EE C245 – ME C218
Introduction to MEMS Design
Fall 2007

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Lecture 23: Electrical Stiffness

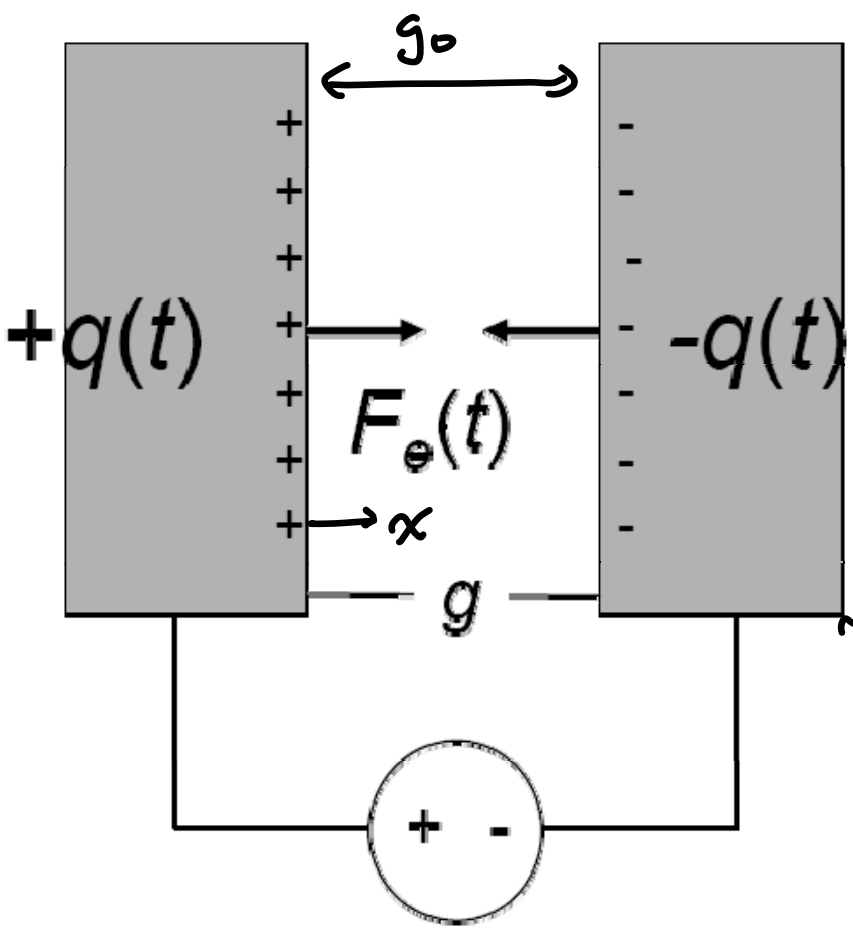
Lecture Outline

- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
    - Charge Control
    - Voltage Control
  - Parallel-Plate Capacitive Transducers
    - Linearizing Capacitive Actuators
    - Electrical Stiffness
  - Electrostatic Comb-Drive
    - 1st Order Analysis
    - 2nd Order Analysis

Linearizing the Voltage-to-Force Transfer Function

Linearizing the Voltage-to-Force T.F.

- Apply a DC bias (or polarization) voltage V_P together with the intended input (or drive) voltage $v_i(t)$, where $V_P >> v_i(t)$



$$v(t) = V_P + v_i(t)$$

$$F_e(t) = \frac{\partial W'}{\partial x} = \frac{\partial}{\partial x}\left(\frac{1}{2} C [v(t)]^2\right)$$

$$= \frac{1}{2}\frac{\partial C}{\partial x}[v(t)]^2 = \frac{1}{2}(V_P + v_i(t))^2 \frac{\partial C}{\partial x}$$

$$= \frac{1}{2}\left[V_P^2 + 2V_P v_i(t) + \cancel{[v_i(t)]^2}\right]\frac{\partial C}{\partial x}$$

$$[V_P >> v_i(t)] \Rightarrow F_e(t) = \underbrace{\frac{1}{2}V_P^2 \frac{\partial C}{\partial x}}_{\text{DC Offset}} + \underbrace{V_P \frac{\partial C}{\partial x} v_i(t)}_{\text{AC Drive Signal}}$$

linear

$V_P \sim 10V$, $|v_i| \sim 10mV$

$$C(x) = \frac{\epsilon A}{g_o - x} = C_o\left(1 - \frac{x}{g_o}\right)^{-1} \approx C_o\left(1 + \frac{x}{g_o}\right)$$

$\frac{\epsilon A}{g_o} = C_o$

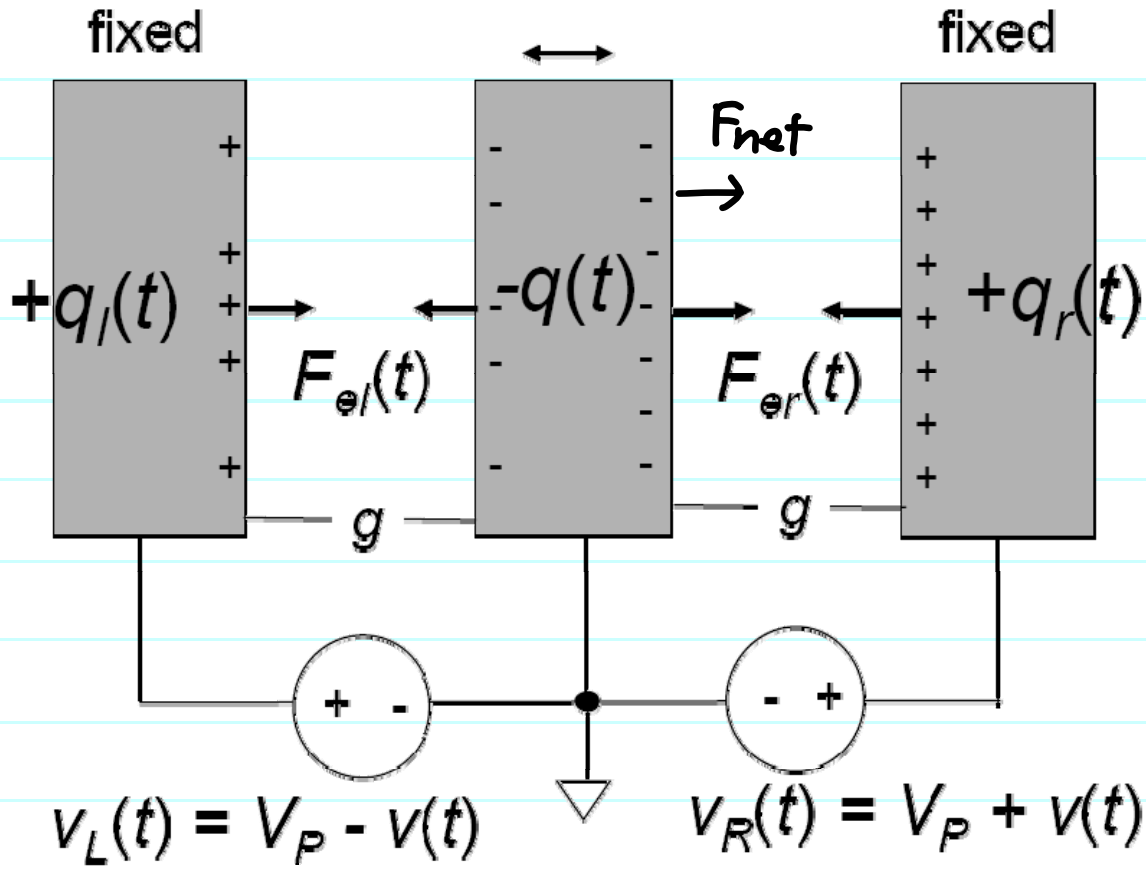
$[x << g_o] \longrightarrow$ ~linear

$$\therefore \frac{\partial C}{\partial x} \approx \frac{C_o}{g_o} \Rightarrow \boxed{F_e(t) = \frac{1}{2}\frac{C_o}{g_o}V_P^2 + V_P \frac{C_o}{g_o} v_i(t)}$$

Differential Capacitive Transducer

- The net force on the suspended center electrode is

$$F_{net} = F_{er}(t) - F_{el}(t)$$



Do the math.

Assume matched gaps.

$$F_{net}(t) = \frac{1}{2}\frac{\partial C}{\partial x}\left\{[v_R(t)]^2 - [v_L(t)]^2\right\}$$

$$= \frac{1}{2}\frac{\partial C}{\partial x}\left\{\left(\cancel{V_P^2} + 2V_P v(t) + \cancel{[v(t)]^2}\right) - \left(\cancel{V_P^2} - 2V_P v(t) + \cancel{[v(t)]^2}\right)\right\}$$

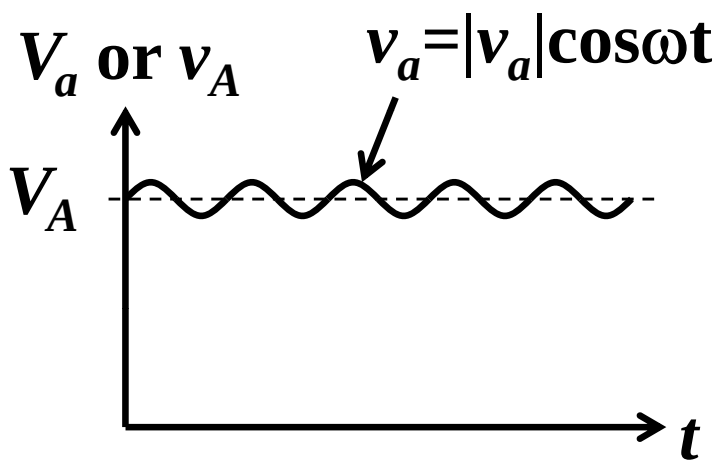
$$\therefore \boxed{F_{net}(t) = 2V_P\frac{\partial C}{\partial x}v(t) = 2V_P\frac{C_o}{g_o}v(t)}$$

$\Rightarrow$ Linear w/ $v(t)$! (gap match limited)

Remaining Nonlinearity (Electrical Stiffness)

Parallel-Plate Capacitive Nonlinearity

- Example: clamped-clamped laterally driven beam with balanced electrodes
- Nomenclature:

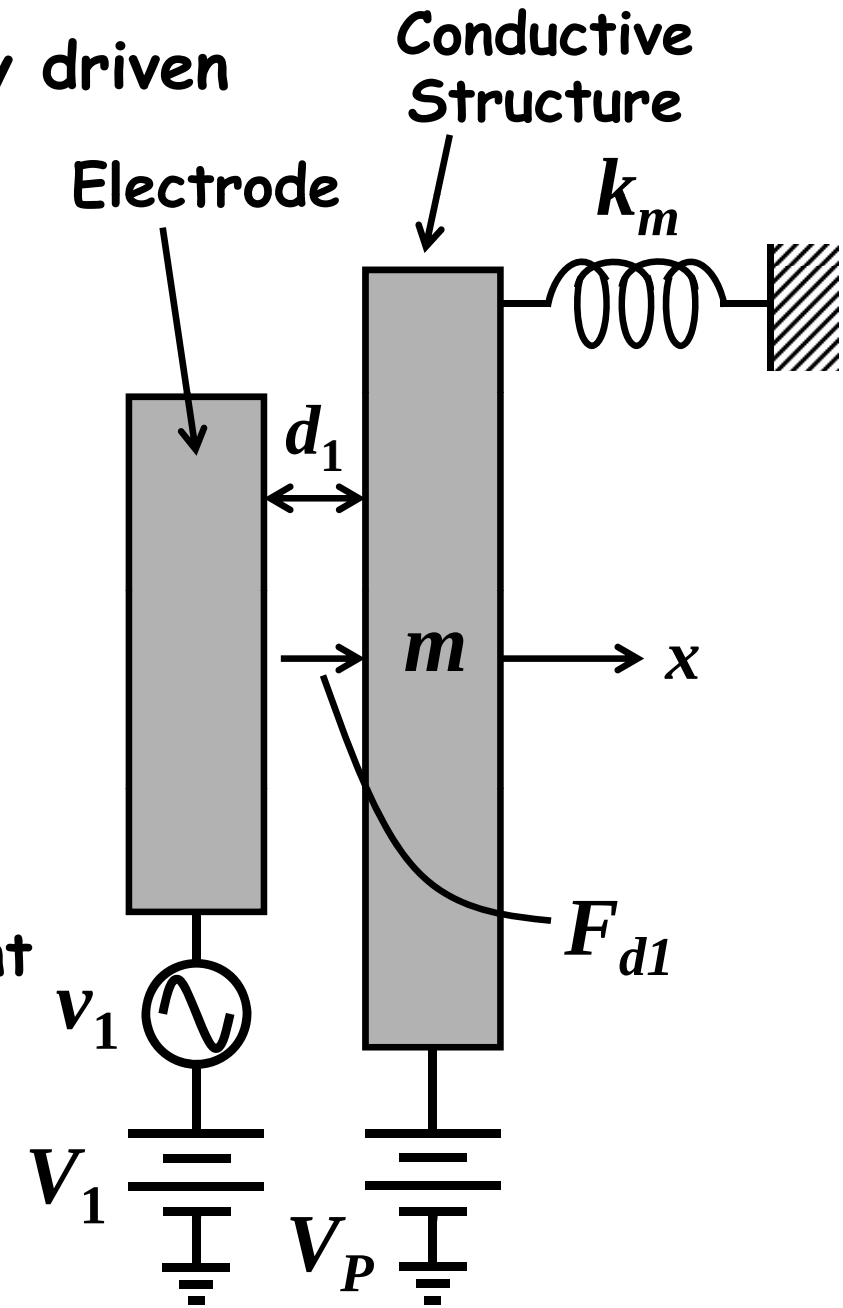


$$V_a \text{ or } v_A = V_A + v_a$$

Total Value

DC Component (upper case variable; upper case subscript)

