PROBLEM SET #1

Issued: Thursday, Sept.3, 2009

Due (at 7 p.m.): Tuesday, Sept. 15, 2009, in the EE C245 HW box in 240 Cory.

This homework assignment is intended to give you some early practice playing with dimensions and exploring how scaling can greatly improve certain performance characteristics of mechanical systems. Don't worry at this point if you do not understand fully some of the physical expressions used. Some of them will be revisited later in the semester.

1. Some of most sensitive MEMS-based sensors utilize changes in a structure's resonance frequency to sense a parameter, e.g., gas concentration, temperature, acceleration, that perturbs the resonance frequency. Such sensors are extremely sensitive because we can often much more precisely detect a shift in frequency than we can a shift in voltage or current. In addition, a frequency output automatically provides a digital output, since one need only count the number of zero crossings of the waveform over a set period of time to provide an integer output.

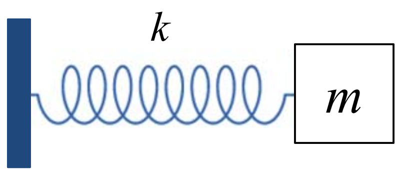

Figure 1

This problem deals with the very simple mass-spring system shown in Figure 1 above, which is intended for use as a gas sensor. Specifically, when a gas is introduced into the system, the gas molecules adsorb onto the mass surface, thereby changing the value of total mass. This then changes the resonance frequency of the system, given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1}$$

Assume for this problem that the mass is a square block constructed of polycrystalline silicon, which has a density of 2,300 kg/m^3. Also, assume that the spring has a stiffness of 0.5 N/m.

Answer the following questions concerning this sensor.

(a) Using (1), derive an expression for the ratio of frequency shift to mass change ($\frac{\Delta f}{\Delta m}$).

(b) You are given two such sensors, one where the side dimensions of the cube element are 1 cm, and another with side dimensions of 10 μm. Suppose you then introduce a gas onto each, and each cube adsorbs a monolayer of this gas with a mass per area density of 5×10^{-16}g/μm^2. (Assume that the spring adsorbs nothing.) Compute the frequency shifts of each sensor system after introduction of the gas.

2. The general equation for transverse free vibrations of a prismatic beam, such as shown in Figure 2, can be expressed as

$$EI\frac{\partial^4 v}{\partial x^4}dx = -\rho A dx\frac{\partial^2 v}{\partial t^2} \tag{2}$$

where E and ρ are the Young's modulus and density of the structural material, respectively; I is the moment of inertia; A is the cross-sectional area of the beam; v is the y-directed displacement variable; x is indicated in the figure, and t is time. You might not understand all of these variables right now, but you will deeper into the course. For now, just treat this problem as a math problem, designed to jog your memory on how to solve differential equations.

Find a general form of the solution to (2) that gives the mode shape of the beam, as indicated in Figure 3. (The mode shape is the shape of the beam at maximum amplitude during resonance.) This should be in terms of x, the resonance frequency ω, and some constants governed by boundary conditions, i.e., it should be expressed as $V = f(x,\omega)$, where V is a function describing the mode shape You need not determine the values of the constants, but you should show them as variables (the same way you've done before in math courses).

[Hint: Assume a solution $v = V\,(C \cos\omega t + D \sin\omega t)$, then find V.]

