PROBLEM SET #6

Issued: Tuesday, Nov. 25, 2008

Due (at 5 p.m.): Tuesday, Dec. 9, 2008 (Advice: get it done earlier to help with your project)

1. Gyroscopes are inertial sensors that measure rotation rate, which is an extremely important variable to know when navigating. In particular, one must know rotation rate (as well as other parameters, e.g., time, linear acceleration, …) in order to determine position accurately (without the aid of GPS). Among the applications that use gyroscopes are airplanes (for navigation), boats (again, for navigation), automobiles (for skid control, among other applications), GPS receivers (to allow position determination during periods when the GPS signal cannot be received), and game controllers (e.g., the Wii). Of these applications, the last three already use MEMS-based gyroscopes, and the first two are presently targeted by MEMS realizations.

Gyroscopes operate by taking of advantage of the conservation of momentum, where an object moving in a given direction with a certain momentum will tend to continue moving in that direction even if its frame of reference is rotated about an axis. This is perhaps best further explained via example.

This problem involves the MEMS-based micro-gyroscope [by Alper & Akin] summarized in Figs. 1-1 to 1-3 and fabricated using a multilayer electroformed-nickel technology (called EFAB). In this device, momentum is generated by driving the outer frame into resonance vibration using the capacitive comb fingers along the y-axis. When the chassis of the device is rotated about the *x*-axis (indicated in Fig. 1-1), the vibrating mass will attempt to preserve its momentum in the original *y*-direction, which will then make the mass appear to deflect in the *z*-direction. This *z*-directed motion is then sensed by parallel-plate capacitive comb fingers to determine the rotation rate. In quantitative terms, the angular velocity is sensed about *x*-axis, which then generates a *Coriolis Force* ($\vec{F}_C = -2m\vec{\omega} \times \vec{v}$) that acts on the inner gimbal along the *z*-direction, which is then picked up by the varying gap capacitive comb fingers.

Several figures are provided to help you answer the questions to follow. In particular, measured or target parameter values for the fabricated device are provided in Table 1. The drive and sense mode shapes as well as the key dimensions are illustrated in Figs. 1-2 (a) and (b), respectively; while zoom-in views of the driving and sensing electrode pairs are shown in Figs. 1-3 (a) and (b), respectively. As indicated, the gaps of each electrode pair are all 4μm. The thicknesses of the proof masses are all 240μm. Finally, assume for this problem that a DC bias voltage of 25V is applied to the movable structure and all electrodes are at DC ground (but some have AC signals applied).

(a) Calculate the values of proof mass and needed spring constants at the electrode locations for (i) the drive; and (ii) the sense modes. Assume for this problem that the volumes of the proof masses are much larger than that of their fingers, so you can neglect the masses of their fingers. Likewise, you can ignore the masses of the springs in this problem.

(b) If the suspension beams for the inner drive-mode mass have a thickness of 4 μm and a width of 40 μm, what suspension beam length would be required to achieve the spring constant calculated in (a)?

(c) Identify the electrodes for the (i) drive and (ii) sense modes and determine the capacitance and change in capacitance per unit displacement for each. (Note that there are two distinct drive mode ports and three distinct sense mode ports.)

(d) Draw and specify (numerically) all element values in the equivalent circuits (transformers + *LCR*) modeling the (i) drive mode, from one electrical port to the other; and (ii) the sense mode, from the center electrical port to the outside ones. Again, for the sense mode, there should be three ports total.

(e) Code the equivalent circuits in (d) into SPICE netlists, add the necessary elements (e.g., a resistor, a capacitor, or an inductor?) to detect displacement, and simulate Bode plots for the voltage-to-displacement transfer functions using SPICE that include the low frequency and resonance responses of the structures. For the sense mode, apply the input voltage to the center port and sense its motion from the outer ports shorted together.

(f) Repeat (e), but for voltage-to-velocity transfer functions. In other words, this time replace the added detector elements in (e) with the necessary elements to detect velocity.