AC or Signal Component (lower case variable; lower case subscript)



Parallel-Plate Capacitive Nonlinearity

- <u>Example</u>: clamped-clamped laterally driven beam with balanced electrodes
- Expression for $\partial C/\partial x$:

$$C_1(x) = \frac{\epsilon A}{d_1 + x} = C_{o1}\left(1 + \frac{x}{d_1}\right)^{-1} \rightarrow \frac{\partial C_1}{\partial x} = -\frac{C_{o1}}{d_1}\left(1 + \frac{x}{d_1}\right)^{-2}$$

$C_{o1} = \frac{\epsilon A}{d_1}$

[Expand the Taylor series further]

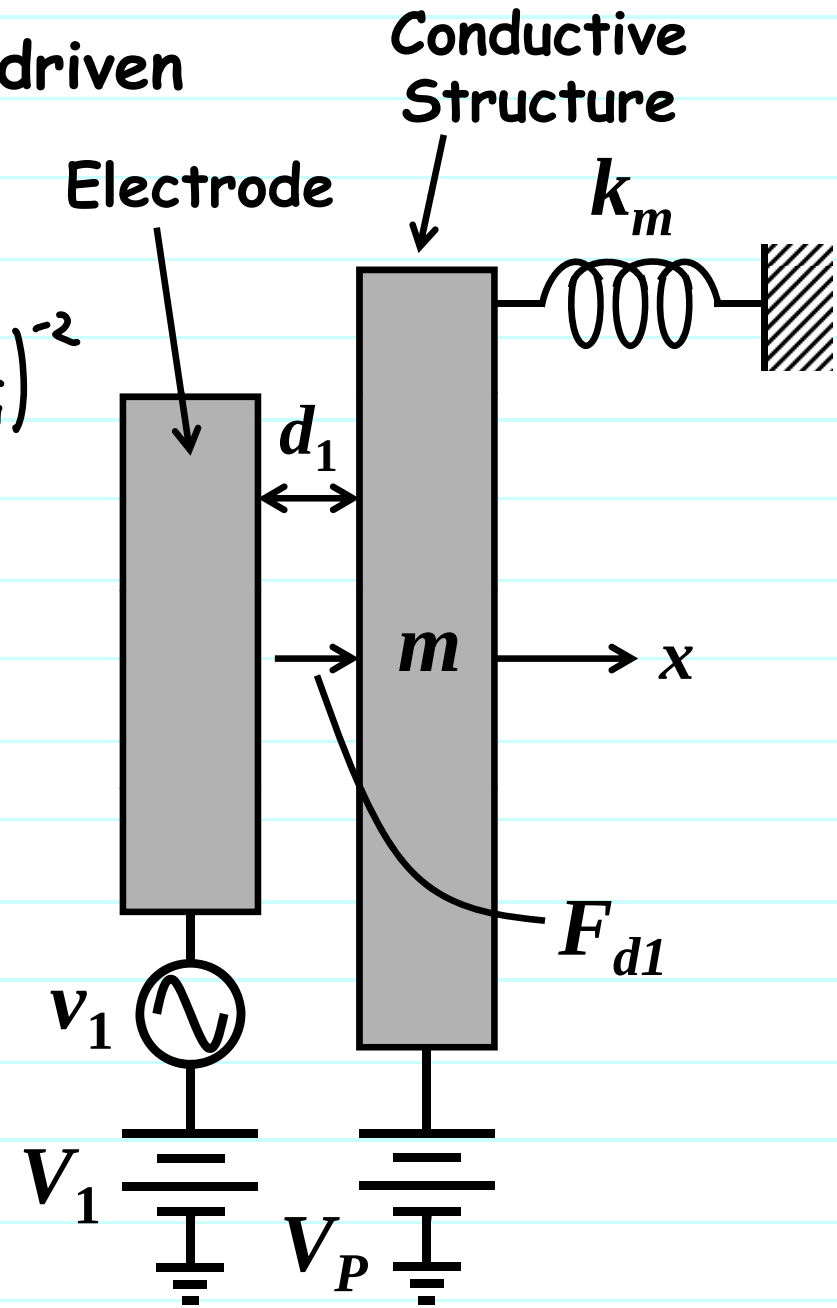
$$\frac{\partial C_1}{\partial x} = -\frac{C_{o1}}{d_1}\left(1 + A_1 x + A_2 x^2 + A_3 x^3 + \cdots\right)$$

where $A_1 = -\frac{2}{d_1}$

$A_2 = \frac{3}{d_1^2}$

$A_3 = -\frac{4}{d_1^3}$

$\vdots$



Parallel-Plate Capacitive Nonlinearity

- Thus, the expression for force from the left side becomes:

$V_{P1} = V_P - V_1$

$$F_{d1} = \frac{1}{2}\frac{\partial C}{\partial x}(V_P - V_1 - v_1)^2 = \frac{1}{2}\frac{\partial C}{\partial x}(V_{P1} - v_1)^2$$

[small displacements: $x \ll d_1$]

$$F_{d1} = \frac{1}{2}\left(-\frac{C_{01}}{d_1}\right)(1 + A_1 x)(V_{P1}^2 - 2V_{P1}v_1 + v_1^2)$$

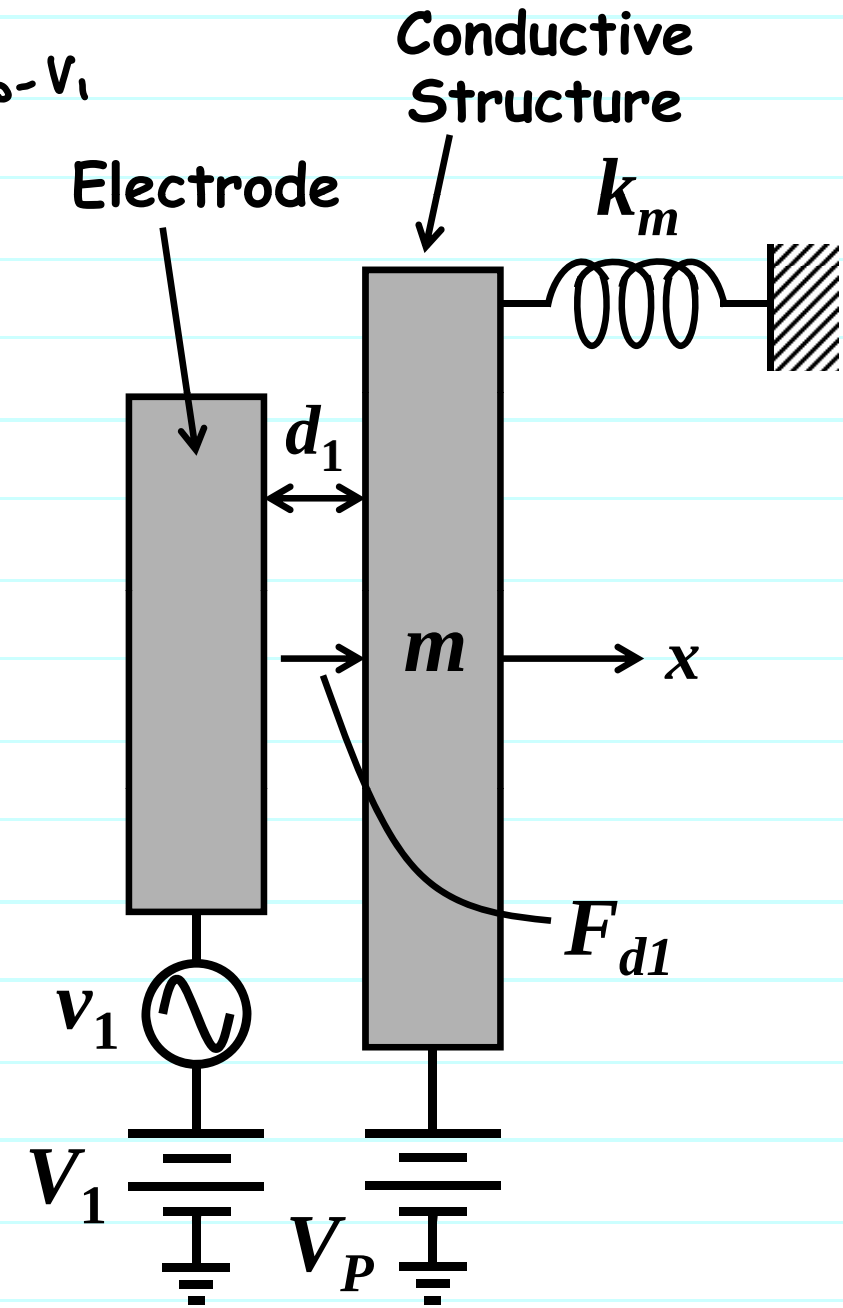
$$= \frac{1}{2}\left(-\frac{C_{01}}{d_1}\right)\left\{V_{P1}^2 - 2V_{P1}v_1 + v_1^2 + A_1 V_{P1}^2 x - 2A_1 V_{P1} x v_1 + A_1 x v_1^2\right\}$$

@ resonance: $x = \frac{QF_{d1}}{jk} \cong \frac{Q}{jk}\frac{\partial C}{\partial x}V_{P1}v_1$

Thus,

$v_1 = |v_1|\cos\omega_0 t \rightarrow x = |x|\sin\omega_0 t$

x 90° phase-shifted from v_1



Parallel-Plate Capacitive Nonlinearity

- Retaining only terms at the drive frequency:

$$F_{d1}\Big|_{\omega_o} = \underbrace{V_{P1}\frac{C_{o1}}{d_1}|v_1|\cos\omega_o t}_{\text{Drive force arising from the input excitation voltage at the frequency of this voltage}} + V_{P1}^2\frac{C_{o1}}{d_1^2}|x|\sin\omega_o t$$

Proportional to displacement

90° phase-shifted from drive, so in phase with displacement

- These two together mean that this force acts against the spring restoring force!
  - A negative spring constant
  - Since it derives from V_P, we call it the electrical stiffness, given by:

$$\boxed{k_e = V_{P1}^2\frac{C_{o1}}{d_1^2} = V_{P1}^2\frac{\varepsilon A}{d_1^3}}$$

Electrical Stiffness, k_e

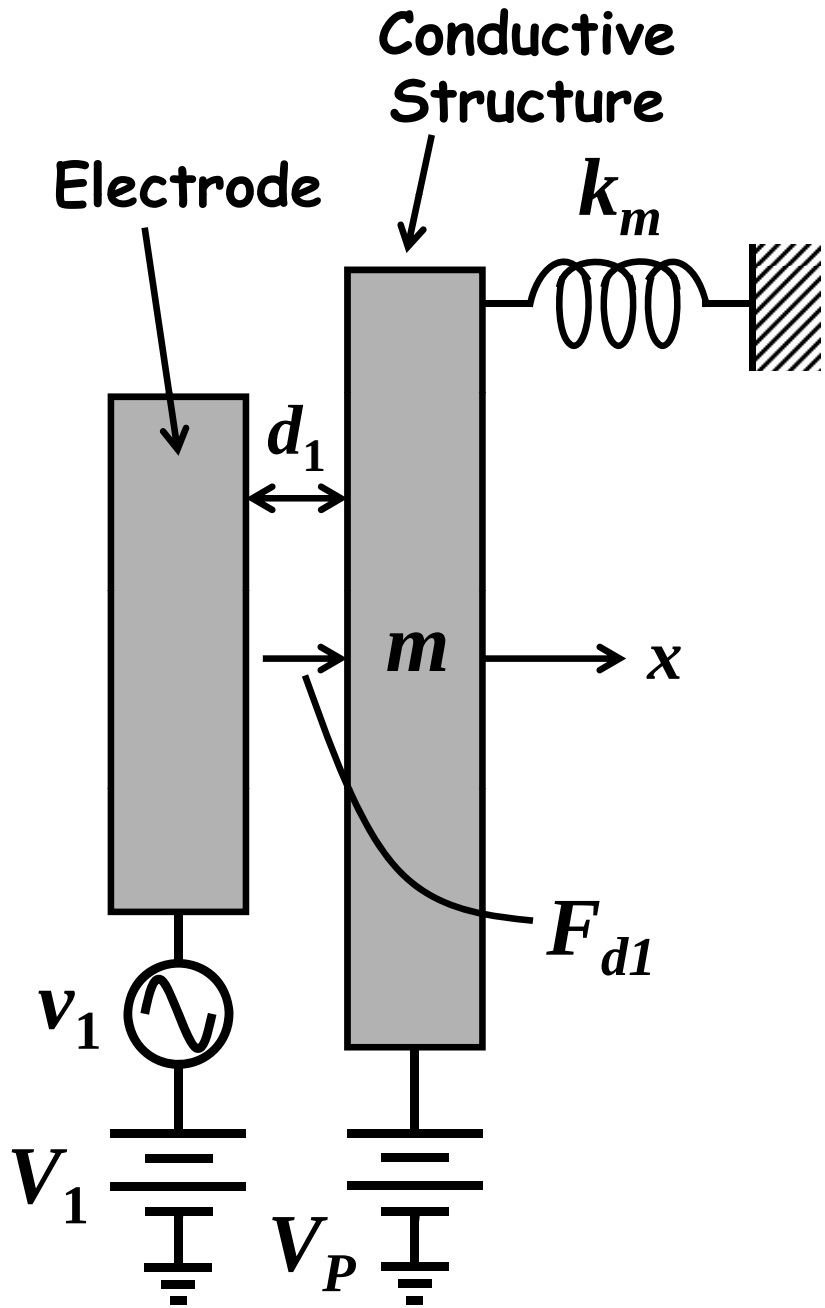
- The electrical stiffness k_e behaves like any other stiffness
- It affects resonance frequency:

