

**PROBLEM SET #5**

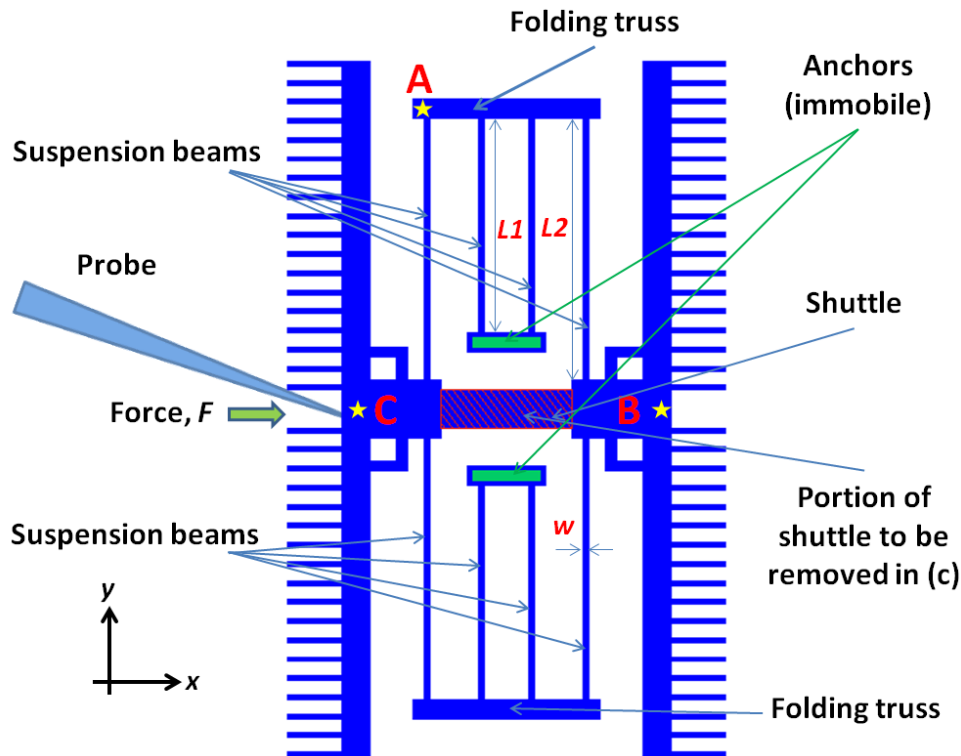
*Issued: Tuesday, Oct. 19, 2010.*

*Due (at 7 p.m.): Thursday, Oct. 28, 2010, in the EE C245 HW box in 240 Cory.*

1. Consider the ratioed folded beam flexure system shown in Figure 1 below. The device is to be fabricated using the following process with a clear field structure mask (blue) and a dark field anchor mask (green).

- Thermally grow  $2\ \mu\text{m}$  of  $\text{SiO}_2$  on a (100) silicon wafer.
- Anisotropically etch anchor holes in the  $\text{SiO}_2$  using RIE and the anchor mask (green).
- Deposit  $2\ \mu\text{m}$  of polysilicon using LPCVD.
- Etch the polysilicon layer using RIE and the structure mask (blue).
- Release the structure by etching the  $\text{SiO}_2$  in hydrofluoric acid (HF)

The result is illustrated in Figure 1 where the suspended structure is shown in blue and is attached to the substrate at the two anchor regions shown in green. The device is to be inspected under a microscope and probed using a sharp metal tip controlled by a micromanipulator.



**Figure 1. A ratioed folded beam flexure system and a probe.**

- (a) Derive the  $x$ -directed stiffness of the folded flexure at point A as a function of  $L_1$  and  $L_2$  in terms of the Young's modulus  $E$ , beam width  $w$ , and film thickness  $t$ , where  $L_1$  and  $L_2$  represent the inner and outer beam lengths respectively. You should assume that the suspension beams are the only elements of the structure that bend significantly.
- (b) Suppose that  $L_1 + L_2 = \gamma$ , where  $\gamma$  is a fixed design parameter, i.e.,  $0 \leq L_1 \leq \gamma$ ,  $L_2 = \gamma - L_1$ . Find a ratio of  $L_1$  to  $L_2$  that yields an  $x$ -directed stiffness at point B equal to  $1/3$  the maximum stiffness attainable by varying  $L_1$  and  $L_2$  only. Note that the positions of the folding trusses

and anchors are dependent on the values of  $L_1$  and  $L_2$ , but these positions do not affect the solution to this problem.

