PROBLEM SET #1

Issued: Thursday, Sept.2, 2010

Due (at 7 p.m.): Tuesday, Sept. 14, 2010, in the EE C245 HW box in 240 Cory.

This homework assignment is intended to give you some early practice playing with dimensions and exploring how scaling can greatly improve certain performance characteristics of mechanical systems. Don't worry at this point if you do not understand fully some of the physical expressions used. Some of them will be revisited later in the semester.

1. Scaling to microscopic dimensions provides benefits in many sensing applications such as pressure and temperature measurement, EM radiation detection, linear acceleration and rotation rate measurement, strain sensing, chemical sensing, and biological sample analysis. For this problem we will focus on the behavior of a resonant cantilever gas sensor.

(a) Consider the fixed-free beam (cantilever) illustrated in Figure 1.

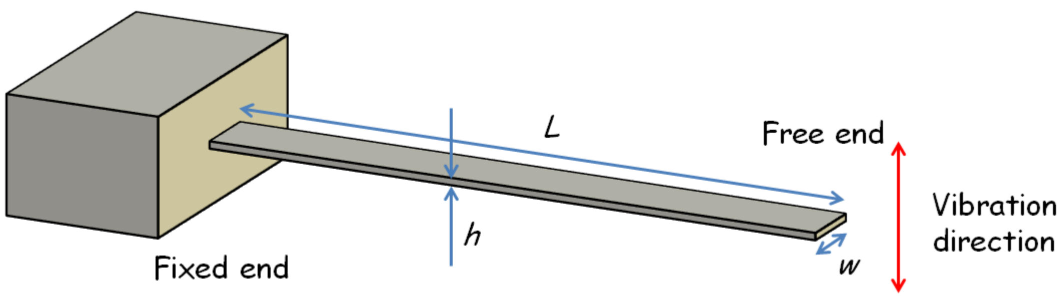

Figure 1

When excited in its fundamental resonant mode, and given that $L >> h$, the tip of the cantilever will vibrate at a frequency given by

$$f = \frac{\frac{1}{2\pi}\sqrt{\frac{35}{33}}\sqrt{\frac{E}{\rho}}\,h}{L^2}$$

where E and ρ are the Young's modulus and density of the structural material, and dimensions are given in Figure 1. If all dimensions are scaled down by a factor of 100, by what factor will the resonant frequency increase? This is an easy exercise intended solely to give perspective on how scaling affects resonant frequency. (5%)

(b) Assume that a 150 mm (6″) diameter wafer has a useful area of 100 mm × 100 mm upon which cantilever sensors can be fabricated. (Here, the edges of the wafer are for handling, so do not yield working devices.) A dicing saw is used to cut the wafer into individual dies and the width of each cut is 50 μm. Each sensor requires a square unit cell with a minimum area of $9L^2$. The cost per sensor is given by $C(n,d) = (\$3000 + \$1 \times n + \$2 \times d)/d$, where n is the number of cuts through the wafer and d is the number of dies. Here, the fixed \$2 cost per sensor is due to post processing, packaging and testing costs. Assume that the minimum die size that can be reliably handled is 1 mm × 1 mm. What is the lowest achievable fabrication cost per sensor (to the nearest cent) and what is the corresponding maximum cantilever size (to the nearest ten microns)? <u>Hint</u>: it would be helpful to define *d(n)* and to find *n*. (15%)

(c) The cantilever is to be used as a sensor for detecting a harmful chemical agent. A small area on the tip of the cantilever is coated with a special polymer to which a monolayer of the chemical agent can bond. When this bonding occurs, the tip's mass increases slightly and there is a corresponding tiny negative shift in the resonant frequency. Recall that $\omega = \sqrt{k/m}$ for a simple mass/spring oscillator. Assuming the shift in frequency is only due to the change in the mass of the tip, will the fractional frequency shift ($\Delta f/f$) be greater for a thicker cantilever (larger h) or a thinner cantilever (smaller h)? Why? (10%)

2. The general equation for the deflection of a thin stretched membrane due to applied pressure, such as shown in Figure 2, can be expressed as the following differential equation:

$$-N\left(\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}\right) = P \text{ with boundary conditions } z(x,0) = z(0,y) = z(L,y) = z(x,L) = 0$$

where N is the tensile stress causing the membrane to stretch; P is the (constant and uniform) applied pressure; z is the displacement function, and x,y are the planar spatial coordinates. You might not understand all of these variables right now, but you will deeper into the course. For now, just treat this problem as a math problem, designed to jog your memory on how to solve differential equations.

Find a general form of the solution to the differential equation above that gives the static displacement shape of the membrane. This should be in terms of x, y, the side length L, and stress parameters N and P. It should be expressed as $Z = f(x,y)$, where Z is a function describing the mode shape. You need not determine the values of the constants, but you should show them as variables (the same way you've done before in math courses). (35%)

<u>Hint #1</u>:

Assume a solution $z(x,y) = \sum_1^\infty Z_{xy}(\sin A_x x)(\sin B_y y)$, then determine $Z_{xy},\ A_x,\ B_y$.

<u>Hint #2</u>:

Recall $\int_0^\pi \sin(ax)\sin(bx)\,dx = 0$ for all integers $a \neq b$. How can you use this to reduce an infinite series to a single element?