$$\omega_o' = \sqrt{\frac{k}{m}} = \sqrt{\frac{k_m - k_e}{m}}$$

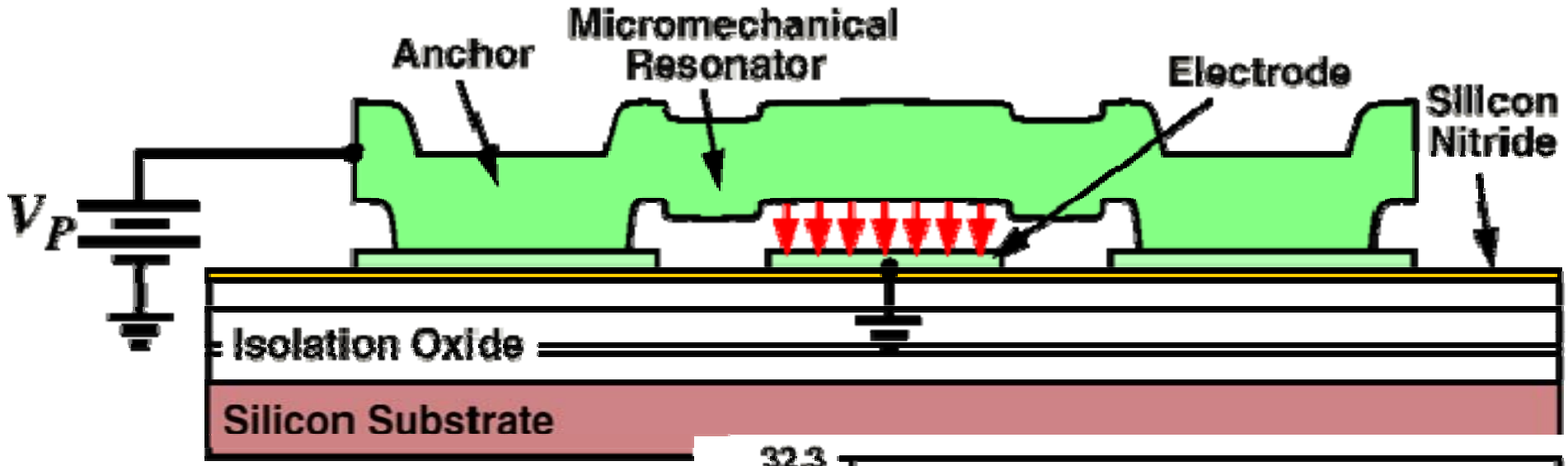
$$= \sqrt{\frac{k_m}{m}}\left(1 - \frac{k_e}{k_m}\right)^{1/2}$$

$$\boxed{\omega_o' = \omega_o\left(1 - \frac{V_{P1}^2}{k_m}\frac{\varepsilon A}{d_1^3}\right)^{1/2}}$$

Frequency is now a function of dc-bias V_{P1}



Voltage-Controllable Center Frequency



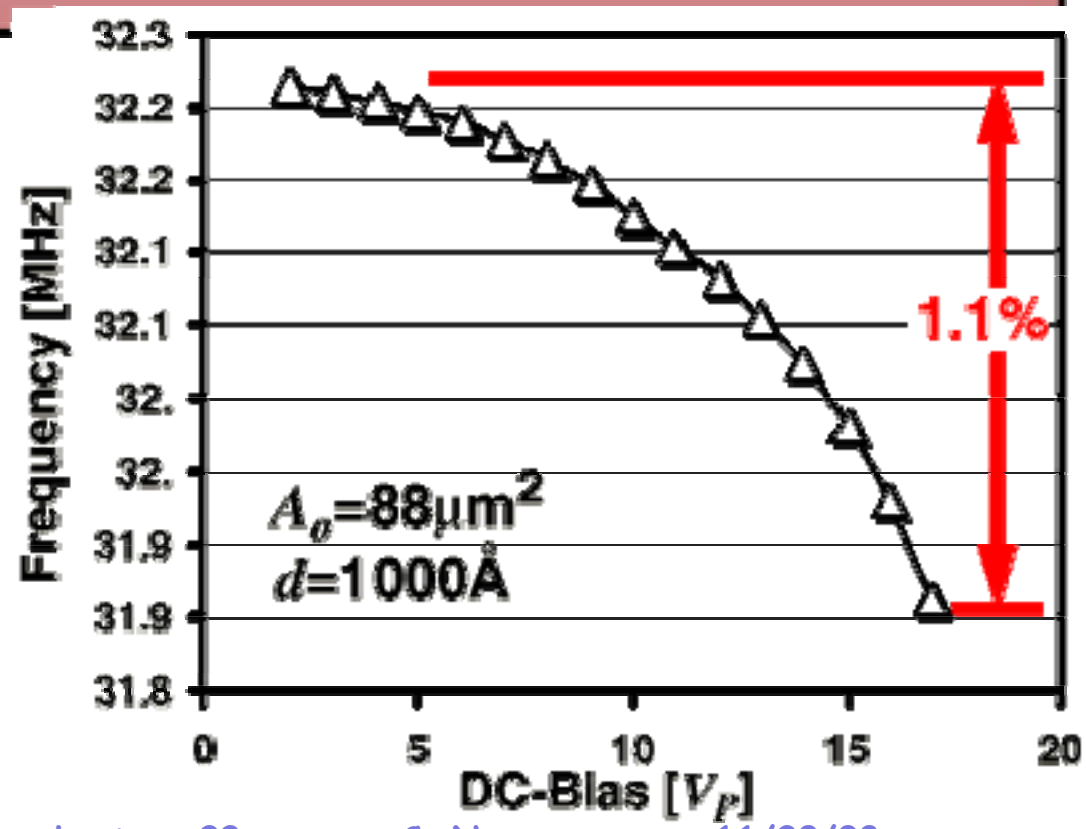
- Quadrature force ➡ voltage-controllable electrical stiffness:

