Q = \frac{R_x}{R_{x+R_0}} \]

Velocity Sensing Circuits
Velocity-to-Voltage Conversion

To convert velocity to a voltage, use a resistive load.

Problems With Purely Resistive Sensing

Now, we get: (approximately)

\[ \frac{V_o}{V_1} \approx \frac{R_o}{R_o + R_D} \]

Depending on both \( R_o + R_D \).
Problems With Purely Resistive Sensing

In general, the sensor output must be connected to the inputs of further signal conditioning circuits → input \( R_i \) of these circuits can load \( R_D \)

These change w/ hook-up → not good.

Problem: need a sensing circuit that is immune to parasitics or loading.

Soln: use op amps.

The TransR Amplifier Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance \( C_p \) and \( R_i \)

- The zero output resistance of the (ideal) op amp can drive virtually anything

\[ V_0 = 0 \Omega \]

Virtual Ground ⇒ No voltage across \( C_p \)

\[ N_0 = \frac{R_2}{R_x} \Theta(s) \]

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Position Sensing Circuits

Position-to-Voltage Conversion

- To sense position (i.e., displacement), use a capacitive load

\[
\frac{N_0}{V_0} = \frac{C_D}{R_x + \frac{1}{C_x} + sL_x + \frac{1}{C_D}}
\]

\[
S = \frac{1}{1 + \frac{5R_xC_x}{11C_D} + \frac{5}{11C_D} + \frac{L_xE_p}{11C_D}}
\]

\[
\left[ \omega_0, \frac{1}{L_xE_p}, a_c(1 + C_D/C_0), b_c(1 + C_D/C_0) \right]
\]

\[
\left[ Q, \omega_0, \frac{L_x}{R_x}, \frac{C_D}{C_0}, C_D(1 + C_D/C_0) \right]
\]
**Position-to-Voltage Conversion**

* To sense position (i.e., displacement), use a capacitive load

\[ V_O = \frac{C_v/C_D}{1 + C_v/C_D} \left( \frac{(\omega_d)^2}{s^2 + \left(\frac{\omega_d}{q}\right)s + (\omega_d)^2} \right) \]

**DC Gain**

To maximize gain \(\rightarrow 1\), need \(C_v \ll C_D\).

**Low-Pass Biquad**

**Note:** Can use similar short-cut to the R case.

1. Get DC response \(\rightarrow C_v\)'s dominate.
2. Then:

\[ V_O(s) = \frac{(DC \text{ Gain})}{s} \cdot \Theta(s, \omega_d, Q) \cdot \omega_d^2 \]

**Problems With Pure-C Position Sensing**

* To sense position (i.e., displacement), use a capacitive load

\[ V_O = \frac{C_v/C_D}{1 + C_v/C_D} \left( \frac{1}{s} \cdot \Theta(s, \omega_d, Q) \cdot \omega_d^2 \right) \]

**Integration yields**

displacement.

To maximize gain, minimize \(C_v\).

\(\Rightarrow\) Problem: parasitic capacitance

\[ C_v \rightarrow C_v + C_{PD} + C_{PB} \]

\[ \text{DC Gain:} \quad \frac{C_v}{1 + C_v/C_D + C_{PB}} \]

**Remedy:** Suppress \(C_v\)

Via use of op amps.
The Op Amp Integrator Advantage

- The virtual ground provided by the ideal op amp eliminates the parasitic capacitance \( C_p \)

\[
\begin{align*}
\text{Electrode 1} & \quad i_1 \quad v_1 \\
\text{Electrode 2} & \quad i_0 \quad v_0 \\
\end{align*}
\]

Differential Position Sensing
**Differential Position Sensing**

- **Example:** ADXL-50

  Tethers with fixed ends

  ![C1](image1) ![C2](image2)

  Fixed Electrodes

  \[ V_p \]

  \[ V_o \]

  \[ -V_p \]

  Proof Mass

  Sense Finger

  Suspension Beam in Tension

  Issues: Parasitic Capacitance

  \[ V_o = -V_p + \frac{(2V_p)C_1C_2}{C_1 + C_2} \]

  \[ V_o = \frac{V_o}{C_1 + C_2} \]

  As before, \( C_p \) reduces gain – Use op amp!

**Buffer-Bootstrapped Position Sensing**

- **Bootstrap the ground lines around the interconnect and bond pads**
  - No voltage across \( C_p \)
  - It's effectively not there!

  Includes capacitance from interconnects, bond pads, and \( C_{gs} \) of the op amp

  \[ CP \]

  \[ CGd \]

  \[ CGd = \text{gate-to-drain capacitance of the input MOS transistor} \]

  Unity Gain Buffer

  Interconnect

  Ground Plane

  \( 1x \)
Effect of Finite Op Amp Gain

\[ +V_p \]

\[ -V_p \]

\[ C_{gd} \]

\[ C_p \]

\[ V_0 \]

\[ \frac{N_0}{N_i} = \frac{A_0(N_i - N_p)}{A_0(N_i - N_p)} \rightarrow \frac{N_0}{N_i} = \frac{A_0}{1 + A_0} \]

\[ G = \frac{N_0}{N_i} = 1 \]

\[ C_{eff} = \frac{C_p}{1 + A_0} \]

\[ \Rightarrow C_{eff} = \frac{2pF}{101} = 20pF \]

\[ \Rightarrow \text{Not negligibly small with ADXL-50 so C_{eff} ~ 100pF}! \]

Integrator-Based Diff. Position Sensing

\[ +V_p \]

\[ -V_p \]

\[ C_F \]

\[ R_2 \]

\[ R_2 \gg \frac{1}{sC_F} \]

\[ V_0 \]

\[ R_o \]

\[ \text{Can drive next stage's} \]

\[ \text{without interference to transfer function!} \]

\[ \frac{N_0}{V_p} = \frac{C_2}{C_F} \]

\[ \Rightarrow \text{A seemingly perfect differential sensor/amplifier output!} \]

\[ \text{...but only when the op amp is ideal...} \]