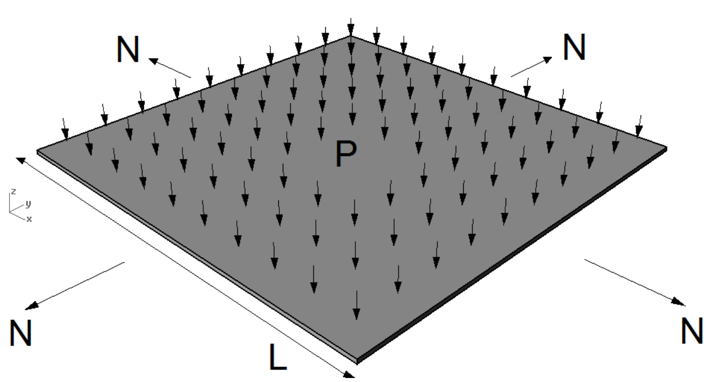

Figure 2

3. Suppose a step-function voltage V_A were suddenly applied across the anchors of a 2 μm thick polysilicon beam and proof mass setup as shown in Figures 3 and 4, which also provide lateral dimensions. For polysilicon, assume the following material properties: Young's modulus E = 150 GPa, density ρ = 2,300 kg/m^3, Poisson ratio ν = 0.226, sheet resistance = 10 Ω/□, specific heat = 0.77 J/(g·K), and thermal conductivity = 30 W/(m·K).

(a) With what time constant will the proof mass reach its steady-state temperature after the voltage V_A steps from 0V to 1V? Give a formula and a numerical answer with units. (10%)

(b) If the final step function value of V_A is 1V, what is the steady-state temperature of the proof mass? Give a formula and a numerical answer with units. (20%)

(c) What effect do you think the applied voltage has on the resonant frequency of the structure in the z-direction (into the page)? Give a brief qualitative explanation. (5%)

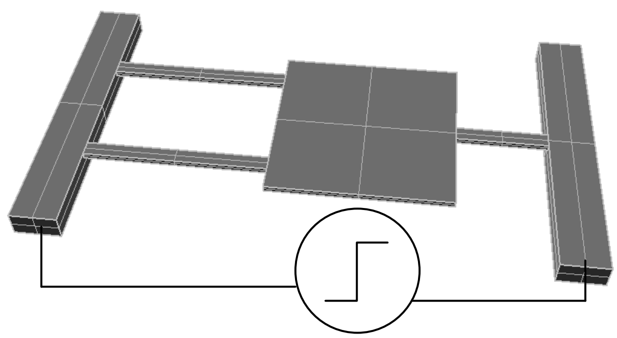

Figure 3

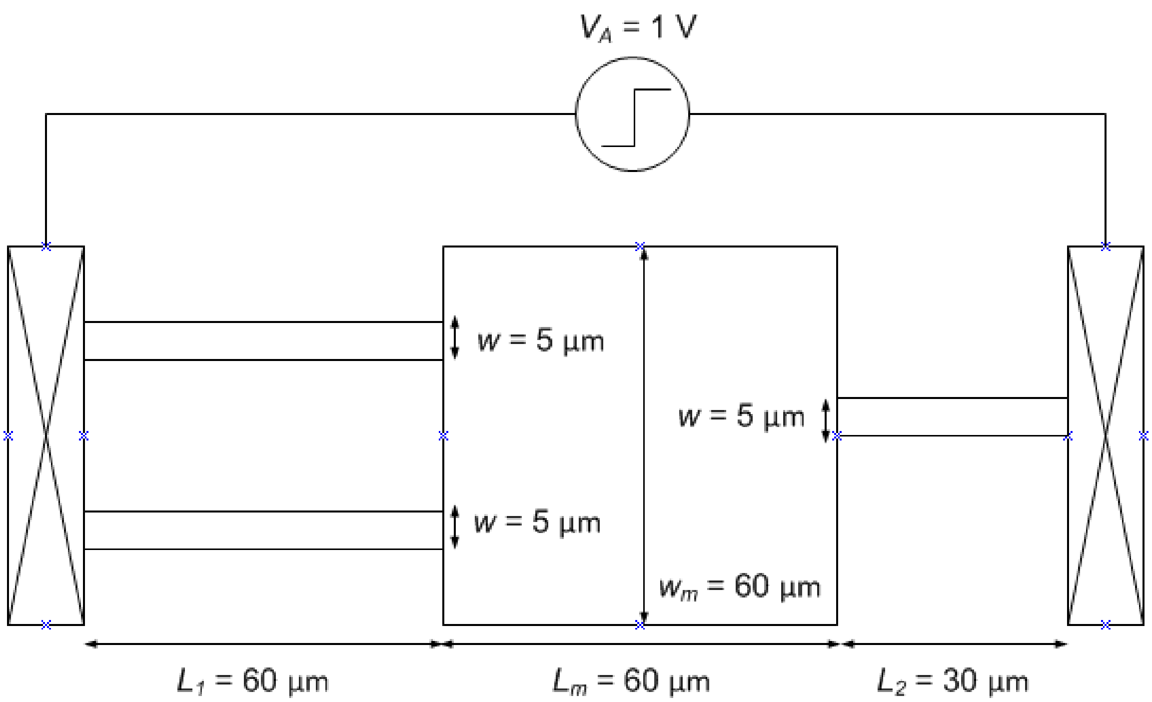

Figure 4