Outine

• Reading: Senturia, Chpt. 8
• Lecture Topics:
  ◦ Stress, strain, etc., for isotropic materials
  ◦ Thin films: thermal stress, residual stress, and stress gradients
  ◦ Internal dissipation
  ◦ MEMS material properties and performance metrics
Vertical Stress Gradients

- Variation of residual stress in the direction of film growth
- Can warp released structures in z-direction

Elasticity
Normal Stress (1D)

If the force acts normal to a surface, then the stress is called a normal stress.

\[ \sigma = \frac{F}{A} \quad [N/m^2 : Pa] \]

Standard mks unit

Microscopic Definition: force per unit area acting on the surface of a differential volume element of a solid body.

Note: assume stress acts uniformly across the entire surface of the element, not at just a point.

Strain (1D)

Sometimes a unit called the "micron" is used, where

\[ \varepsilon = \frac{\Delta L}{L} \quad [\text{unitless}] \]

In the elastic regime (i.e., for "small" stresses at "low" temperatures), strain is found to be proportional to stress:

\[ \sigma = E \varepsilon \]

For solids: MPa → GPa

Slope: \( E \) = Young's modulus of elasticity

Thus, the units of \( E \) are the same as \( \sigma \rightarrow Pa \).
The Poisson Ratio

Apply normal stress to a free-standing object but also get contraction in directions transverse to the uniaxial strain.

⇒ contraction creates a (-) strain:

\[ \varepsilon_y = \frac{W-W'}{W} = -\nu \varepsilon_x \]

\( \nu \) = Poisson ratio [unitless]

- Typical values: 0 → 0.5
- Inorganic solids: 0.2 → 0.3
- Elastomers (e.g., rubber): < 0.5

Shear Stress & Strain (1D)

Note: Assume compensating forces are applied to the vertical faces to avoid a net torque. (This by convention.)

Shear Stress = \{ Force per Unit Area Parallel to the Surfaces \} = \tau = \frac{F}{A} \quad [\text{Pa}]

Generates a shear strain:

\[ \text{Shear Strain} = \theta = \frac{\tau}{G} \]

\( G \) = shear modulus

\[ G = \frac{E}{2(1+\nu)} \]
2D and 3D Considerations

- Important assumption: the differential volume element is in static equilibrium → no net forces or torques (i.e., rotational movements)
- Every σ must have an equal σ in the opposite direction on the other side of the element
- For no net torque, the shear forces on different faces must also be matched as follows:
  \[ \tau_{xy} = \tau_{yx} \quad \tau_{xz} = \tau_{zx} \quad \tau_{yz} = \tau_{zy} \]

2D Strain

- In general, motion consists of
  - rigid-body displacement (motion of the center of mass)
  - rigid-body rotation (rotation about the center of mass)
  - Deformation relative to displacement and rotation

- Must work with displacement vectors
- Differential definition of axial strain:
  \[ \varepsilon_x = \frac{u_x(x+\Delta x) - u_x(x)}{\Delta x} = \frac{\partial u_x}{\partial x} \]
### 2D Shear Strain

For shear strain, must remove any rigid body rotation that accompanies the deformation.

Use a symmetric definition of shear strain:

\[ \gamma_{xy} = \frac{\Delta u_y}{\Delta y} = \frac{\Delta u_x}{\Delta x} \]

For small amplitude deformations.

### Volume Change for a Uniaxial Stress

Given an x-directed uniaxial stress, \( \sigma_x \):

\[ \Delta x \rightarrow \Delta x (1 + \epsilon_x) \]
\[ \Delta y \rightarrow \Delta y (1 - \gamma \epsilon_x) \]
\[ \Delta z \rightarrow \Delta z (1 - \gamma \epsilon_x) \]

The resulting change in volume \( \Delta V \)

\[ \Delta V = \Delta x \Delta y \Delta z \left( \frac{(1+\epsilon_x) (1-\gamma \epsilon_x)^2 - 1}{1+\epsilon_x} \right) \]

Assume small strains:

\[ \Delta V = \Delta x \Delta y \Delta z \left( \frac{(1+\epsilon_x) (1-\gamma \epsilon_x)^2 - 1}{1+\epsilon_x} \right) \]

\[ \Delta V \approx \Delta x \Delta y \Delta z (1-2\gamma) \epsilon_x \]

