PROBLEM SET #7

Issued: Tuesday, Nov. 23, 2010.

Due (at 7 p.m.): Thursday, Dec. 9, 2010, in the EE C245 HW box in 240 Cory.

1. Gyroscopes are inertial sensors that measure rotation rate, which is an extremely important variable to know when navigating. One must know rotation rate (as well as other parameters, e.g., time, linear acceleration, etc) in order to determine position accurately (without the aid of GPS). Among the applications that use gyroscopes are airplanes (for navigation), boats (again, for navigation), automobiles (for skid control, among other applications), GPS receivers (to allow position determination during periods when the GPS signal cannot be received), and game controllers (e.g., the Wii). Of these applications, the last three already use MEMS-based gyroscopes, and the first two are presently targeted by MEMS realizations.

Gyroscopes operate by taking of advantage of the conservation of momentum, where an object moving in a given direction with a certain momentum will tend to continue moving in that direction even if its frame of reference is rotated about an axis. This is perhaps best further explained via example.

This problem involves the MEMS-based micro-gyroscope shown in Figure 1 and explained further in the figures that follow. This device is fabricated using the three-mask polysilicon process of HW #6—where a thin interconnect polysilicon layer is deposited and etched, a sacrifical silicon dioxide layer is deposited, anchor holes are etched, 4 μm of polysilicon is deposited and etched to define the structure, then the sacrificial layer is completely etched away.

In this device, momentum is generated by driving the proof mass into resonance vibration using the capacitive comb fingers along the *x*-axis. When the device is rotated about the *z*-axis (indicated in Fig. 2), the vibrating mass will attempt to preserve its momentum in the original *x*-direction, which will then make the mass appear to deflect in the *y*-direction. This *y*-directed motion is then sensed by parallel-plate capacitances to determine the rotation rate. In quantitative terms, the angular velocity is sensed about *z*-axis, which then generates a *Coriolis Force* ($F = 2m_s \dot{x}_d \times \Omega$) that acts on the proof mass along the *y*-direction, which is then picked up by the varying gap capacitances.

Several figures are provided to support the questions that follow. In particular, Table 1 provides measured or target parameter values for the fabricated device. Figs. 2-7 then identify different parts of the structure, indicate which portions are freely suspended and which are anchored (the blue regions are anchored), and provide key dimensions. As indicated, the gaps of comb fingers are all 1μm and the gaps of parallel-plate capacitive fingers are all 2μm. The thicknesses of the structures are all 4μm. Use a density $\rho = 2300 \; kg/m^3$ and Young's Modulus $E = 150 \; GPa$. The movable structure is DC biased relative to all electrodes at 10 V.