$$k_e = \frac{\varepsilon_o A_o}{d^3} V_P^2$$

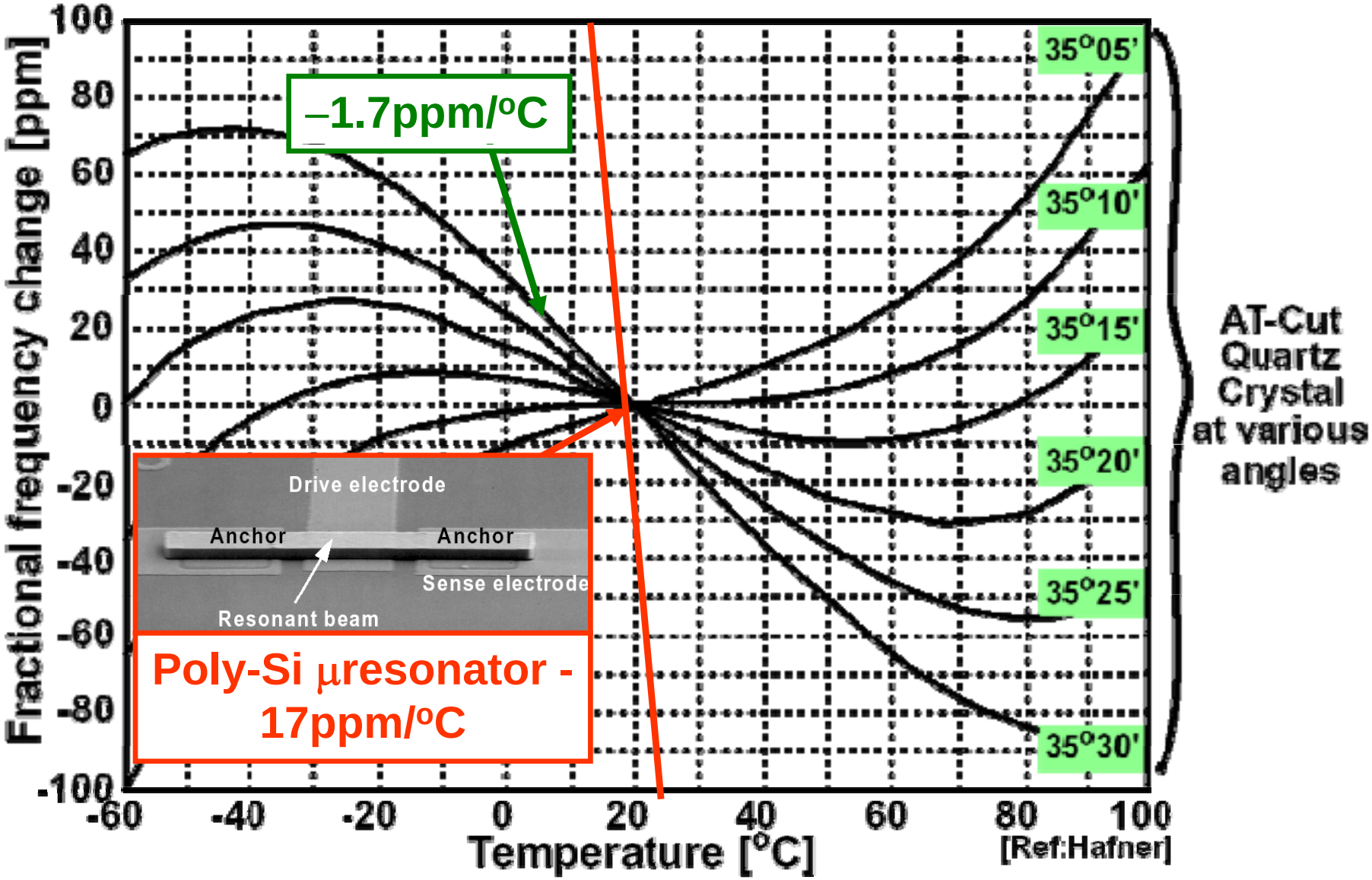
Electrode Overlap Area

Gap

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}}$$



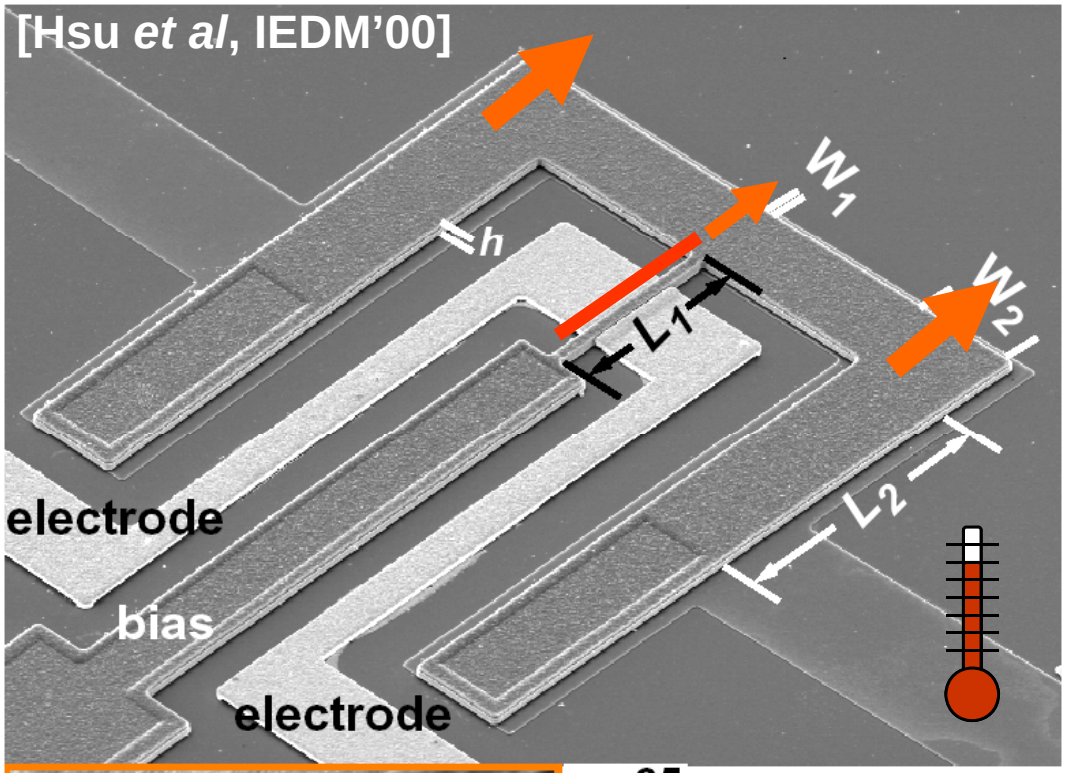
Microresonator Thermal Stability

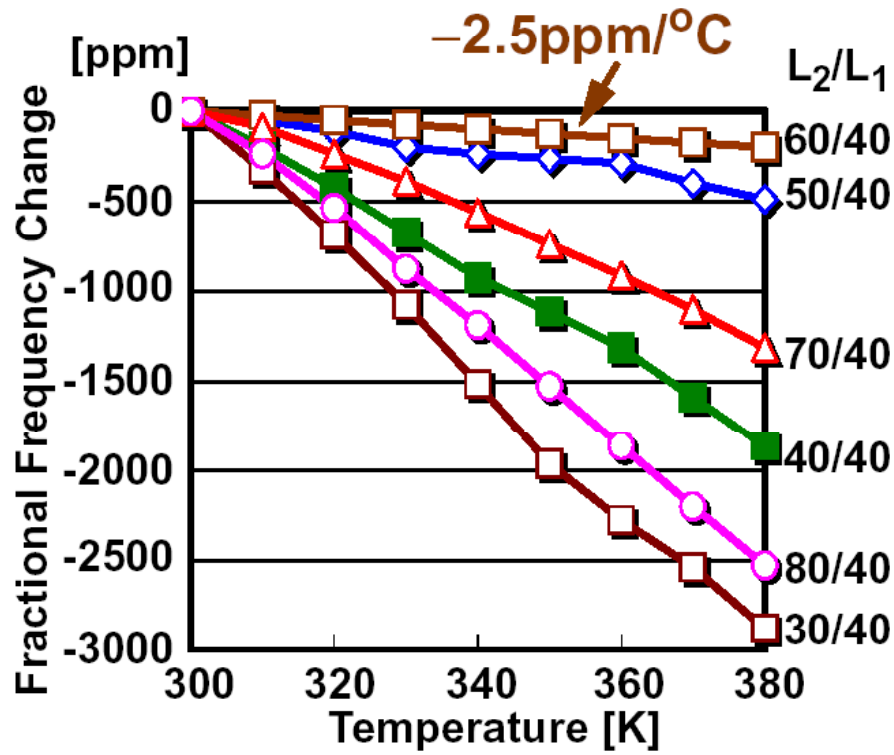


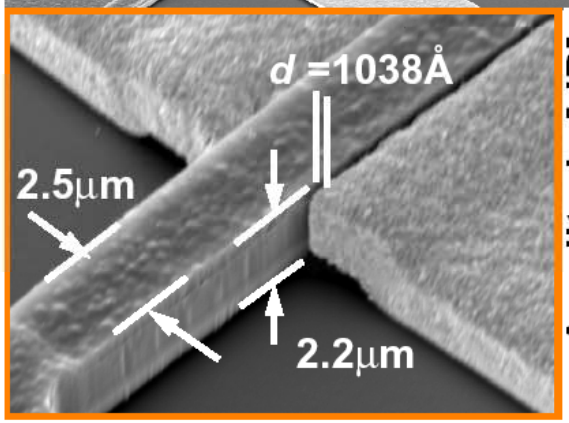
- Thermal stability of poly-Si micromechanical resonator is 10X worse than the worst case of AT-cut quartz crystal

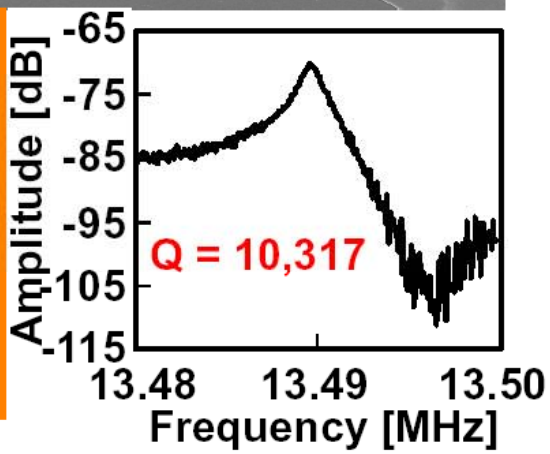
Geometric-Stress Compensation

- Use a temperature dependent mechanical stiffness to null frequency shifts due to Young's modulus thermal dep.





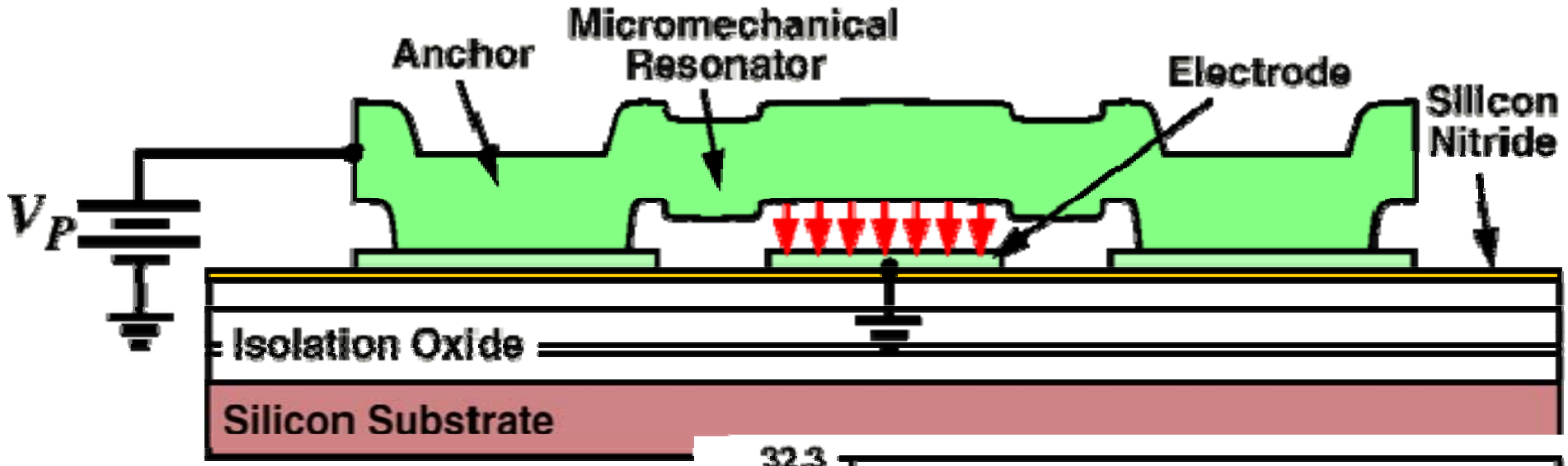




- ***Problems***:
  - stress relaxation
  - compromised design flexibility

[Hsu *et al* IEDM 2000]

Voltage-Controllable Center Frequency



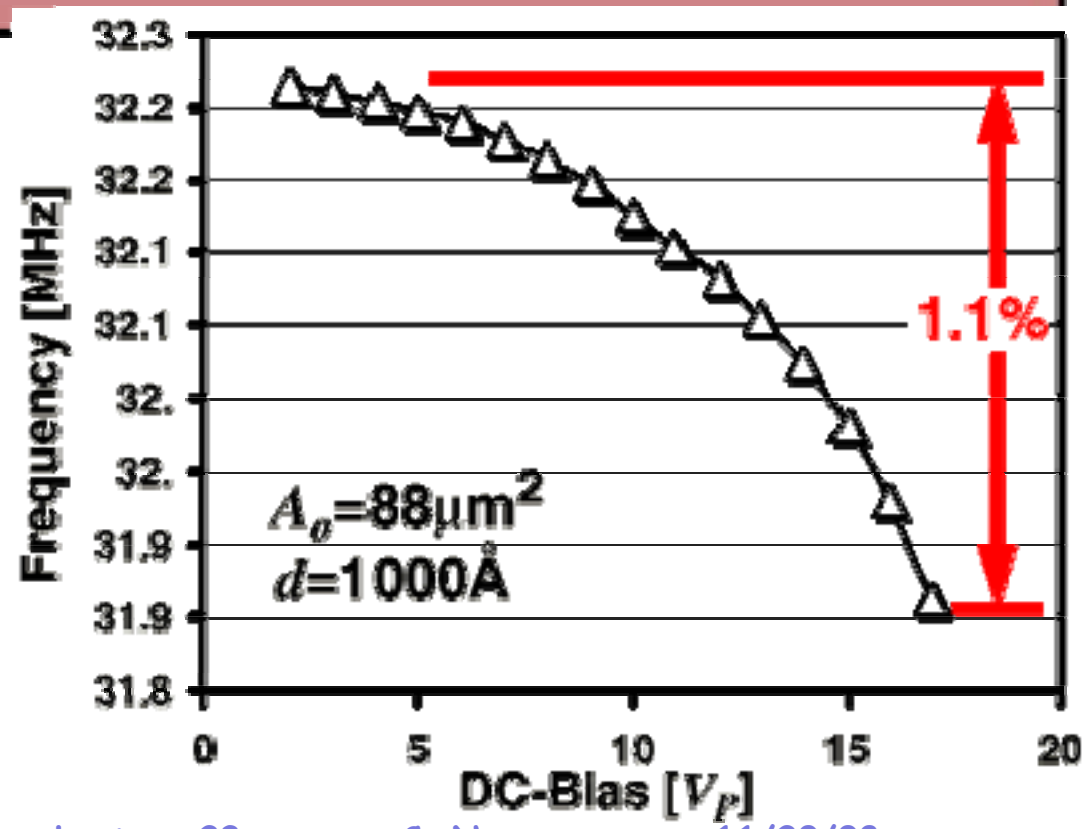
- Quadrature force ➡ voltage-controllable electrical stiffness:

$$k_e = \frac{\varepsilon_o A_o}{d^3} V_P^2$$

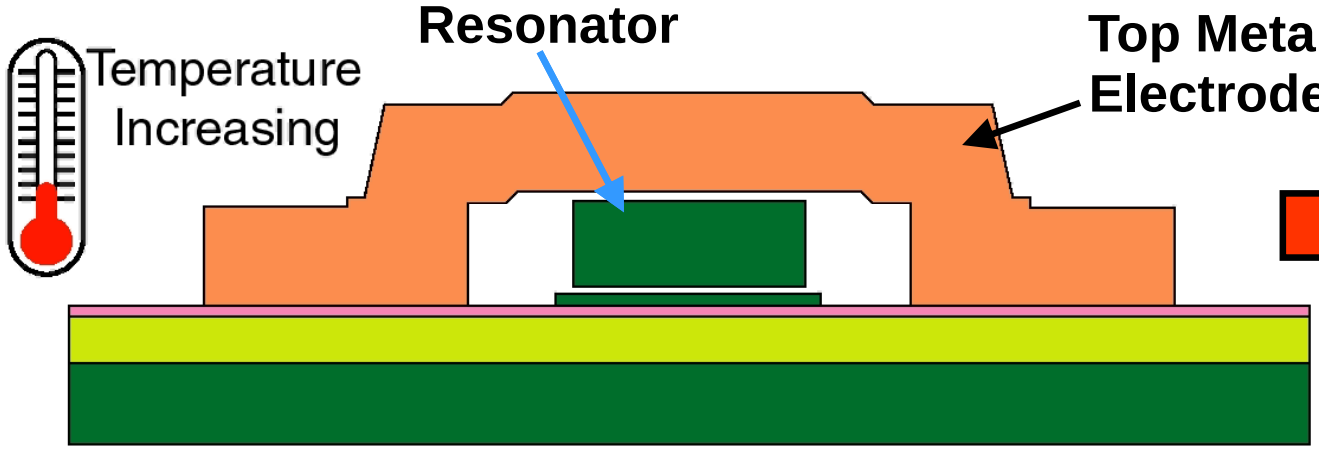
Electrode Overlap Area

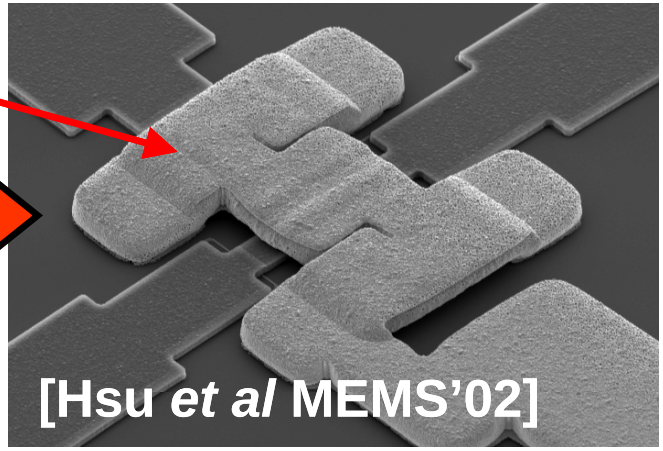
Gap

$$f_o = \frac{1}{2\pi}\sqrt{\frac{k_m - k_e}{m_r}}$$



Excellent Temperature Stability



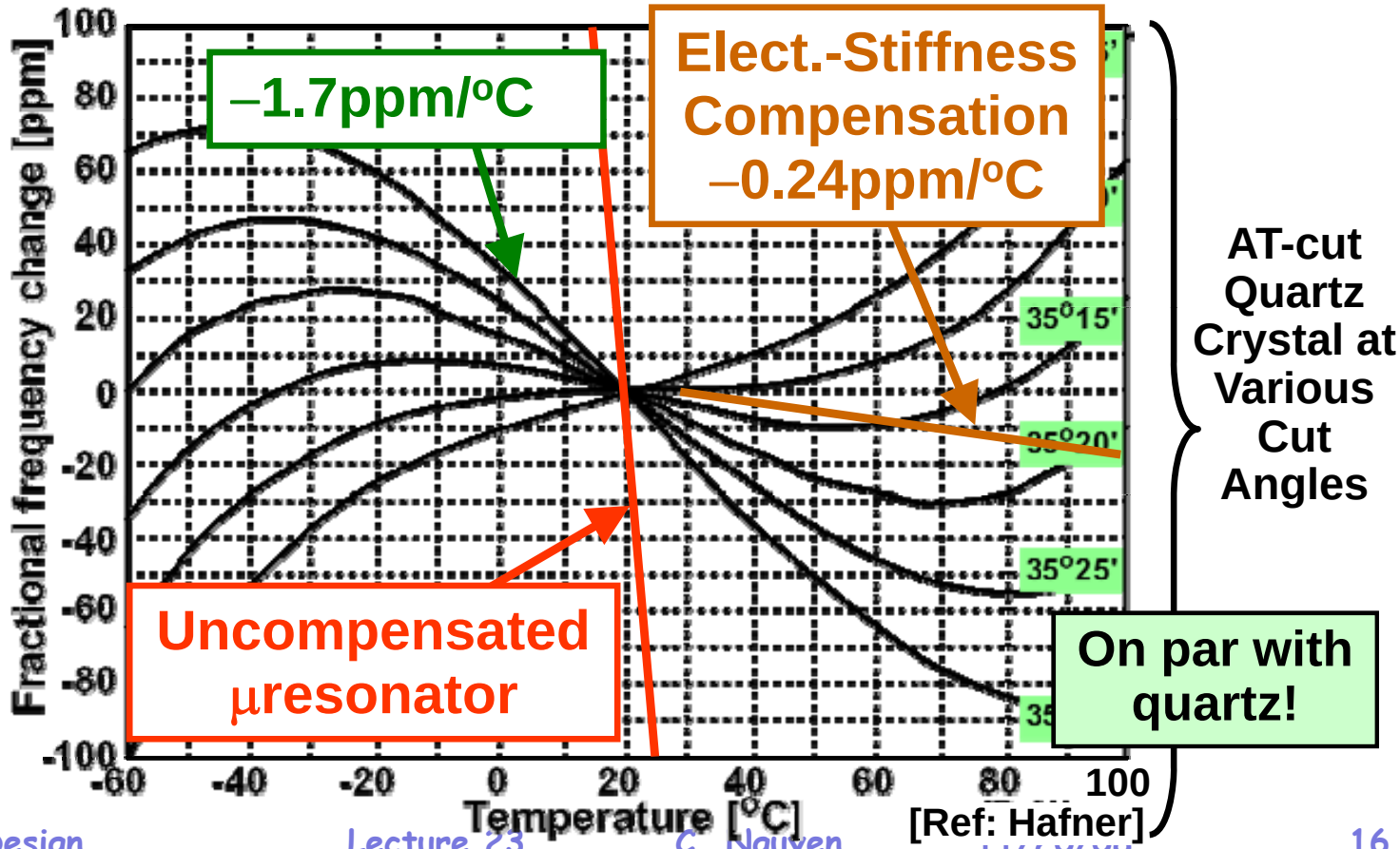


Top Electrode-to-Resonator Gap ⇧

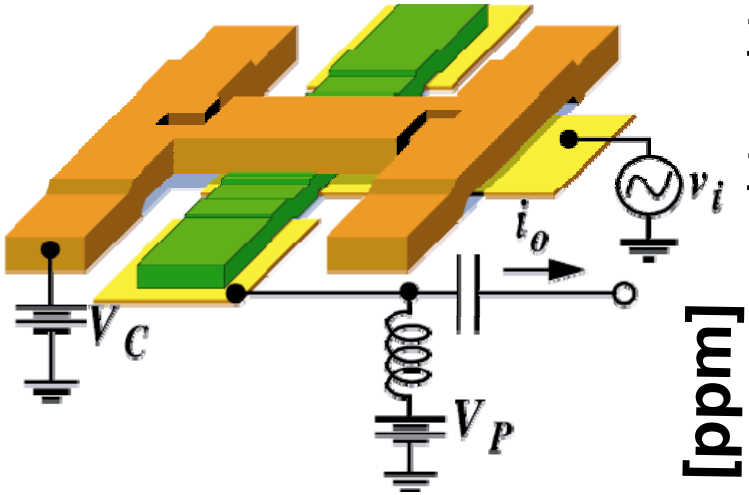
Elect. Stiffness: $k_e \sim 1/d^3$ ⇩

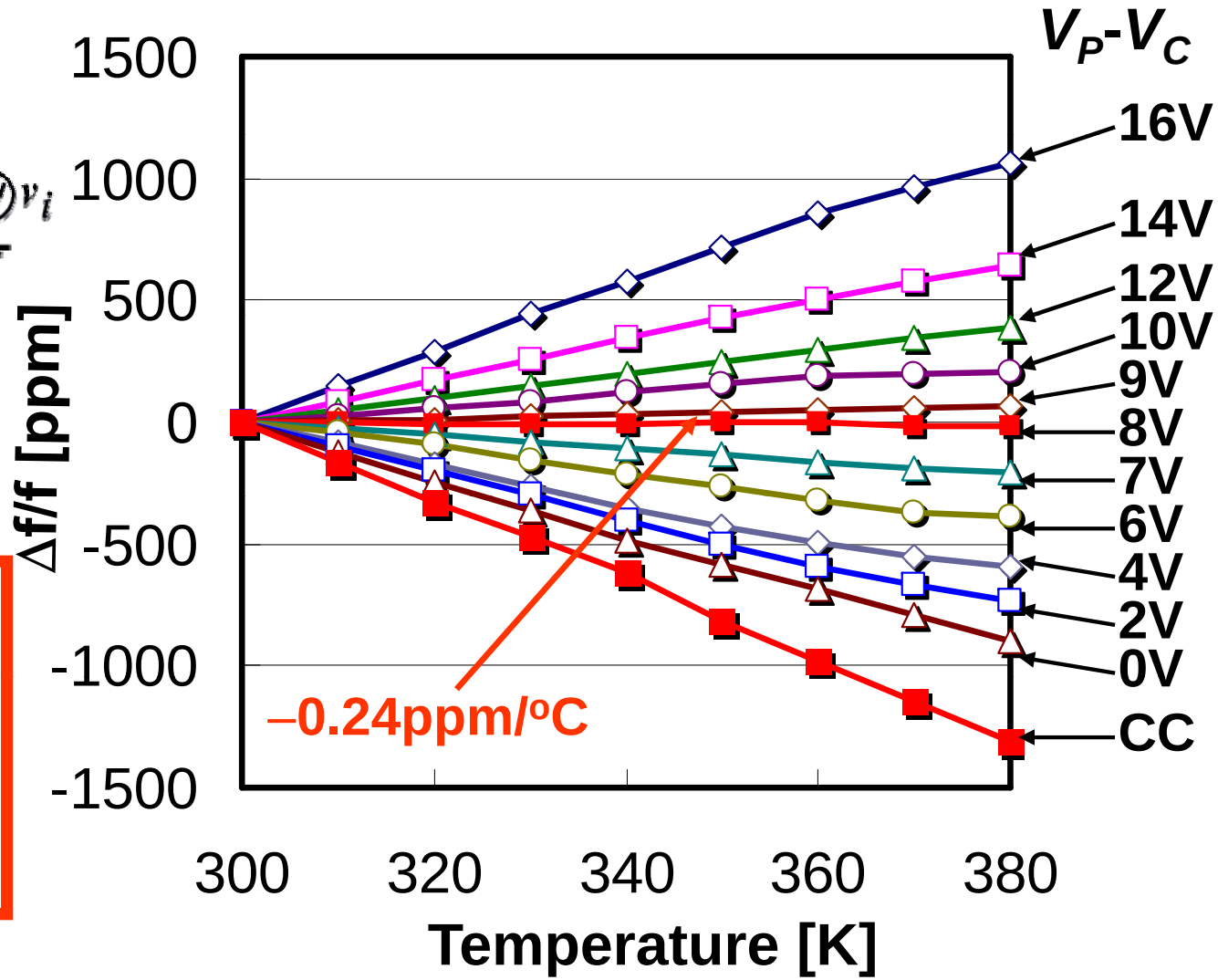
Frequency: $f_o \sim (k_m - k_e)^{0.5}$ ⇧

Counteracts reduction in frequency due to Young's modulus temp. dependence



Measured $\Delta f/f$ vs. T for k_e-Compensated μResonators





Design/Performance:

f_o =10MHz, Q=4,000
V_P=8V, h_e=4μm
d_o=1000Å, h=2μm
W_r=8μm, L_r=40μm

[Hsu *et al* MEMS'02]

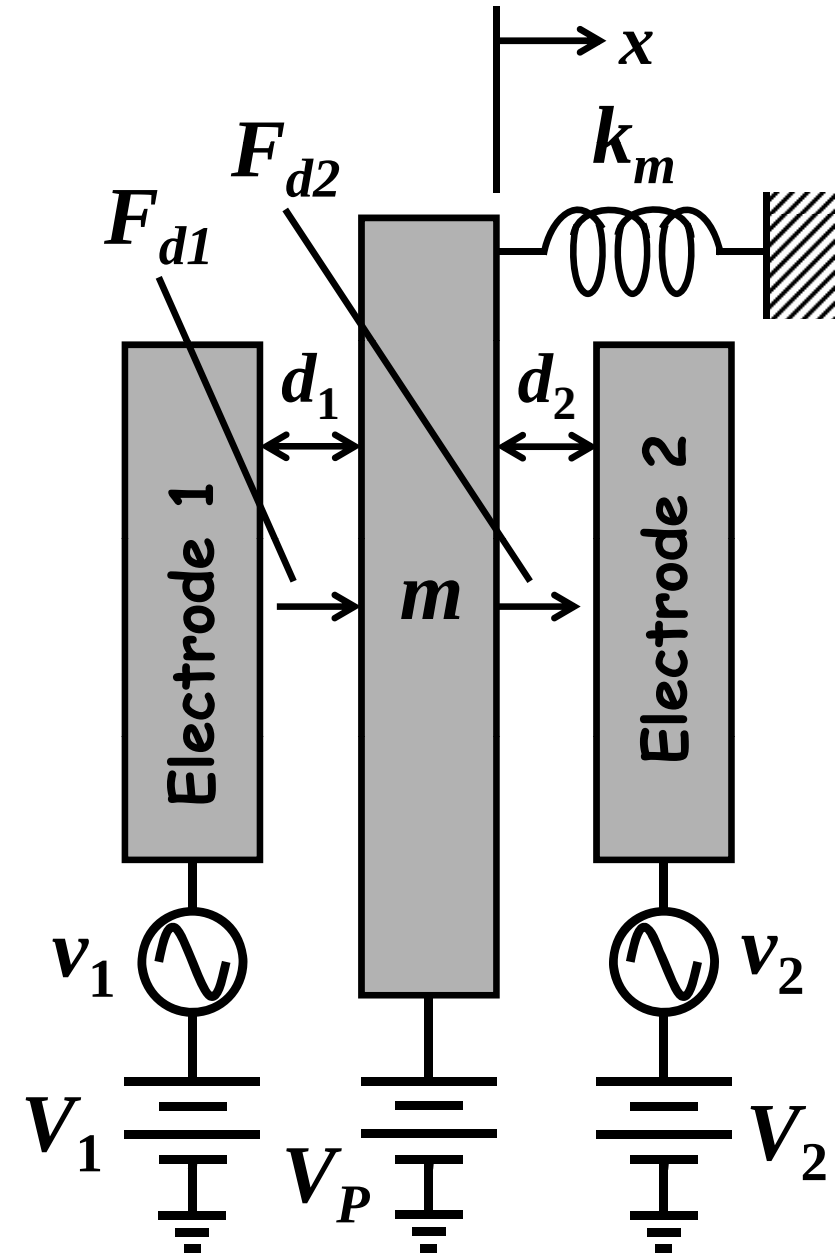
- Slits help to release the stress generated by lateral thermal expansion ⇨ linear TC_f curves ⇨ –0.24ppm/°C!!!

Can One Cancel k_e w/ Two Electrodes?