For \( \gamma < 0.5 \) (rubber) \( \rightarrow \) no \( \Delta V \)

\( \gamma < 0.5 \) \( \rightarrow \) finite \( \Delta V \)
Isotropic Elasticity in 3D

• Isotropic = same in all directions
• The complete stress-strain relations for an isotropic elastic solid in 3D: (i.e., a generalized Hooke’s Law)

\[
\varepsilon_x = \frac{1}{E} \left[ \sigma_x - \nu(\sigma_y + \sigma_z) \right] \quad \gamma_{xy} = \frac{1}{G} \tau_{xy}
\]

\[
\varepsilon_y = \frac{1}{E} \left[ \sigma_y - \nu(\sigma_z + \sigma_x) \right] \quad \gamma_{yz} = \frac{1}{G} \tau_{yz}
\]

\[
\varepsilon_z = \frac{1}{E} \left[ \sigma_z - \nu(\sigma_x + \sigma_y) \right] \quad \gamma_{zx} = \frac{1}{G} \tau_{zx}
\]

Basically, add in off-axis strains from normal stresses in other directions

Important Case: Plane Stress

• Common case: very thin film coating a thin, relatively rigid substrate (e.g., a silicon wafer)

• At regions more than 3 thicknesses from edges, the top surface is stress-free \( \sigma_z = 0 \)

• Get two components of in-plane stress:

\[
\varepsilon_x = (1/E)[\sigma_x - \nu(\sigma_y + 0)]
\]

\[
\varepsilon_y = (1/E)[\sigma_y - \nu(\sigma_x + 0)]
\]
Important Case: Plane Stress (cont.)

- Symmetry in the $xy$-plane $\Rightarrow \sigma_x = \sigma_y = \sigma$
- Thus, the in-plane strain components are: $\varepsilon_x = \varepsilon_y = \varepsilon$

where

$$\varepsilon_x = \frac{1}{E} [\sigma - \nu \sigma] = \frac{\sigma}{E/((1-\nu))} = \frac{\sigma}{E'}$$

and where

Biaxial Modulus $E' = \frac{E}{1-\nu}$

Edge Region of a Tensile ($\sigma > 0$) Film

Net non-zero in-plane force (that we just analyzed)

At free edge, in-plane force must be zero

Film must be bent back, here

There's no Poisson contraction, so the film is slightly thicker, here

Shear stresses

Extra peel force

Discontinuity of stress at the attached corner $\Rightarrow$ stress concentration

Peel forces that can peel the film off the surface
Linear Thermal Expansion

- As temperature increases, most solids expand in volume
- Definition: linear thermal expansion coefficient

\[ \alpha_T = \frac{d\varepsilon_x}{dT} \text{ [Kelvin}^{-1}\text{]} \]

Remarks:
- \(\alpha_T\) values tend to be in the 10\(^{-6}\) to 10\(^{-7}\) range
- Can capture the 10\(^{-6}\) by using dimensions of \(\mu\)strain/K, where 10\(^{-6}\) K\(^{-1}\) = 1 \(\mu\)strain/K
- In 3D, get volume thermal expansion coefficient

\[ \frac{\Delta V}{V} = 3\alpha_T \Delta T \]

- For moderate temperature excursions, \(\alpha_T\) can be treated as a constant of the material, but in actuality, it is a function of temperature

\[ \alpha_T \text{ As a Function of Temperature} \]

[Cubic symmetry implies that \(\alpha\) is independent of direction

[Madou, Fundamentals of Microfabrication, CRC Press, 1998]
Thin-Film Thermal Stress

- Assume film is deposited stress-free at a temperature $T_r$, then the whole thing is cooled to room temperature $T_r$.
- Substrate much thicker than thin film implies substrate dictates the amount of contraction for both it and the thin film.