Table 1: Gyroscope Modeling Parameters

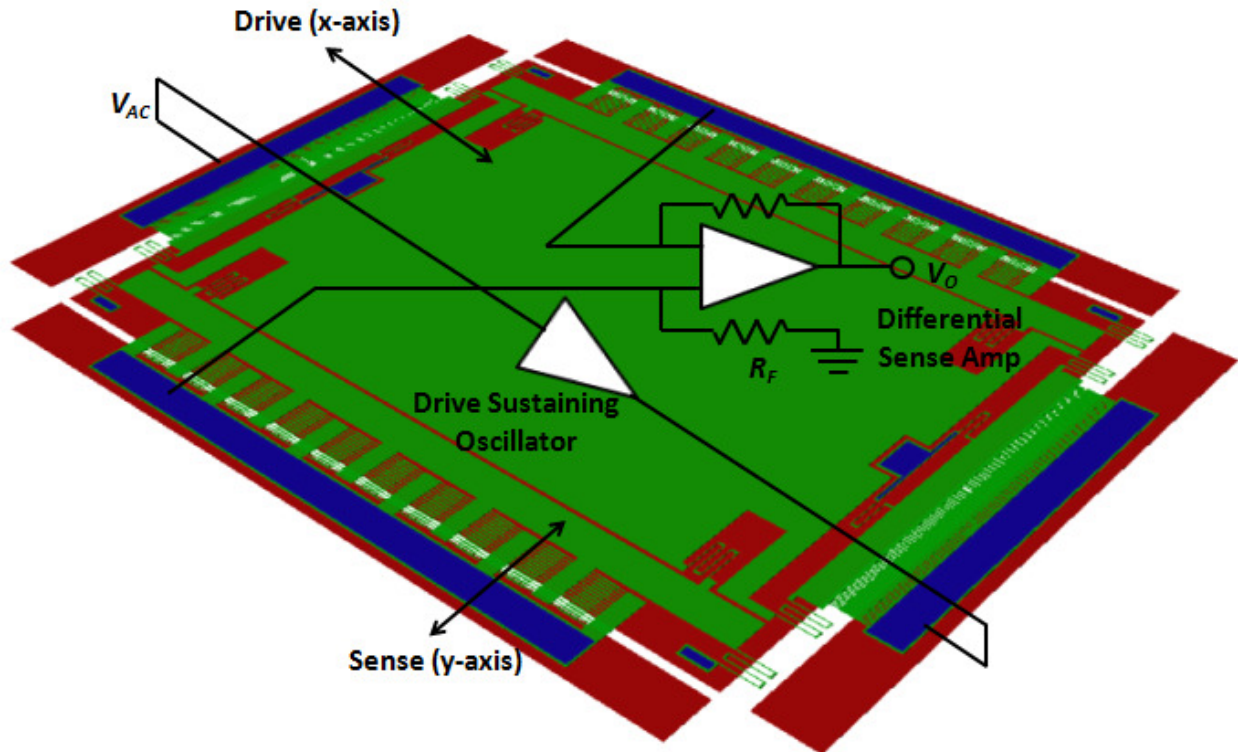|	Target Resonant Frequency (f_0)	Measured Quality Factor (Q)
Drive Mode	1000 Hz	100
Sense Mode	1200 Hz	50

Figure 1. Isometric View of a surface micromachined MEMS gyroscope.

(a) Use the surface area information given in the figures to determine the needed spring constants at the electrode locations for (i) the drive and (ii) the sense modes. Ignore the masses of the springs in this problem.

(b) What suspension beam lengths, L_D and L_S, are required to achieve the needed spring constants for (i) the drive and (ii) the sense modes determined in (a)?

(c) Identify the electrodes for the (i) drive and (ii) sense modes and determine the capacitance and change in capacitance per unit displacement for each. Calculate values for $x = 0$.

(d) Draw and specify (numerically) all element values in the equivalent circuits (transformers + LCR) modeling the (i) drive mode and (ii) the sense mode in (a).

(e) Code the equivalent circuits in (d) into SPICE netlists, add the necessary elements (e.g., a voltage source, a resistor, a capacitor, or an inductor) to drive at one end and detect velocity at the other, and simulate Bode plots for the voltage-to-velocity transfer functions using SPICE that include the low frequency and resonance responses of the structures.