- (c) Suppose that the red shaded region of the structure is removed,  $L_1 = 110 \mu\text{m}$  and  $L_2 = 90 \mu\text{m}$ . The Young's modulus of polysilicon is 150 GPa and the beam width  $w = 2.0 \mu\text{m}$ . A force of  $1 \mu\text{N}$  is applied at point C. Calculate the  $x$ -directed displacement of point B.
2. Suppose that the polysilicon cantilever illustrated in Figures 2 and 3 was fabricated using the process described in Problem 1. Before HF release, the cantilever had the stress profile shown in Figure 4. The Poisson's ratio of polysilicon,  $\nu = 0.22$ . What point force  $F$  should be applied to the tip after release so that the deflection at the tip is zero? Explain why the cantilever will not be perfectly flat even when a point load is applied at the tip.

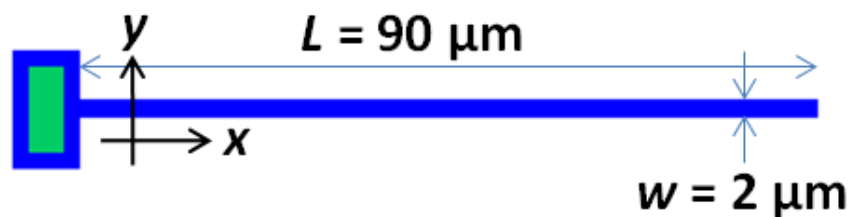


Figure 2. Layout view of a polysilicon cantilever

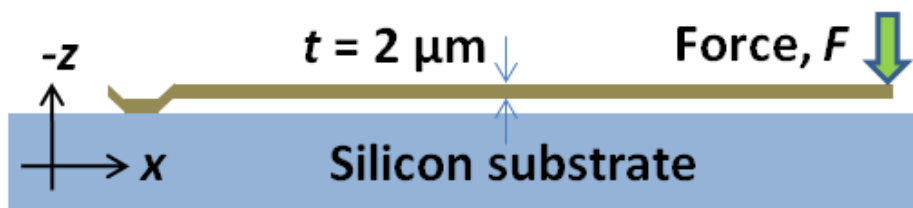


Figure 3. Cross-section of the cantilever after release but before bending (right).

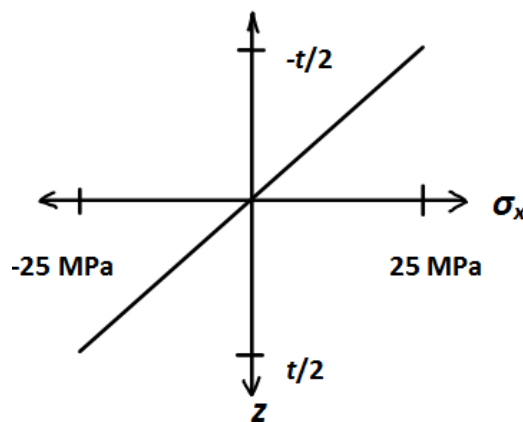


Figure 4. Stress profile in the cantilever before bending.

3. Consider a fixed-fixed beam of width  $W$  and height  $H$  under the loading conditions shown in Figure 5 below.
- (a) Calculate the small-deflection stiffness of the beam by integration of the moment.
  - (b) Calculate the small-deflection stiffness of the beam using equivalent spring circuits.
  - (c) Calculate the large-deflection stiffness of the beam using virtual work principles. [Hint: Use a cosine trial function  $\hat{w} = \frac{c}{2} \left( 1 + \cos \frac{2\pi x}{L} \right)$ , where  $c$  is the displacement at the center of the beam.]

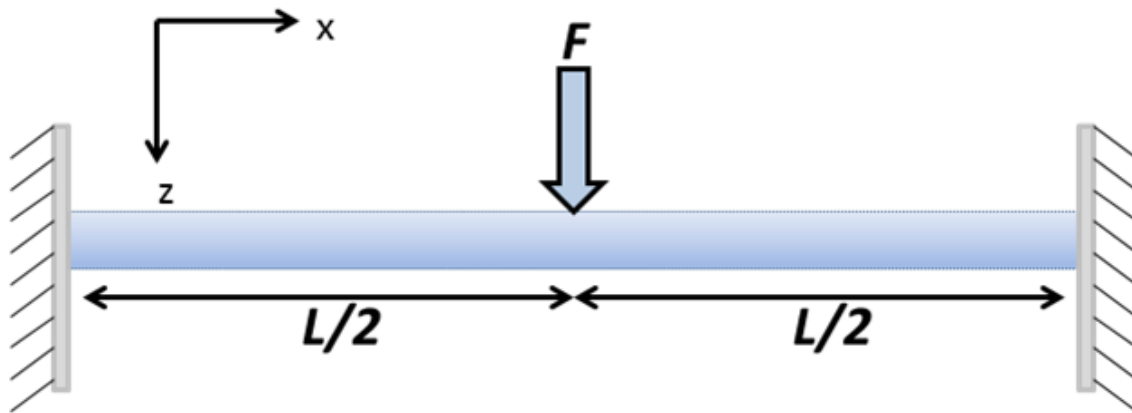


Figure 5. A fixed-fixed beam with a point load  $F$  applied at its center.