Thermal strain of the substrate: (in one in-plane dimension)
$$\varepsilon_s = \alpha_s \Delta T$$, where $\Delta T = T_d - T_r$

If the film were not attached to the substrate: $\varepsilon_{f,\text{free}} = -\alpha_f \Delta T$

Silicon Substrate ($\alpha_s = 2.8 \times 10^{-6} \text{K}^{-1}$)

Thin Film ($\alpha_f$)

Substrate much thicker than thin film

Linear Thermal Expansion

But the film is attached to the substrate, so the actual strain in the film is the same as that in the substrate:
$$\varepsilon_{f,\text{attached}} = -\alpha_f \Delta T$$

Thus:

Thermal Mismatch Strain = $\varepsilon_{f,\text{mismatch}} = (\alpha_f - \alpha_s) \Delta T$

Note that this is biaxial strain and it can only be developed by an in-plane biaxial stress:

$$\sigma\text{f,mismatch} = \left(\frac{E_f}{1-\nu}\right)\varepsilon_{f,\text{mismatch}}$$

Ex. Thin film is polyimide, $\alpha_{f} = 7 \times 10^{-6} \text{K}^{-1}$, $E_f = 46GPa$

Deposited at 250°C, then cooled to RT: 25°C $\Delta T = 225K$

$$\varepsilon_{f,\text{mismatch}} = (70-2.8) \times 18.5 \times 10^{-2}$$

$$\sigma_{f,\text{mismatch}} = (46)(1.5\times10^{5}) = 69.5 \text{MPa}$$

Stress is in (-), so tensile

[1] Coulomb
MEMS Material Properties

### Material Properties for MEMS

<table>
<thead>
<tr>
<th>Material</th>
<th>Density, ρ, Kg/m³</th>
<th>Modulus, E, GPa</th>
<th>E/ρ, GN/kg-m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon</td>
<td>2330</td>
<td>165</td>
<td>72</td>
</tr>
<tr>
<td>Silicon Oxide</td>
<td>2200</td>
<td>73</td>
<td>36</td>
</tr>
<tr>
<td>Silicon Nitride</td>
<td>3300</td>
<td>304</td>
<td>92</td>
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<tr>
<td>Nickel</td>
<td>8900</td>
<td>207</td>
<td>23</td>
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<tr>
<td>Aluminum</td>
<td>2710</td>
<td>69</td>
<td>25</td>
</tr>
<tr>
<td>Aluminum Oxide</td>
<td>3970</td>
<td>393</td>
<td>99</td>
</tr>
<tr>
<td>Silicon Carbide</td>
<td>3300</td>
<td>430</td>
<td>130</td>
</tr>
<tr>
<td>Diamond</td>
<td>3510</td>
<td>1035</td>
<td>295</td>
</tr>
</tbody>
</table>

Units: (m/s)²

$\sqrt{(E/\rho)}$ is acoustic velocity

[Mark Spearing, MIT]
Young's Modulus Versus Density

Lines of constant acoustic velocity

[Ashby, Mechanics of Materials, Pergamon, 1992]

Yield Strength

• Definition: the stress at which a material experiences significant plastic deformation (defined at 0.2% offset pt.)
• Below the yield point: material deforms elastically → returns to its original shape when the applied stress is removed
• Beyond the yield point: some fraction of the deformation is permanent and non-reversible

Yield Strength: defined at 0.2% offset pt.
Elastic Limit: stress at which permanent deformation begins
Proportionality Limit: point at which curve goes nonlinear
True Elastic Limit: lowest stress at which dislocations move
**Yield Strength (cont.)**

- **Below:** typical stress vs. strain curves for brittle (e.g., Si) and ductile (e.g., steel) materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Modulus, $E$, GPa</th>
<th>Useful Strength*, $\sigma_f$, MPa</th>
<th>$\sigma_f/E$ x $10^3$</th>
<th>$O_f^2/E$</th>
<th>Stored mechanical energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silicon</td>
<td>165</td>
<td>4000</td>
<td>24</td>
<td>97</td>
<td></td>
</tr>
<tr>
<td>Silicon Oxide</td>
<td>73</td>
<td>1000</td>
<td>13</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Silicon Nitride</td>
<td>304</td>
<td>1000</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Nickel</td>
<td>207</td>
<td>500</td>
<td>2</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>69</td>
<td>300</td>
<td>4</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td>Aluminum Oxide</td>
<td>393</td>
<td>2000</td>
<td>5</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Silicon Carbide</td>
<td>430</td>
<td>2000</td>
<td>4</td>
<td>9.3</td>
<td></td>
</tr>
<tr>
<td>Diamond</td>
<td>1035</td>
<td>1000</td>
<td>1</td>
<td>0.9</td>
<td></td>
</tr>
</tbody>
</table>