(g) Using the analytical transfer function for the sense mode, find the percent change in scale factor when the temperature changes from 0°C to 70°C. For this problem, assume that *Q* does not change with temperature.

Table 1

	Parameters	**Measured/Target Value**
	Young's Modulus (GPa)	200
	Density (g/cm^3)	8.908
	Thermal expansion coefficient (10^{-6}/K)	1.5
	Thermal expansion coefficient of substrate (10^{-6}/K)	0.7
	Thermal coefficient of Young's modulus (10^{-6}/K)	-330
Drive Mode	Resonance Frequency (Hz)	7000
	Quality Factor	100
Sense Mode	Resonance Frequency (Hz)	8250
	Quality Factor	20

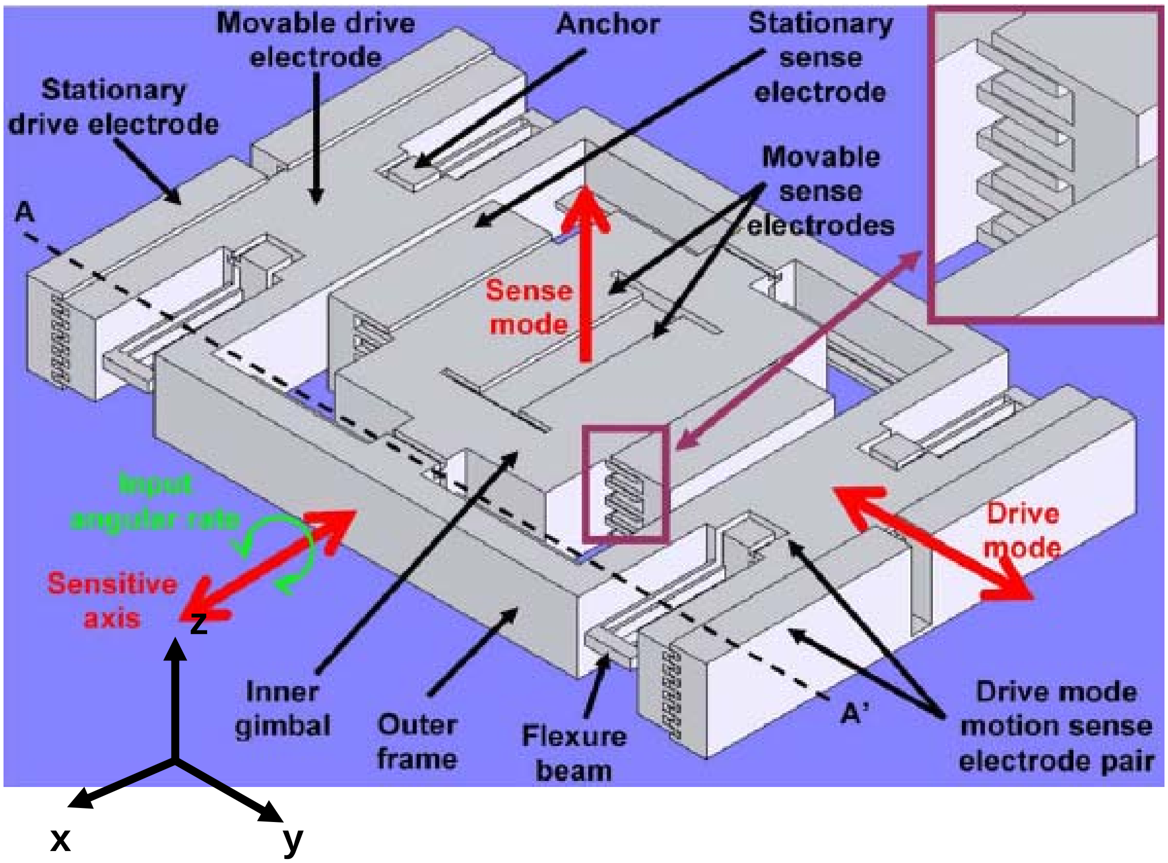

Fig. 1-1 (a): Isometric view

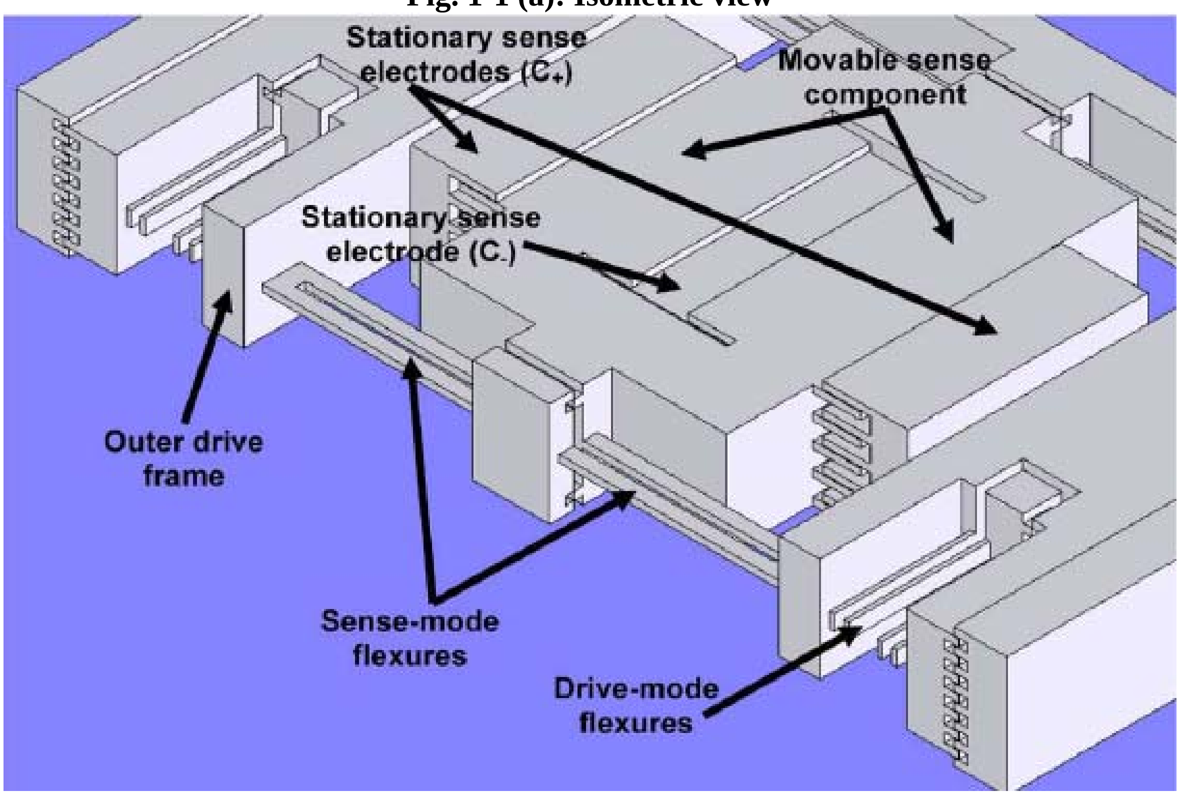

Fig. 1-1 (b): A-A' cross section of (a)