- What if we don't like the dependence of frequency on V_P?
- Can we cancel k_e via a differential input electrode configuration?
- If we do a similar analysis for F_{d2} at Electrode 2:

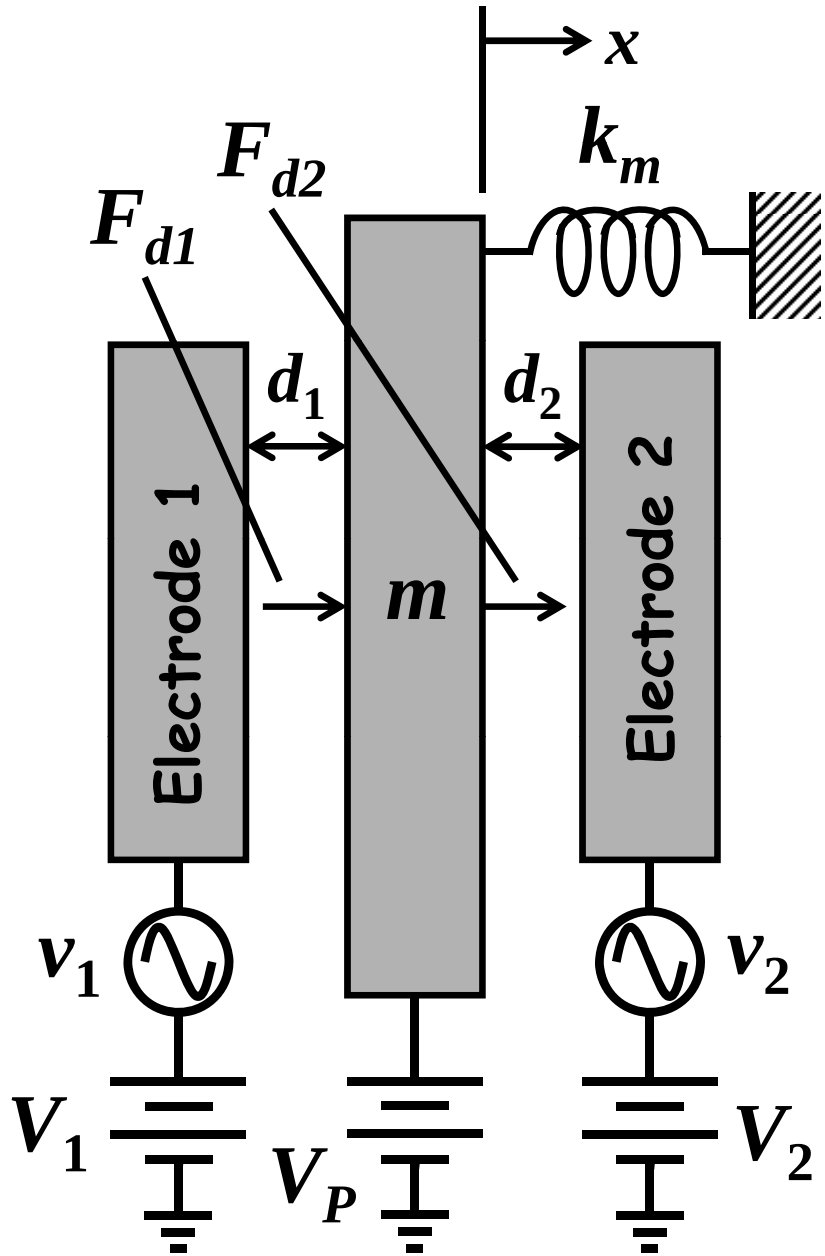
$$F_{d2}\Big|_{\omega_o} = \overbrace{-V_{P2}\frac{C_{o2}}{d_2}|v_2|\cos\omega_o t}^{\text{Subtracts from the } F_{d1} \text{ term, as expected}}$$

$$\underbrace{+V_{P2}^2\frac{C_{o2}}{d_2^2}|x|\sin\omega_o t}_{\text{Adds to the quadrature term} \rightarrow k_e\text{'s add, no matter the electrode configuration!}}$$



Problems With Parallel-Plate C Drive

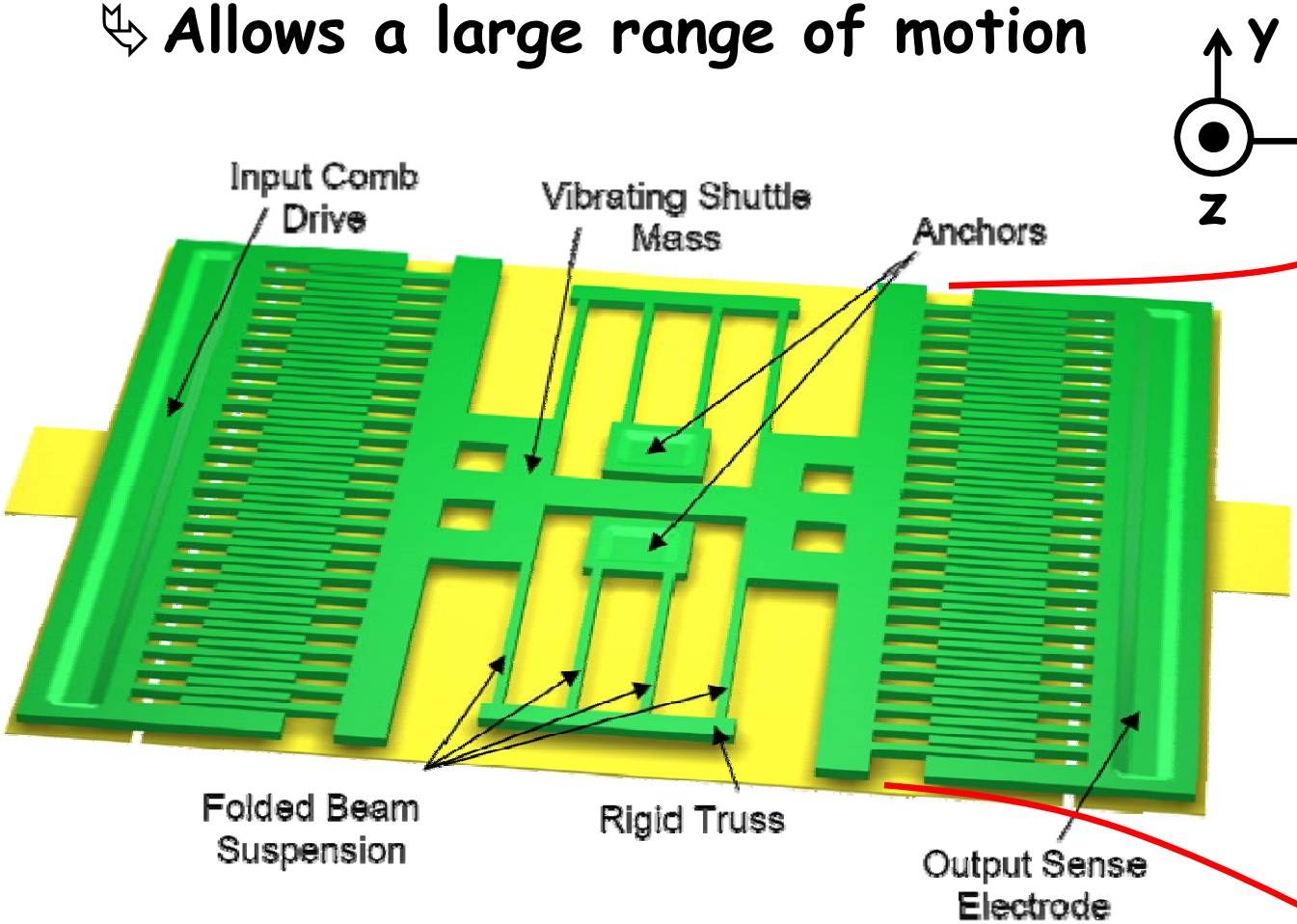
- Nonlinear voltage-to-force transfer function
  - Resonance frequency becomes dependent on parameters (e.g., bias voltage V_P)
  - Output current will also take on nonlinear characteristics as amplitude grows (i.e., as x approaches d_o)
  - Noise can alias due to nonlinearity
- Range of motion is small
  - For larger motion, need larger gap … but larger gap weakens the electrostatic force
  - Large motion is often needed (e.g., by gyroscopes, vibromotors, optical MEMS)

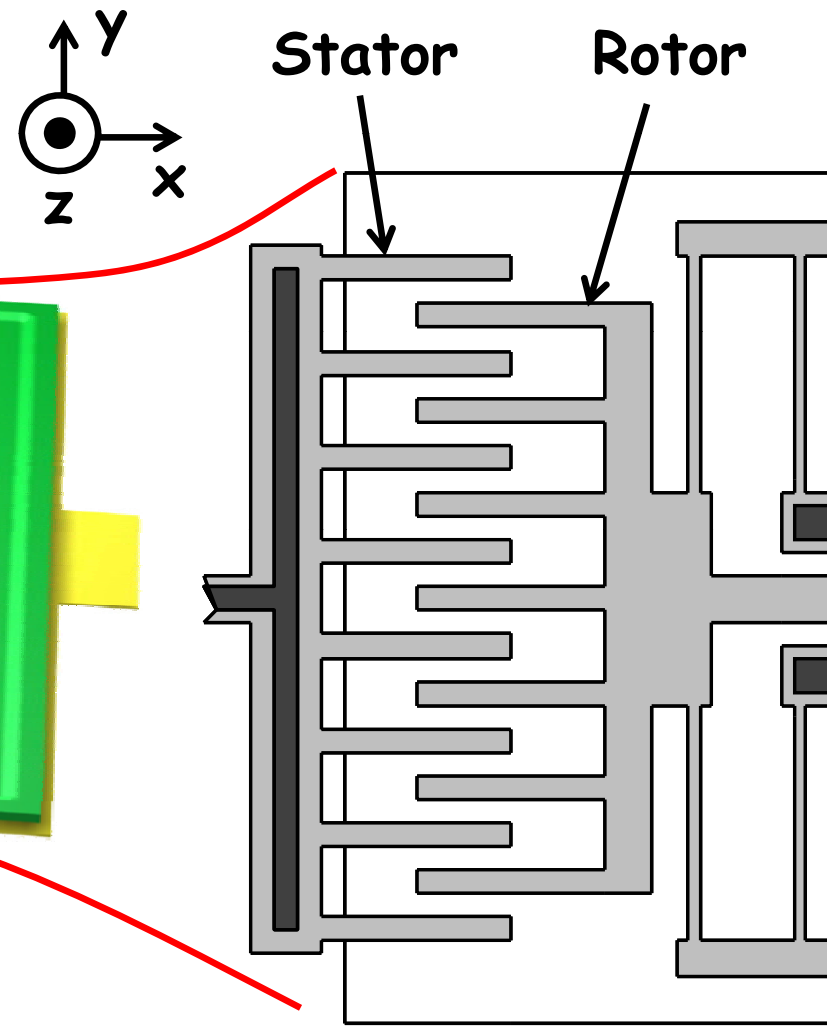


Electrostatic Comb Drive

Electrostatic Comb Drive

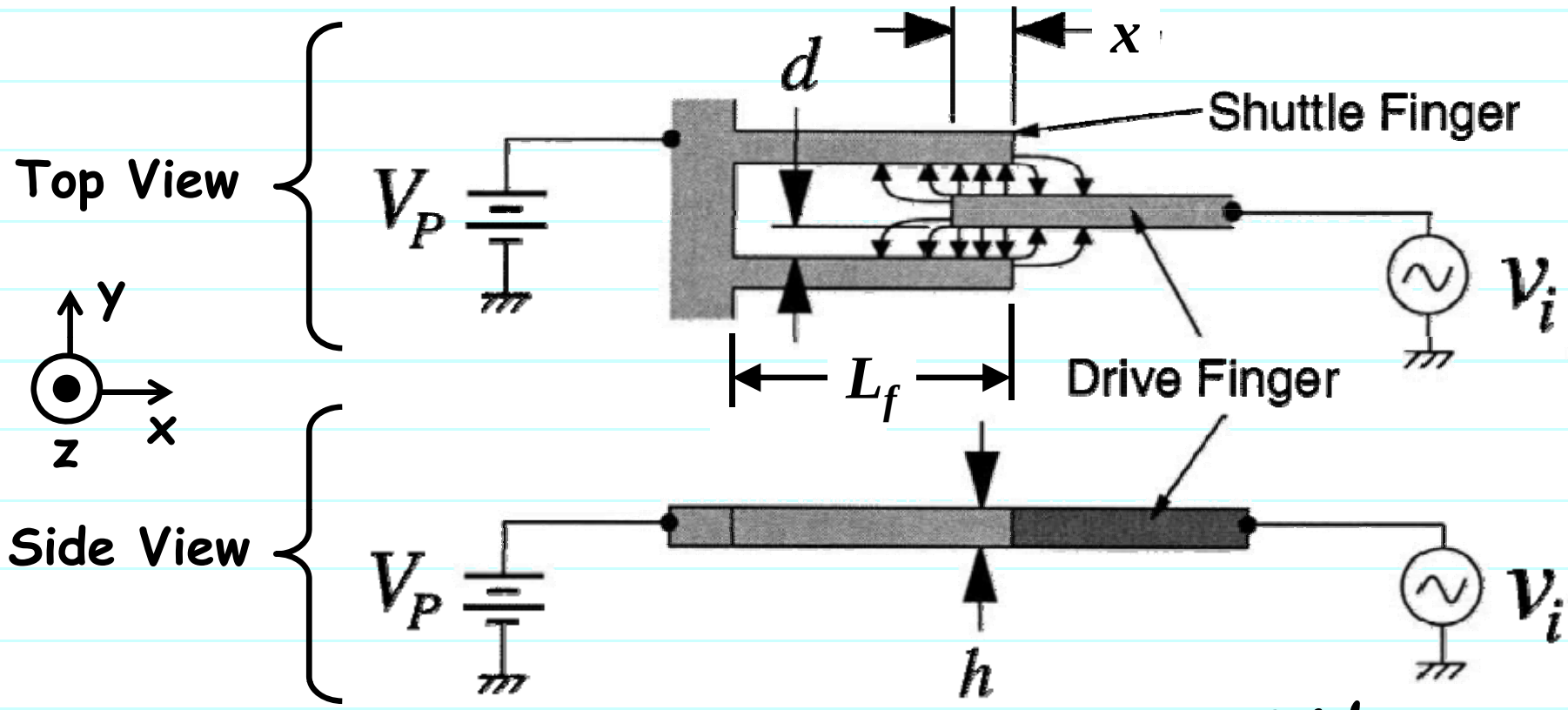
- Use of comb-capacitive tranducers brings many benefits
  - Linearizes voltage-generated input forces
  - (Ideally) eliminates dependence of frequency on dc-bias
  - Allows a large range of motion