**Young's Modulus Versus Strength**

Lines of constant maximum strain

[Ashby, Mechanics of Materials, Pergamon, 1992]

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**Quality Factor (or Q)**
**Clamped-Clamped Beam μResonator**

- **Resonator Beam**
- **Electrode**
- **Frequency:**
  - Stiffness: $f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m_r}} = 1.03 \frac{E}{\rho L_r^2}$
  - Young's Modulus
  - Density
  - Mass: (e.g., $m_r = 10^{-13}$ kg)
  - Smaller mass $\Rightarrow$ higher freq. range and lower series $R_x$  

**Quality Factor (or $Q$)**

- **Measure of the frequency selectivity of a tuned circuit**
- **Definition:**
  $$Q = \frac{\text{Total Energy Per Cycle}}{\text{Energy Lost Per Cycle}} = \frac{f_0}{BW_{3\text{dB}}}$$

- **Example: series LCR circuit**
  $$Q = \frac{\text{Im}(Z)}{\text{Re}(Z)} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

- **Example: parallel LCR circuit**
  $$Q = \frac{\text{Im}(Y)}{\text{Re}(Y)} = \frac{\omega_0 C}{G} = \frac{1}{\omega_0 LG}$$
Selective Low-Loss Filters: Need $Q$

- In resonator-based filters: high tank $Q \leftrightarrow$ low insertion loss
- At right: a 0.1% bandwidth, 3-res filter @ 1 GHz (simulated) \(\Rightarrow\) heavy insertion loss for resonator $Q < 10,000$

Oscillator: Need for High $Q$

- **Main Function:** provide a stable output frequency
- **Difficulty:** superposed noise degrades frequency stability
### Attaining High Q

- **Problem:** IC’s cannot achieve Q’s in the thousands
  - transistors ⇒ consume too much power to get Q
  - on-chip spiral inductors ⇒ Q’s no higher than ~10
  - off-chip inductors ⇒ Q’s in the range of 100’s
- **Observation:** vibrating mechanical resonances ⇒ Q > 1,000
- **Example:** quartz crystal resonators (e.g., in wristwatches)
  - extremely high Q’s ~ 10,000 or higher (Q ~ 10^6 possible)
  - mechanically vibrates at a distinct frequency in a thickness-shear mode

### Energy Dissipation and Resonator Q

- **Material Defect Losses**
- **Gas Damping**
  \[
  \frac{1}{Q} = \frac{1}{Q_{\text{defects}}} + \frac{1}{Q_{\text{TED}}} + \frac{1}{Q_{\text{viscous}}} + \frac{1}{Q_{\text{support}}}
  \]
- **Thermoelastic Damping (TED)**
- **Anchor Losses**
  - At high frequency, this is our big problem!
  - Bending CC-Beam
  - Tension ⇒ Cold Spot
  - Heat Flux (TED Loss)
  - Compression ⇒ Hot Spot
  - Elastic Wave Radiation (Anchor Loss)
Thermoelastic Damping (TED)

- Occurs when heat moves from compressed parts to tensioned parts → heat flux = energy loss

\[ \zeta = \frac{1}{2Q} \]

\[ \Gamma(T) = \frac{\alpha^2 TE}{4\rho C_p} \]

\[ \Omega(f_o) = 2 \left[ \frac{f_{TED} f}{f_{TED}^2 + f^2} \right] \]

\[ f_{TED} = \frac{\pi K}{2\rho C_p h^2} \]