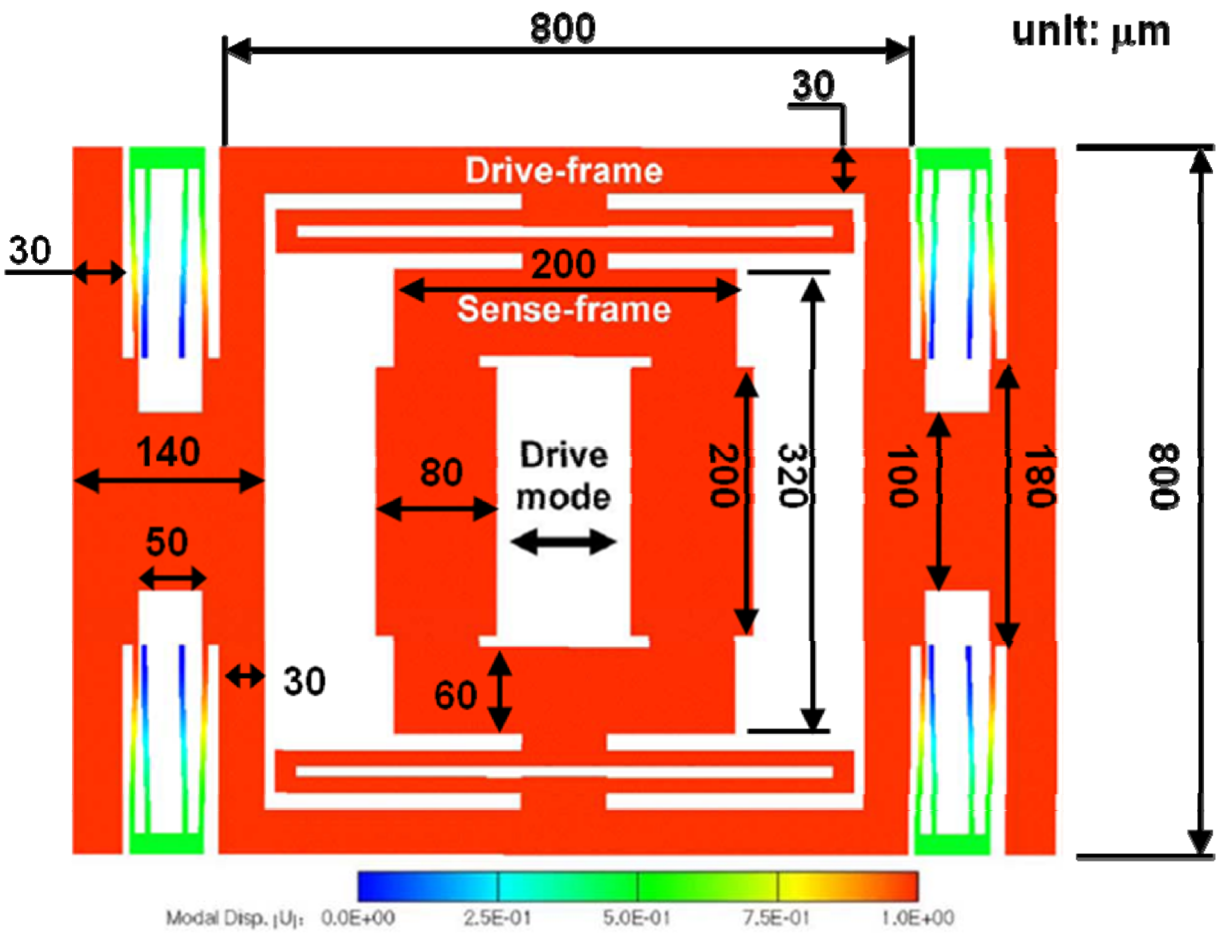

Fig. 1-2 (a): Layout showing drive-mode displacement in the *y* direction