(f) Assume that during steady-state oscillation along the drive axis the drive amplifier delivers an ac voltage v_d with an amplitude of 1.5V and a frequency equal to the resonance frequency of the drive mode. Also, assume that the input of the drive amplifier detects velocity and that its input resistance is very small. Determine the rotation rate-to-output current scale factor for this gyroscope. Give an expression and calculate its numerical value.
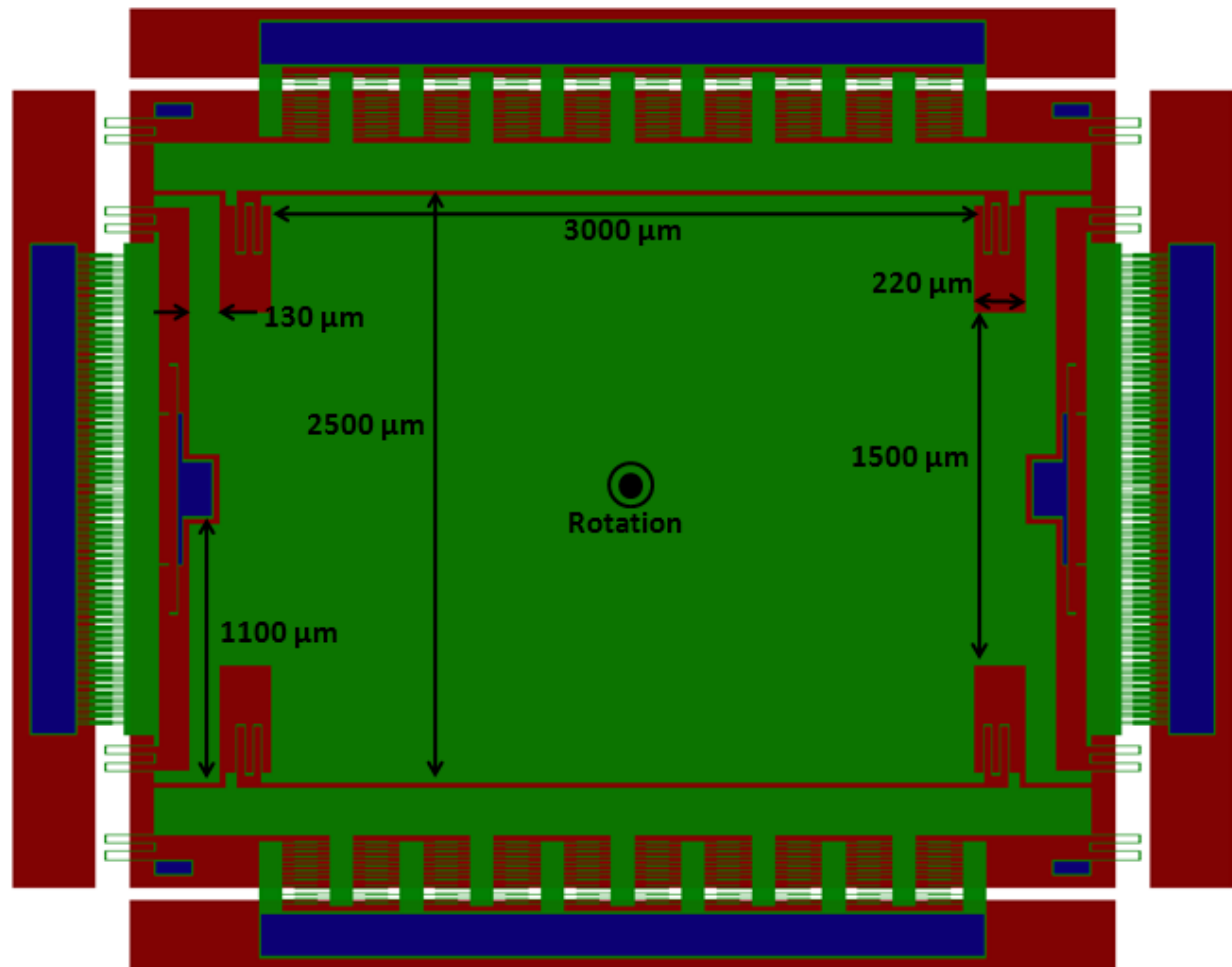
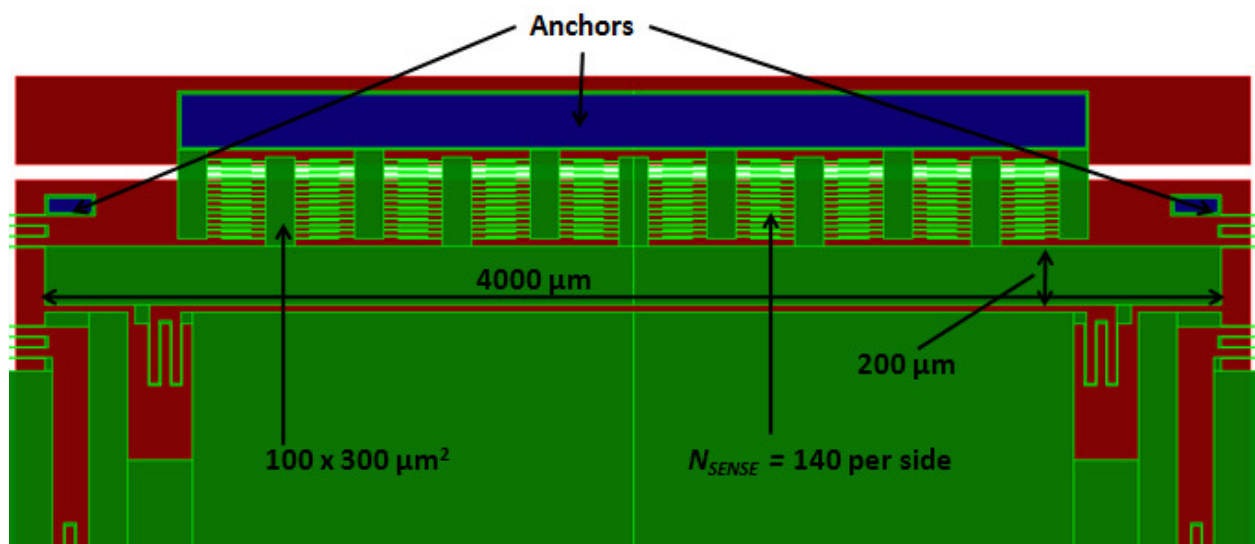**Figure 2: Top View with Proof Mass Dimensions**
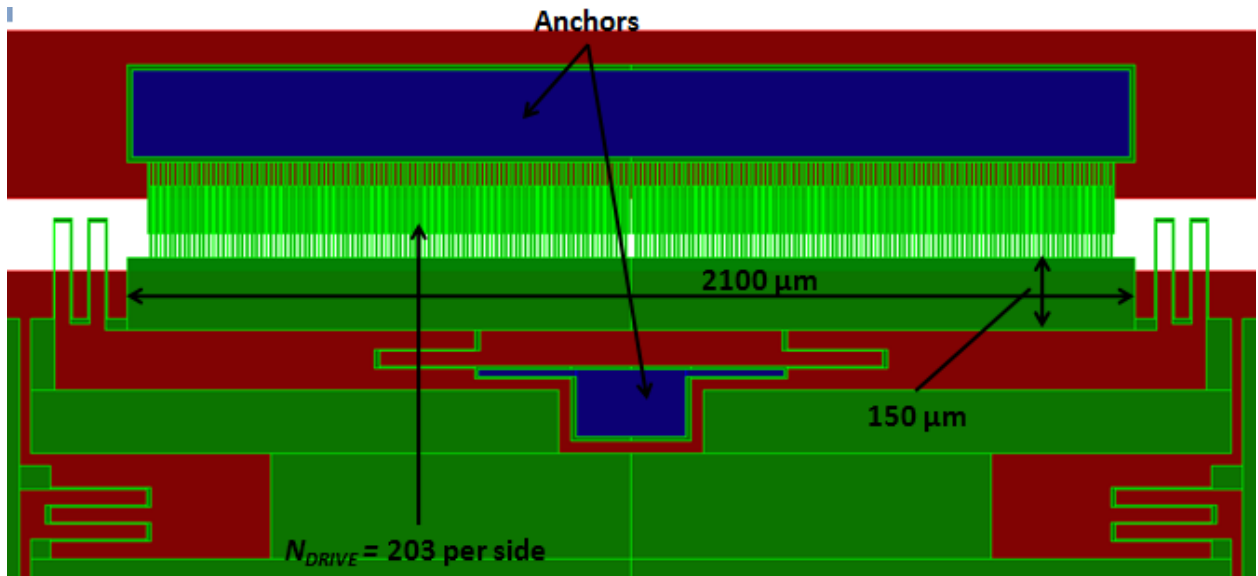