Comb-Driven Folded Beam Actuator

Comb-Drive Force Equation (1st Pass)



$$C(x) = \frac{2\epsilon x h}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2\epsilon h}{d}$$

assuming symmetrical electrodes

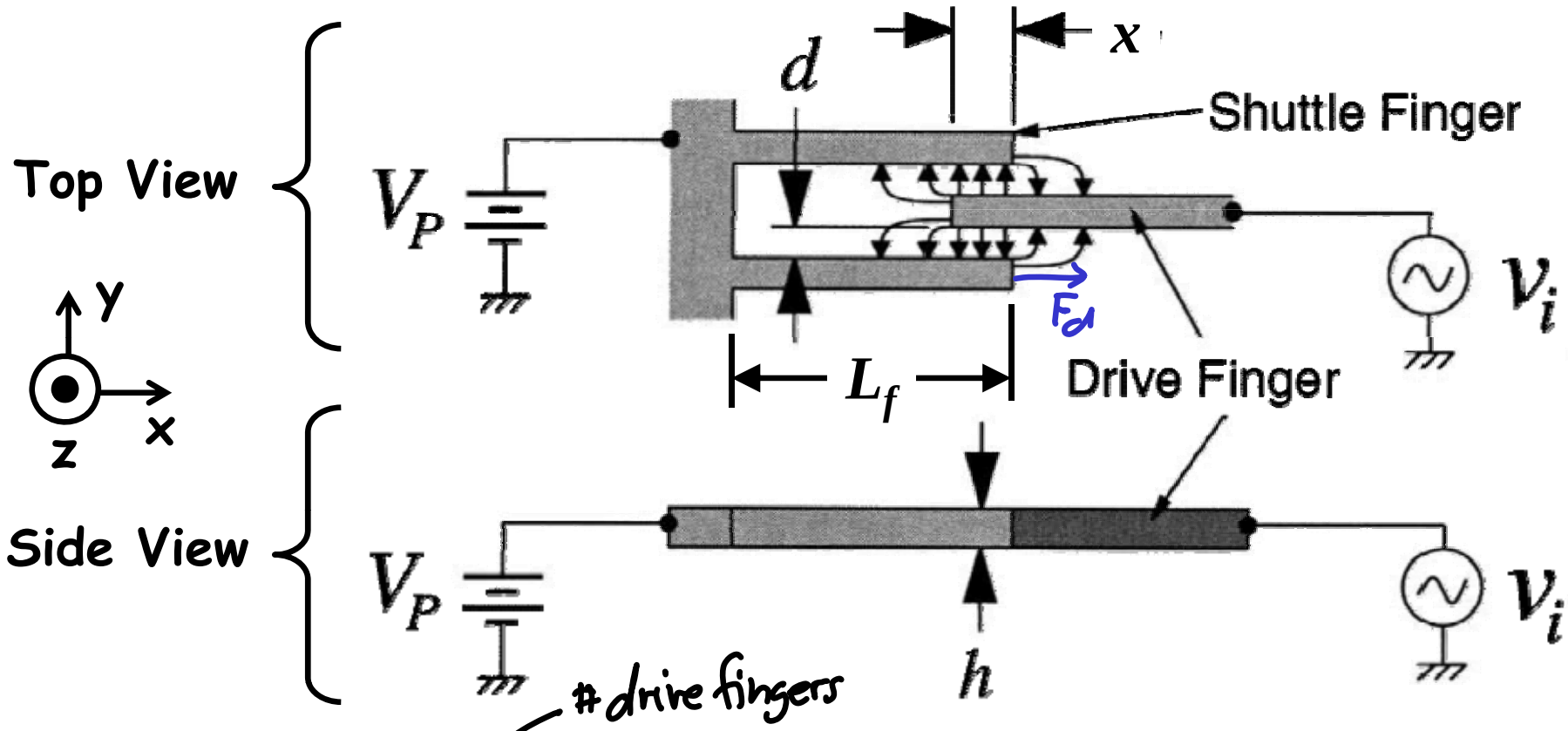
when $v_i = (+) \rightarrow F_d = (-)$

$$F_d = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{\partial C}{\partial x}(V_P - v_i)^2 = \frac{2}{2}\frac{\epsilon h}{d}(V_P^2 - 2V_P v_i + v_i^2) \approx \boxed{-2V_P \frac{\epsilon h}{d} v_i = F_d}$$

(No dependence on x! LINEAR!)

But wait! This ignores some practical effect!

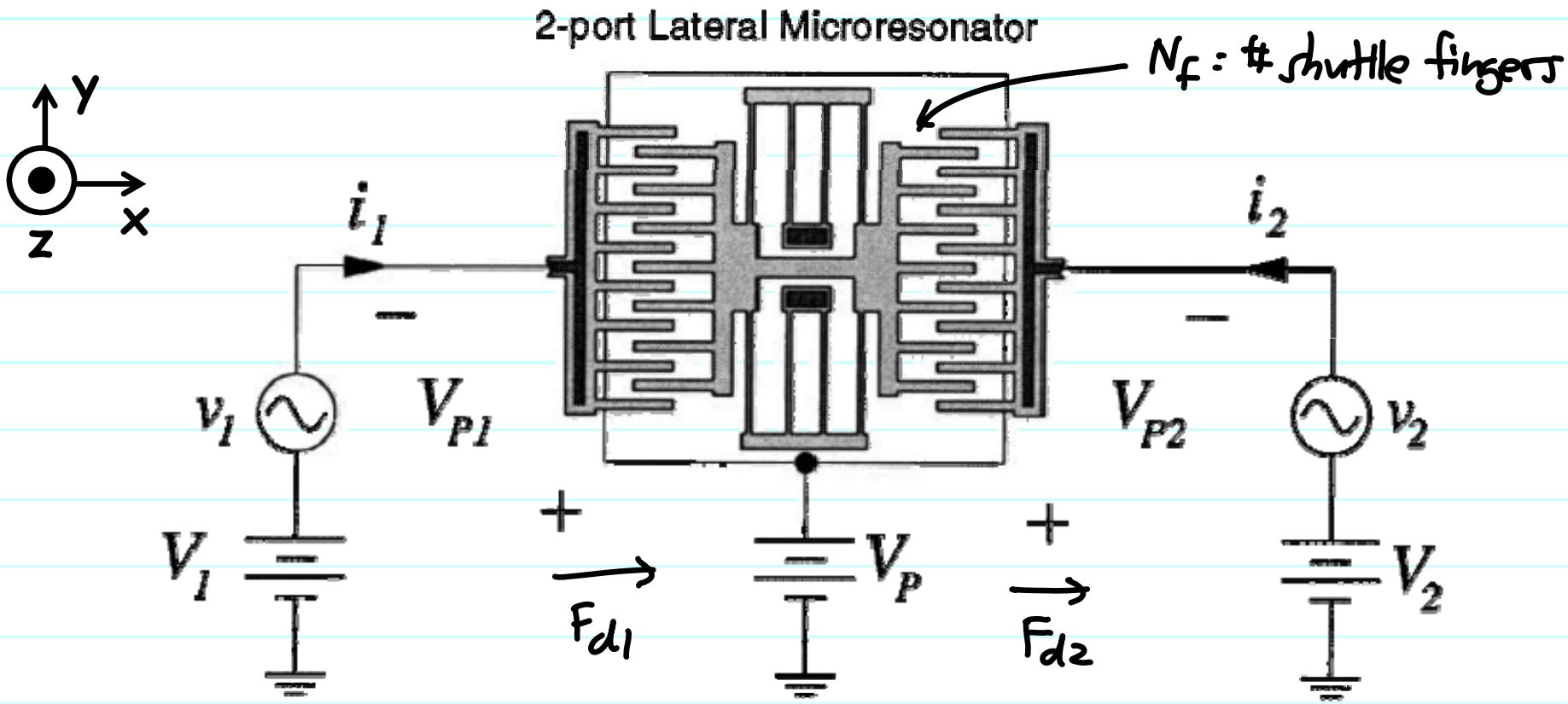
Lateral Comb-Drive Electrical Stiffness



- Again: $C(x) = \frac{2N\varepsilon hx}{d} \rightarrow \frac{\partial C}{\partial x} = \frac{2N\varepsilon h}{d}$

- No ($\partial C/\partial x$) x-dependence → no electrical stiffness: $k_e = 0$!
- Frequency immune to changes in V_P or gap spacing!

Typical Drive & Sense Configuration



Simple Analysis:

$$F_{d1} = \frac{1}{2}\frac{\partial C_1}{\partial x}(v_1 - V_{P1})^2 = \frac{1}{2}\left(-\frac{\epsilon_0 h}{d_1}\right)(v_1^2 - 2V_{P1}v_1 + V_{P1}^2)(2N_f)$$

$$F_{d2} = \frac{1}{2}\frac{\partial C_2}{\partial x}(v_2 - V_{P2})^2 = \frac{1}{2}\left(\frac{\epsilon_0 h}{d_2}\right)(v_2^2 - 2V_{P2}v_2 + V_{P2}^2)(2N_f)$$

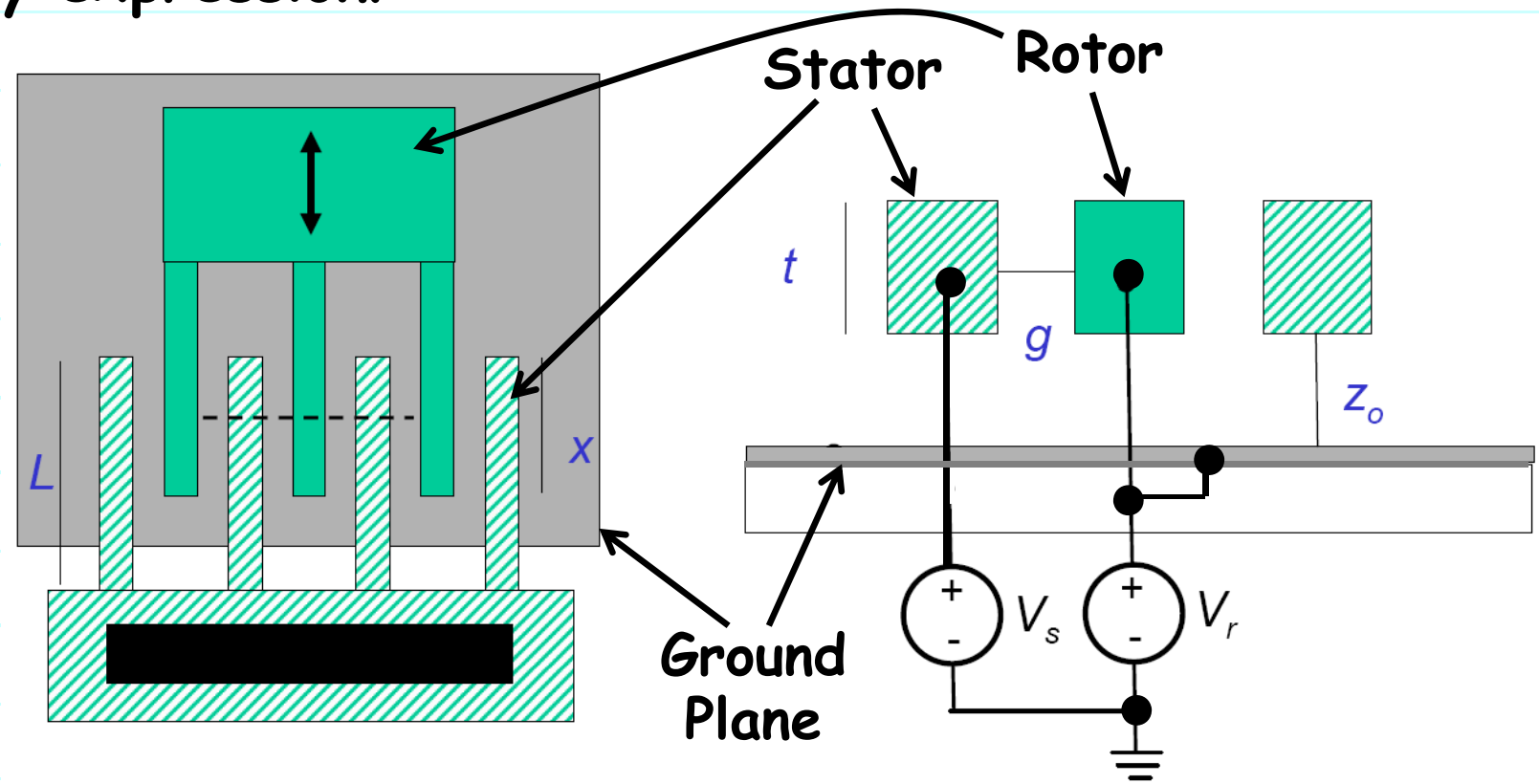
$$\therefore\ F_{net} = F_{d1} + F_{d2} = \frac{1}{2}\left(\frac{\epsilon_0 h}{d}\right)(v_2^2 - v_1^2 - 2(V_{P2}v_2 - V_{P1}v_1) + V_{P2}^2 - V_{P1}^2)(2N_f)$$

For $V_1 = V_2$, $v_1 = -v_2$

$$\boxed{F_{net} = 2(2N_f)\left(\frac{\epsilon_0 h}{d}\right)V_{P1}v_1}$$

Comb-Drive Force Equation (2nd Pass)