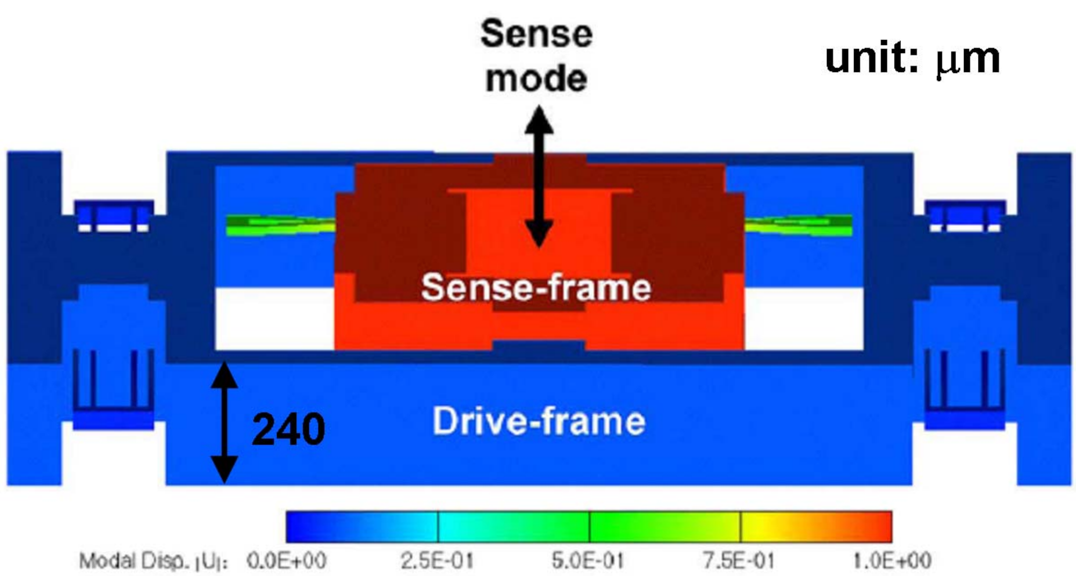

Fig. 1-2 (b) 3-D view showing sense-mode displacement in the *z* direction