Figure 3: Sense Axis Detail

Figure 4: Drive Axis Detail (note this figure is rotated with respect to the others)
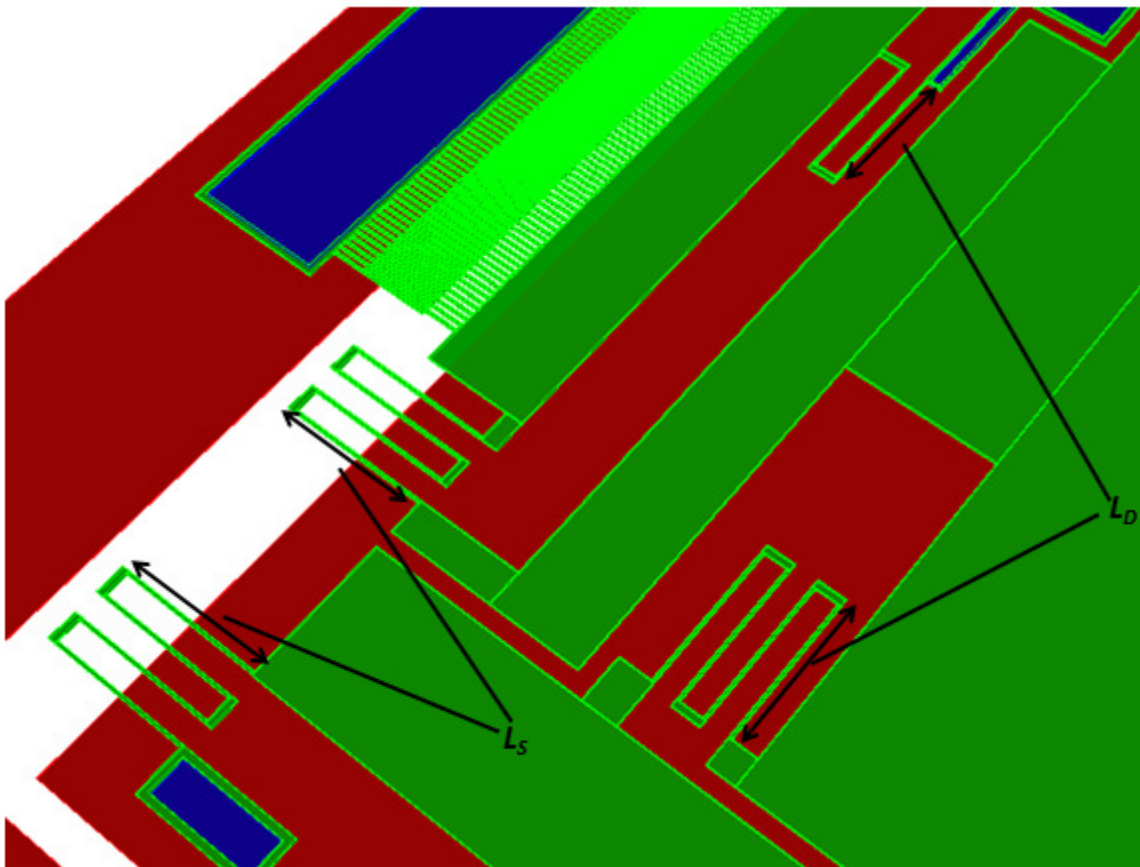
Figure 5: L_S denotes sense-directed