TED Characteristic Frequency

- Governed by
  - Resonator dimensions
  - Material properties

**TABLE 1. MATERIAL PROPERTIES**

<table>
<thead>
<tr>
<th>Property</th>
<th>Silicon</th>
<th>Quartz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal expansion</td>
<td>2.60</td>
<td>13.70</td>
</tr>
<tr>
<td>Elastic modulus</td>
<td>1.70</td>
<td>0.75</td>
</tr>
<tr>
<td>Material density</td>
<td>1.33</td>
<td>2.60</td>
</tr>
<tr>
<td>Heat capacity</td>
<td>0.70</td>
<td>0.75</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>1.00</td>
<td>0.10</td>
</tr>
<tr>
<td>Peak damping @ 300°C</td>
<td>1.06</td>
<td>11.34</td>
</tr>
</tbody>
</table>

\[ \rho = \text{material density} \]

\[ C_p = \text{heat capacity at const. pressure} \]

\[ K = \text{thermal conductivity} \]

\[ h = \text{beam thickness} \]

\[ f_{TED} = \text{characteristic TED frequency} \]

Critical Damping Factor, \( \zeta \)

<table>
<thead>
<tr>
<th>Relative Frequency, ( f/f_{TED} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \zeta )</td>
</tr>
<tr>
<td>0.1</td>
</tr>
<tr>
<td>0.2</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.4</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.6</td>
</tr>
<tr>
<td>0.7</td>
</tr>
<tr>
<td>0.8</td>
</tr>
<tr>
<td>0.9</td>
</tr>
<tr>
<td>1.0</td>
</tr>
</tbody>
</table>

[from Roszhart, Hilton Head 1990]
Q vs. Temperature

Quartz Crystal
- Q ~5,000,000 at 30K
- Q ~300,000,000 at 4K

Aluminum Vibrating Resonator
- Q ~500,000 at 30K
- Q ~1,250,000 at 4K

Mechanism for Q increase with decreasing temperature thought to be linked to less hysteretic motion of material defects ⇒ less energy loss per cycle

Even aluminum achieves exceptional Q's at cryogenic temperatures

Polysilicon Wine-Glass Disk Resonator

Compound Mode (2,1)

Resonator Data
- R = 32 μm, h = 3 μm
- d = 80 nm, V_p = 3 V
- f_0 = 61.37 MHz
- Q = 145,780

[from Braginsky, Systems With Small Dissipation]
1.51-GHz, $Q=11,555$ Nanocrystalline Diamond Disk $\mu$Mechanical Resonator

- Impedance-mismatched stem for reduced anchor dissipation
- Operated in the 2nd radial-contour mode
- $Q \approx 11,555$ (vacuum): $Q \approx 10,100$ (air)
- Below: 20 $\mu$m diameter disk

**Design/Performance:**

$R=10\mu$m, $t=2.2\mu$m, $d=800\AA$, $V_p=7V$

$f_o=1.51$ GHz (2nd mode), $Q=11,555$

$Q = 10,100$ (air)

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**Disk Resonator Loss Mechanisms**

- Gas Damping (Not Dominant in Vacuum)
- Hysteretic Motion of Defect (Dwarfed By Substrate Loss)
- Electronic Carrier Drift Loss (Dwarfed By Substrate Loss)
- Substrate Loss Thru Anchors (Dominates)
- No motion along the nodal axis, but motion along the finite width of the stem

$\lambda/4$ helps reduce loss, but not perfect

**Substrate**

**Disk**

**Stem**

**Nodal Axis**

**Strain Energy Flow**

**Mixed Amplitude [dB]**

$Q = 10,100$ (air)
MEMS Material Property Test Structures

Stress Measurement Via Wafer Curvature

- Compressively stressed film → bends a wafer into a convex shape
- Tensile stressed film → bends a wafer into a concave shape
- Can optically measure the deflection of the wafer before and after the film is deposited
- Determine the radius of curvature $R$, then apply:

$$\sigma = \frac{E' h^2}{6Rt}$$

Where:
- $\sigma$ = film stress [Pa]
- $E'$ = $E/(1-v)$ = biaxial elastic modulus [Pa]
- $h$ = substrate thickness [m]
- $t$ = film thickness
- $R$ = substrate radius of curvature [m]
MEMS Stress Test Structure

• **Simple Approach:** use a clamped-clamped beam
  - Compressive stress causes buckling
  - Arrays with increasing length are used to determine the critical buckling load, where

  \[
  \sigma_{\text{critical}} = \frac{\pi^2 \cdot E \cdot h^2}{3 \cdot L^2}
  \]