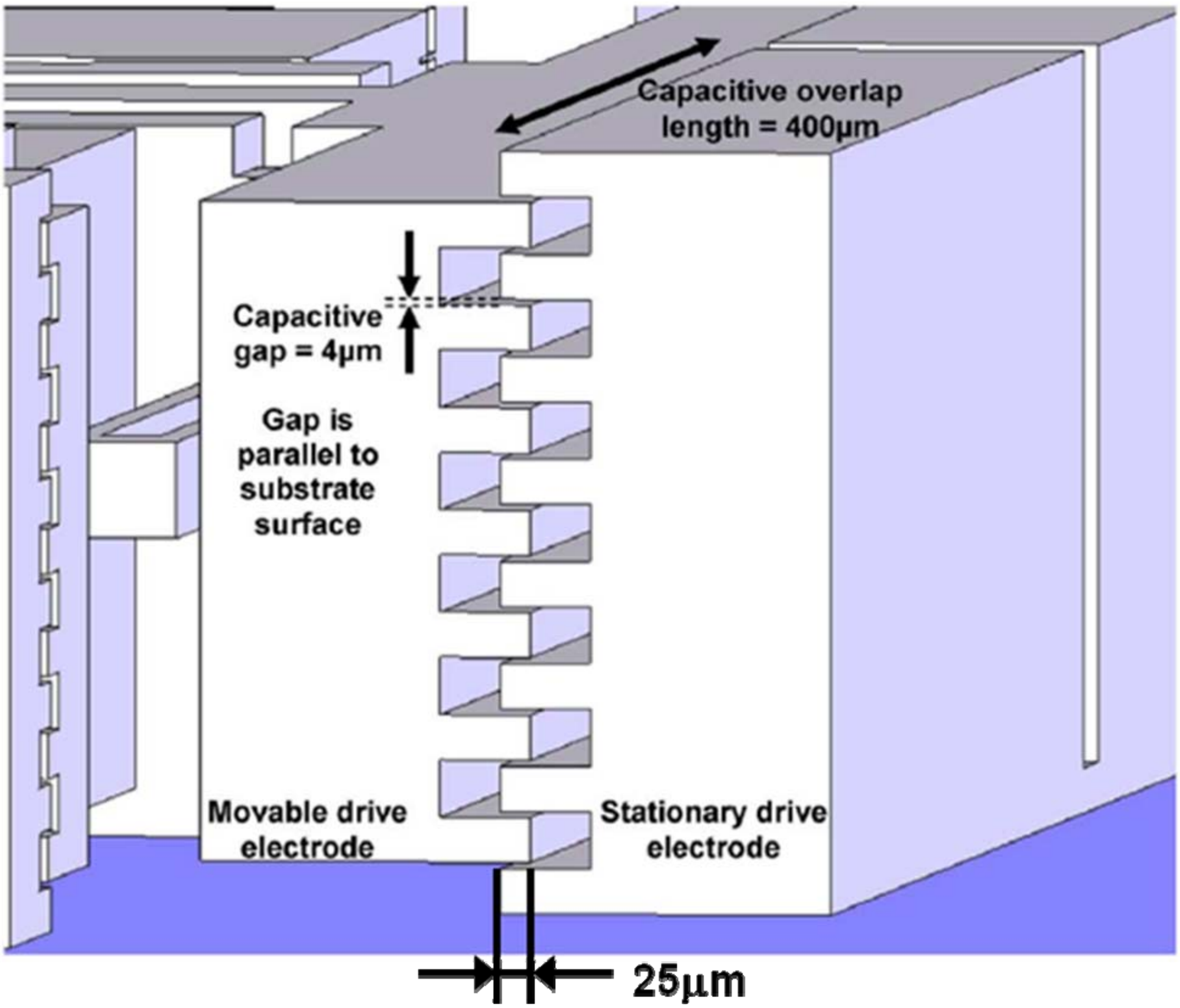

Fig. 1-3 (a): Drive electrode

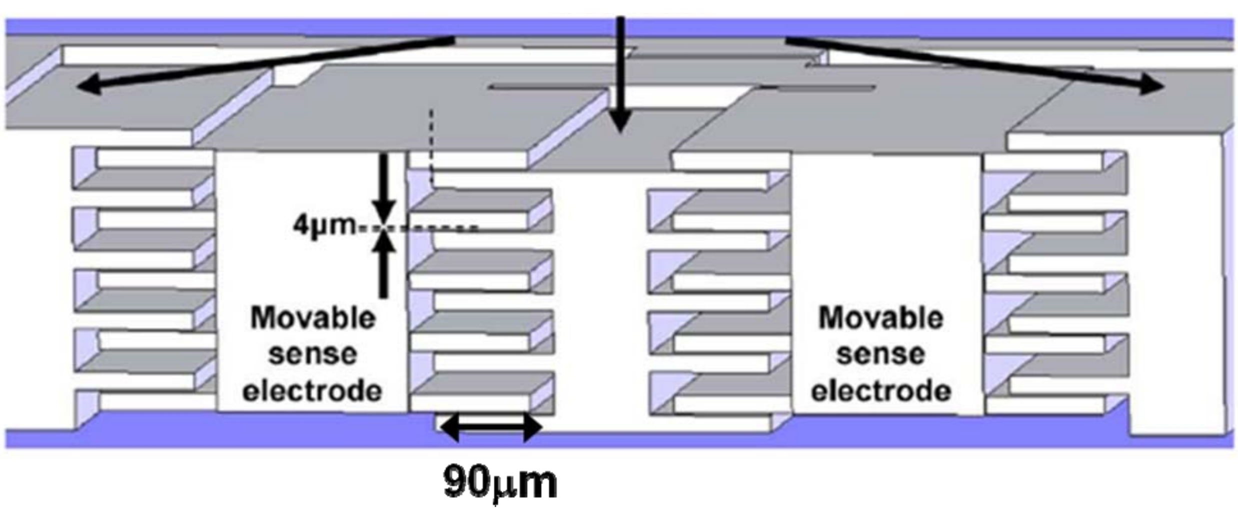

Fig. 1-3 (b): Sense electrodes