springs, L_D denotes drive-directed springs.
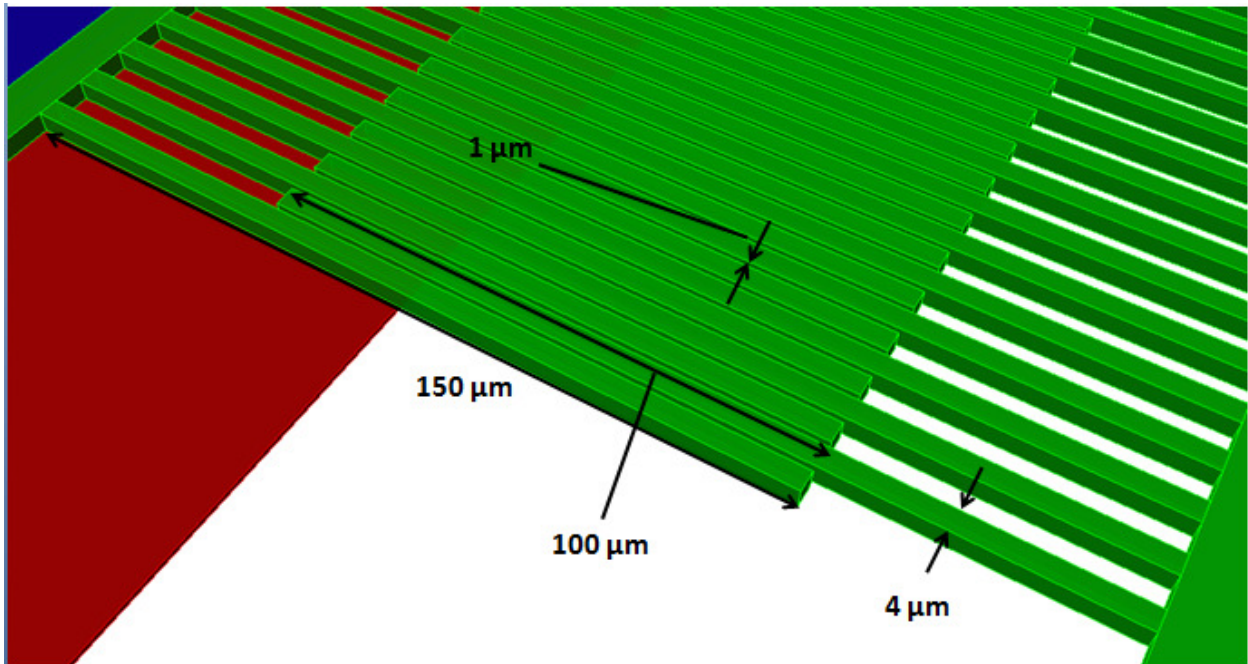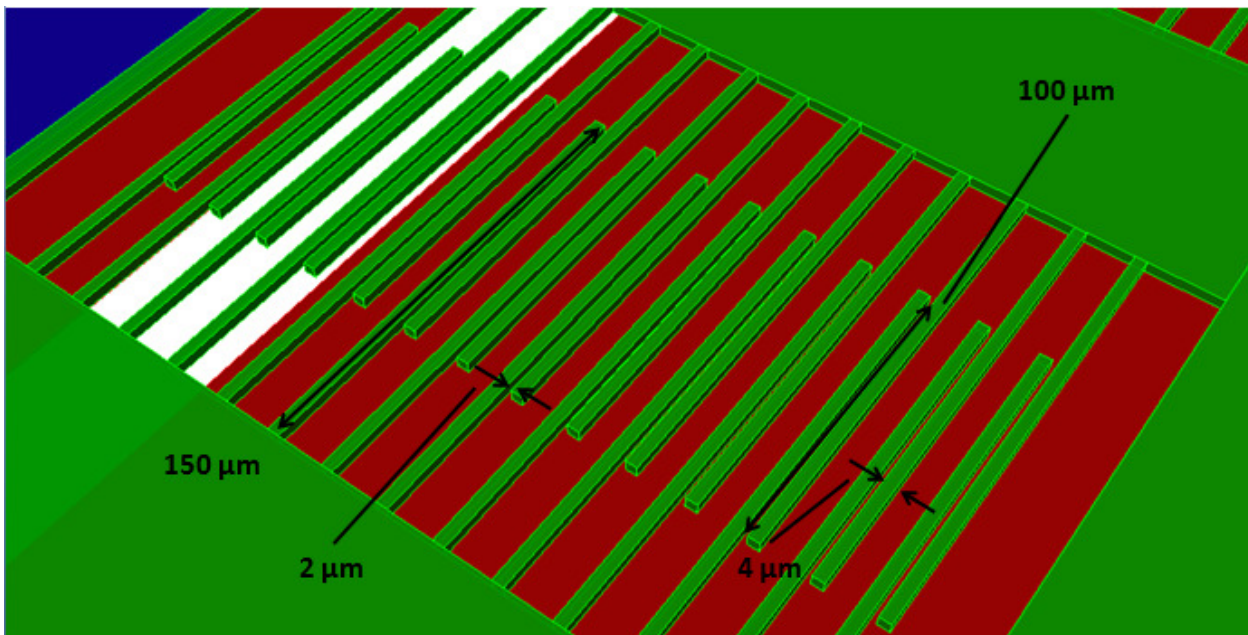
Figure 6: Drive Finger Detail

Figure 7: Sense Finger Detail