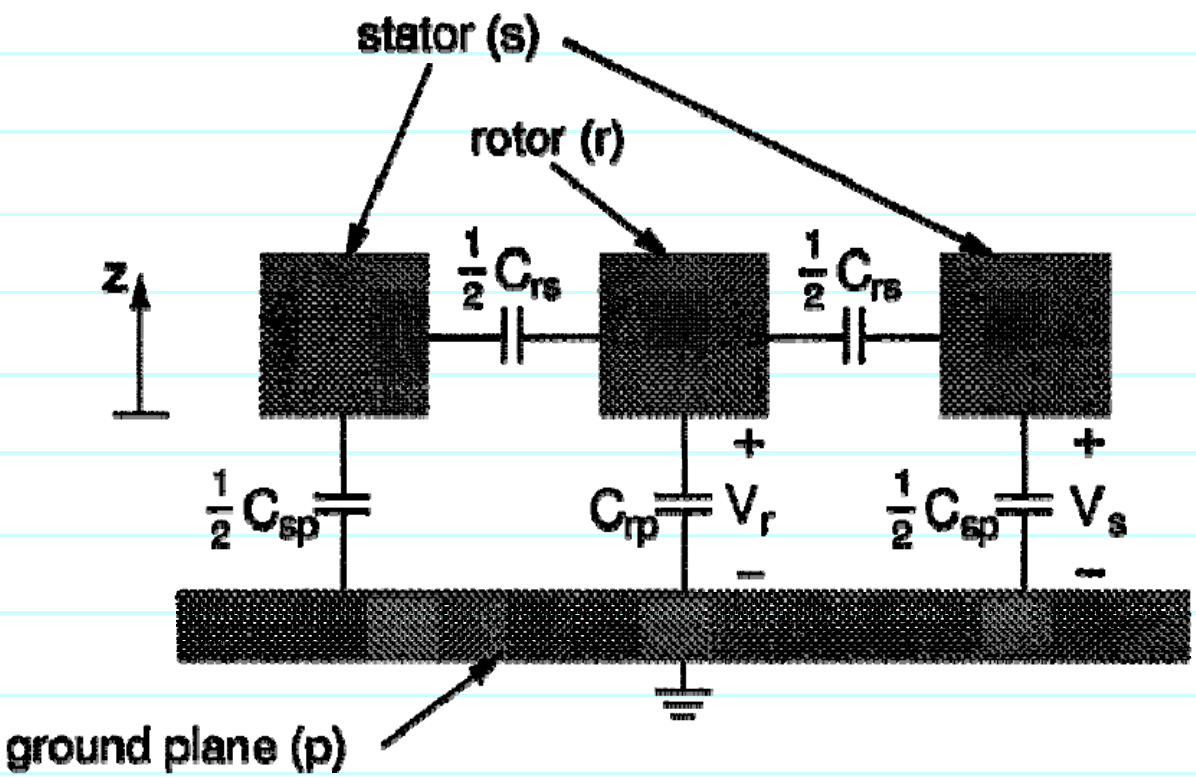
- In our 1st pass, we neglected
  - Fringing fields
  - Parallel-plate capacitance between stator and rotor
  - Capacitance to the substrate
- All of these capacitors must be included when evaluating the energy expression!



Comb-Drive Force With Ground Plane Correction

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{dC_{sp}}{dx}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dx}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dx}(V_s - V_r)^2$$



[Gary Fedder, Ph.D., UC Berkeley, 1994]

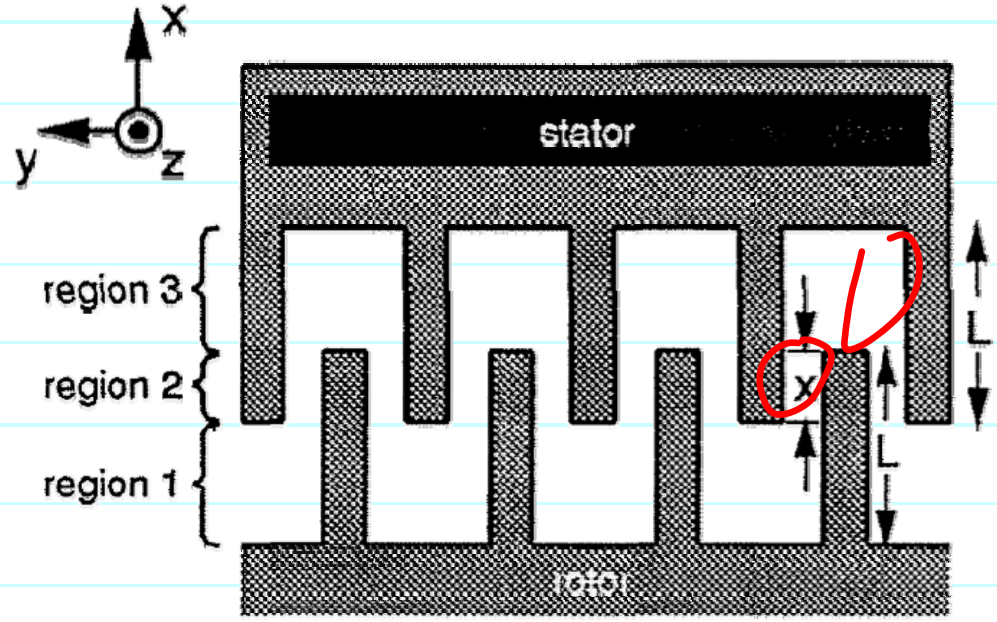
Capacitance Expressions

- Case: $V_r = V_P = 0V$
- C_{sp} depends on whether or not fingers are engaged

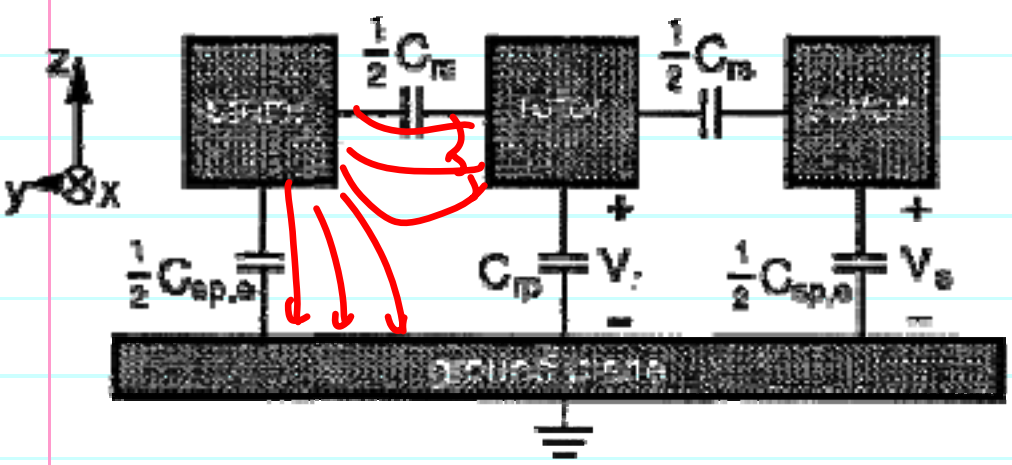
$$C_{sp} = N[C'_{sp,e}x + C'_{sp,u}(L-x)]$$

$$C_{rs} = NC'_{rs}x$$

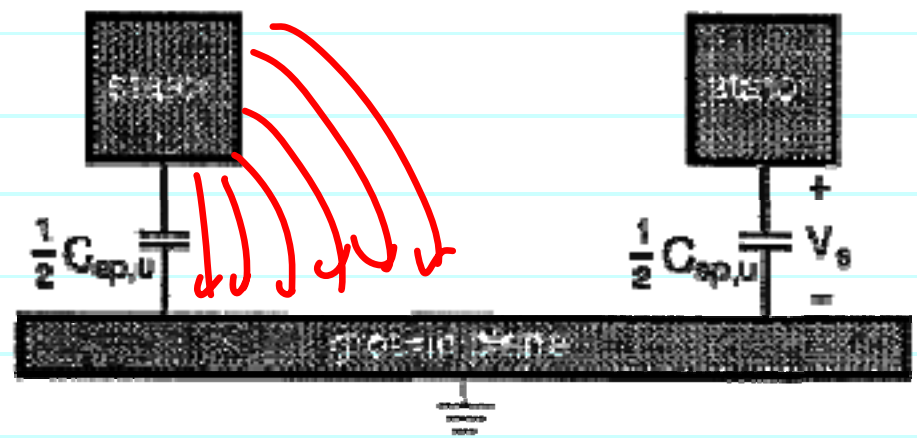
Capacitance per unit length



Region 2



Region 3



[Gary Fedder, Ph.D., UC Berkeley, 1994]

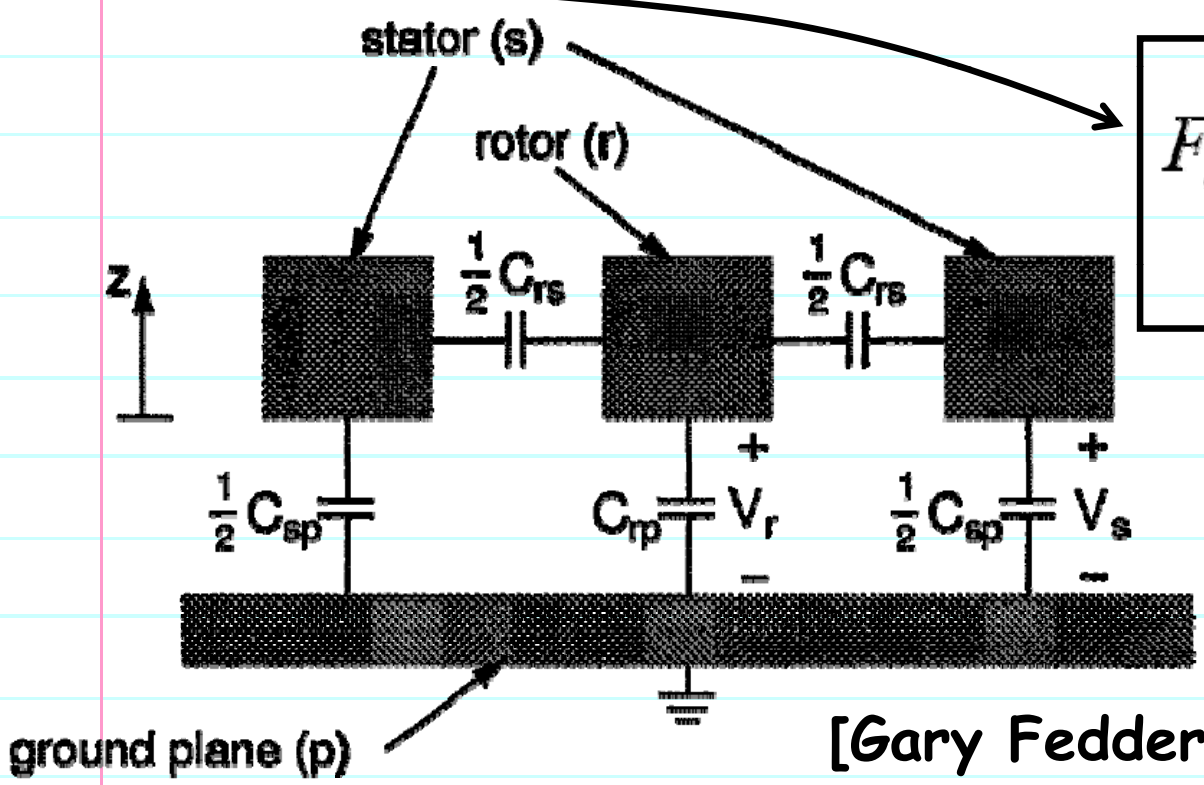
Comb-Drive Force With Ground Plane Correction

- Finger displacement changes not only the capacitance between stator and rotor, but also between these structures and the ground plane → modifies the capacitive energy

$$F_{e,x} = \frac{\partial W'}{\partial x} = \frac{1}{2}\frac{dC_{sp}}{dx}V_s^2 + \frac{1}{2}\frac{dC_{rp}}{dx}V_r^2 + \frac{1}{2}\frac{dC_{rs}}{dx}(V_s - V_r)^2$$

$$F_{e,x} = \frac{N}{2}(C'_{rs} + C'_{sp,e} - C'_{sp,u})V_s^2$$

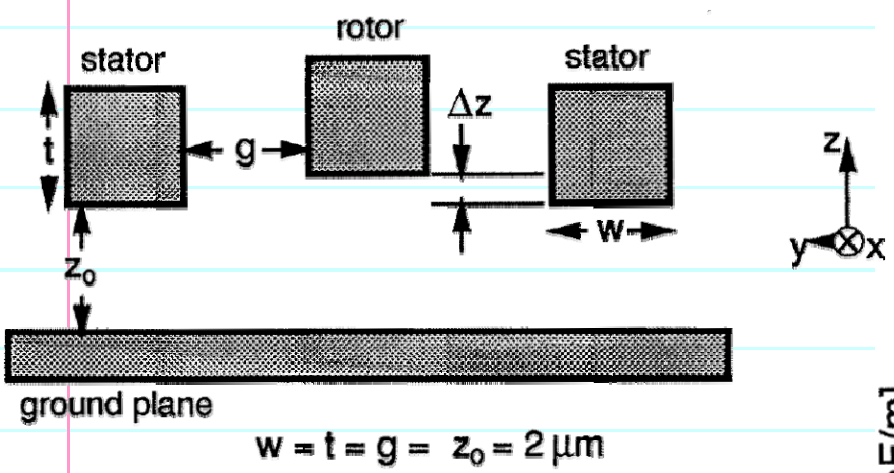
(for $V_r = V_p = 0$)



[Gary Fedder, Ph.D., UC Berkeley, 1994]

Simulate to Get Capacitors → Force

- **Below**: 2D finite element simulation



$$F_{e,x} = \frac{N}{2}(C'_{rs} + \underbrace{C'_{sp,e} - C'_{sp,u}})V_s^2$$

20-40% reduction of $F_{e,x}$