  1. \(E\) = Young’s modulus [Pa]
  2. \(I = \frac{1}{12}Wh^3\) = moment of inertia
  3. \(L, W, h\) indicated in the figure

• **Limitation:** Only compressive stress is measurable

More Effective Stress Diagnostic

• Single structure measures both compressive and tensile stress
• Expansion or contraction of test beam \(\rightarrow\) deflection of pointer
• Vernier movement indicates type and magnitude of stress
**Q Measurement Using Resonators**

**Compound Mode (2,1)**

- **Wine Glass Disk Resonator**
  - \( R = 32 \mu m \)

**Resonator Data**
- \( R = 32 \mu m, h = 3 \mu m \)
- \( d = 80 \) nm, \( V_p = 3 \) V

**Frequency [MHz]**
- \( f_0 = 61.37 \) MHz
- \( Q = 145,780 \)

**Unmatched Transmission [dB]**
- \(-40\) dB to \(-100\) dB

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**Folded-Beam Comb-Drive Resonator**

- **Issue w/ Wine-Glass Resonator**: non-standard fab process
- **Solution**: use a folded-beam comb-drive resonator

**Capacitive-Comb Transducer**
- Ground Plane
- Inner Beam
- Outer Beam
- Coupling Beam (not part of resonator)

**Input Force, \( f_i \)**
- (Port 3)
- Shuttle

**Amplitude [dB]**
- \( f_c = 342.5 \) kHz
- \( Q = 41,000 \)

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**Comb-Drive Resonator in Action**

- Below: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach

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**Folded-Beam Comb-Drive Resonator**

- **Issue w/ Wine-Glass Resonator**: non-standard fab process
- **Solution**: use a folded-beam comb-drive resonator

---

**8.3 Hz**

\[ f = \frac{Q}{2\pi} \]

\[ f = 342.5 \text{kHz} \]

\[ Q = 41,000 \]

\[ Q = \frac{f_0}{8.3} \]
**Measurement of Young's Modulus**

- Use micromechanical resonators
  - Resonance frequency depends on $E$
  - For a folded-beam resonator:
    
    \[
    f_o = \left( \frac{4Eh(W/L)^3}{M_{eq}} \right)^{1/2}
    \]

    where $h = $ thickness

- Extract $E$ from measured frequency $f_o$
- Measure $f_o$ for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters

**Anisotropic Materials**
Elastic Constants in Crystalline Materials

- Get different elastic constants in different crystallographic directions → 81 of them in all
  - Cubic symmetries make 60 of these terms zero, leaving 21 of them remaining that need be accounted for
- Thus, describe stress-strain relations using a 6x6 matrix

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yz} \\
\tau_{zx} \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\
C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\
C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36} \\
C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46} \\
C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56} \\
C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix}
\]

Stresses  Stiffness Coefficients  Strains

Stiffness Coefficients of Silicon

- Due to symmetry, only a few of the 21 coefficients are non-zero
- With cubic symmetry, silicon has only 3 independent components, and its stiffness matrix can be written as:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yz} \\
\tau_{zx} \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & 0 & 0 & 0 & 0 \\
C_{12} & C_{22} & 0 & 0 & 0 & 0 \\
C_{11} & C_{12} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{zx} \\
\gamma_{xy}
\end{bmatrix}
\]

where \( C_{11} = 165.7 \text{ GPa} \)
\( C_{12} = 63.9 \text{ GPa} \)
\( C_{44} = 79.6 \text{ GPa} \)
Young's Modulus in the (001) Plane

Poisson Ratio in (001) Plane
Anisotropic Design Implications

• Young's modulus and Poisson ratio variations in anisotropic materials can pose problems in the design of certain structures
  • E.g., disk or ring resonators, which rely on isotropic properties in the radial directions
    • Okay to ignore variation in RF resonators, although some Q hit is probably being taken
  • E.g., ring vibratory rate gyroscopes
    • Mode matching is required, where frequencies along different axes of a ring must be the same
    • Not okay to ignore anisotropic variations, here