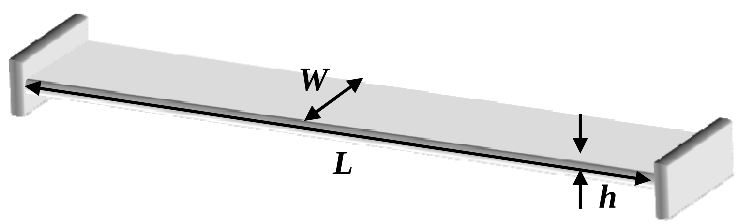

Figure 2

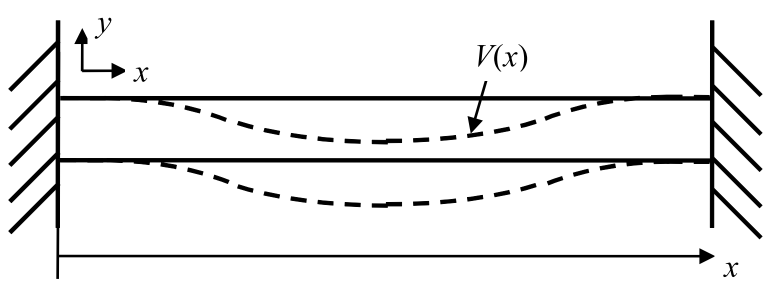

Figure 3

3. Suppose a step-function voltage V_A were suddenly applied across the anchors of a 2μm thick polysilicon fixed-fixed beam and proof mass as shown in Figures 4 and 5, which also provide lateral dimensions. For polysilicon, assume the following material properties: Young's modulus E = 150 GPa, density ρ = 2,300 kg/m^3, Poisson ratio ν = 0.226, sheet resistance = 10 Ω/□, specific heat = 0.77 J/(g·K), and thermal conductivity = 30 W/(m·K).

(a) With what time constant will the proof mass reach its steady-state temperature after the voltage V_A steps from 0V to 1V? Give a formula and a numerical answer with units.

(b) If the final step function value of V_A is 1V, what is the steady-state temperature of the proof mass? Give a formula and a numerical answer with units.

(c) If each supporting beam is considered as consisting of two sections as shown in Figure 6. Find the steady-state temperature of each part if the final step function value of V_A is 1V.

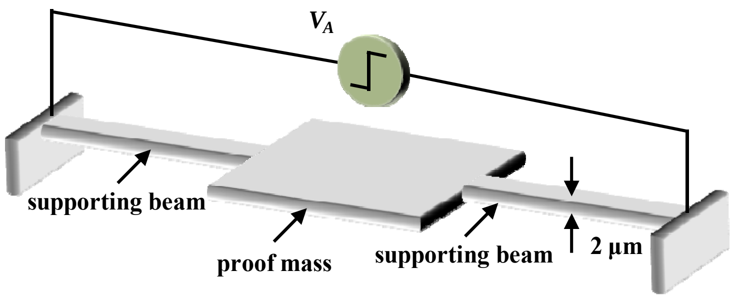

Figure 4

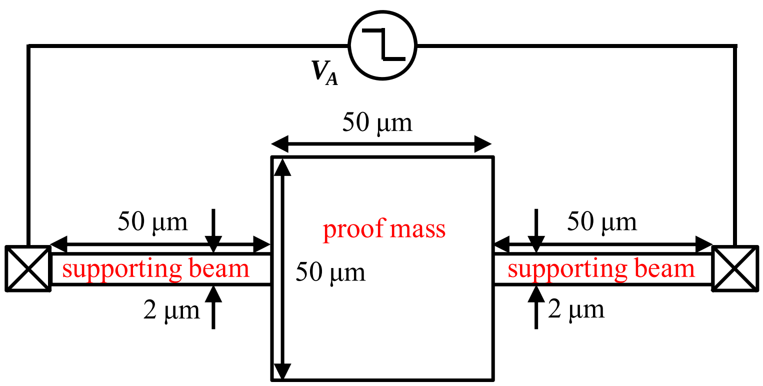

Figure 5

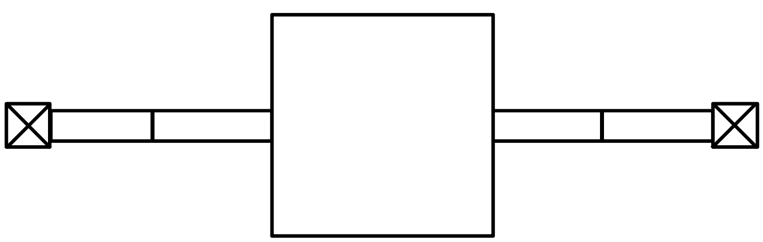

Figure 6