Lecture 12: Mechanics of Materials I

- **Announcements:**
- Module 7 on Mechanics of Materials online
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- Reading: Senturia Chpt. 3, Jaeger Chpt. 11, Handouts: "Bulk Micromachining of Silicon"
- Lecture Topics:
  - Bulk Micromachining
  - Anisotropic Etching of Silicon
  - Boron-Doped Etch Stop
  - Electrochemical Etch Stop
  - Isotropic Etching of Silicon
  - Deep Reactive Ion Etching (DRIE)
  - Wafer Bonding
- -----------------------------------
- Finish up bulk micromachining Module 6
- Start through material of Module 7: Mechanics of Materials, but lectures themselves will be mostly handwritten
- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
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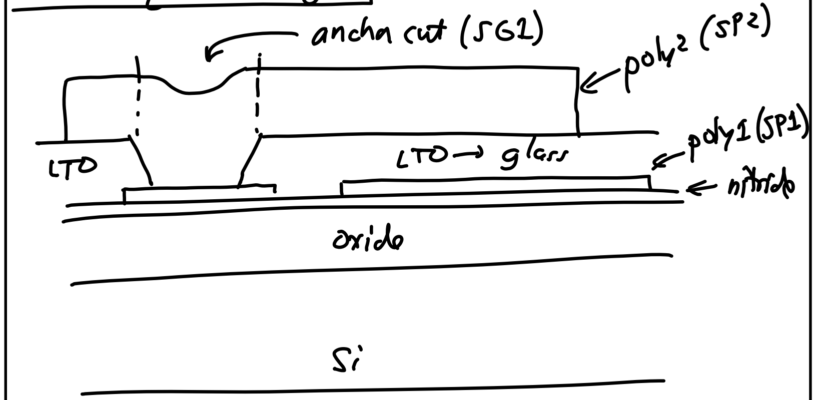
- **Last Time:**
- Going thru Bulk Micromachining Module 6
- Finish this now

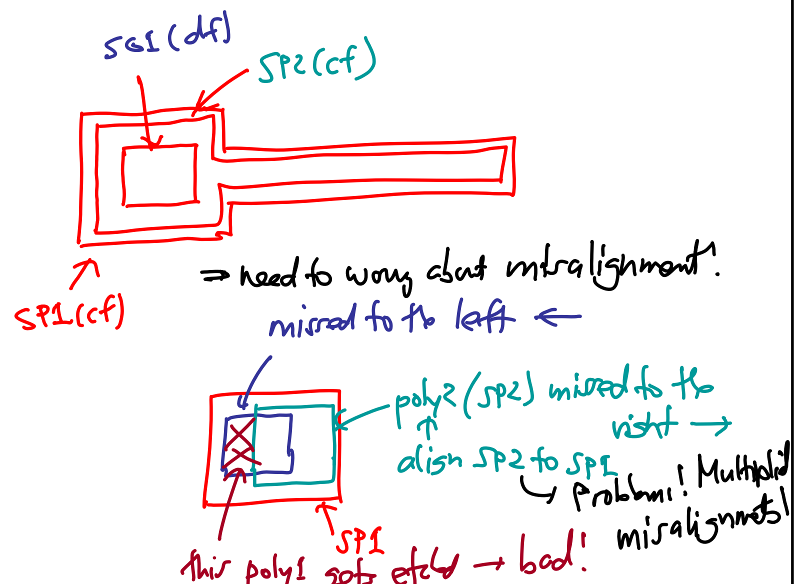
Mark Sequence Design



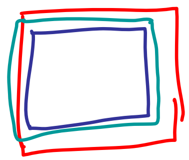
SP1 → 1st mark → no alignment

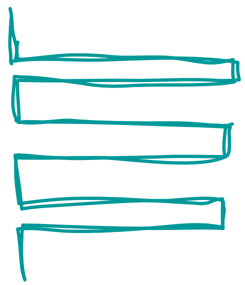
SG1 → align to SP1 (by default)

SP2 → align to either SP1 or SG1 → ???

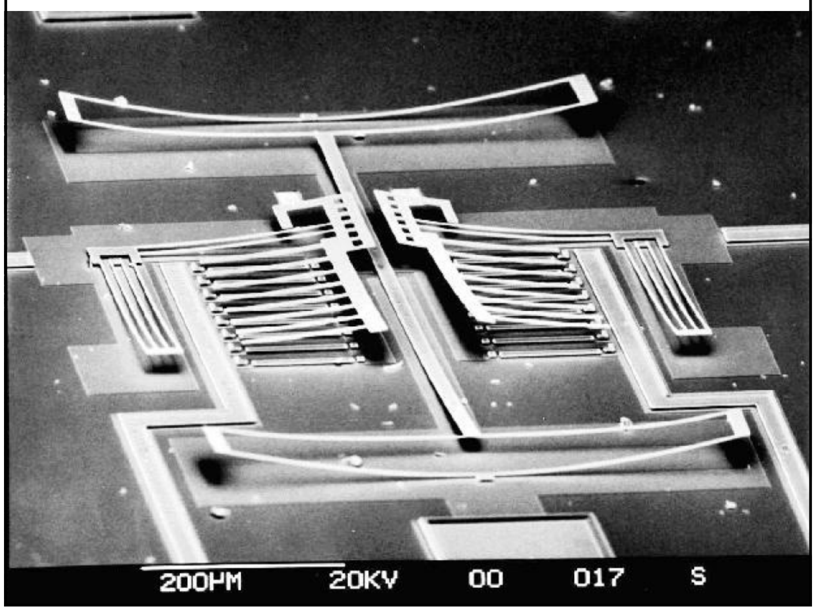


To fix this: align SP2 → SG1





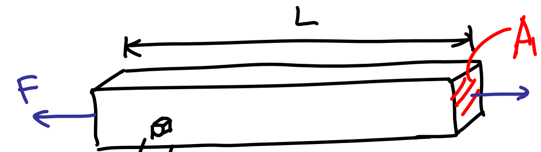
- We need to be able to model and understand this:



$\boxed{\text{Stress}}$

⇒ in 1-D:

- If F is normal to the surface → "normal stress"



assume F is uniform over A

$$\text{Stress} = \left\{ \begin{matrix} \text{Force per} \\ \text{Unit Area} \end{matrix} \right\} = \sigma = \frac{F}{A} \quad [N/m^2 \rightarrow Pa]$$

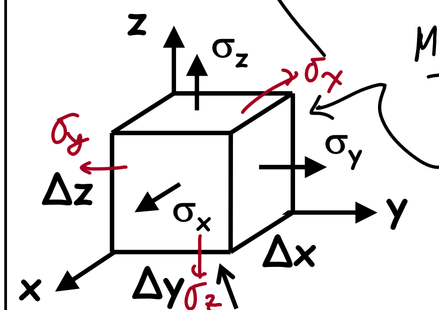
standard mks unit

Microscopic Definition:

applied to a differential volume element

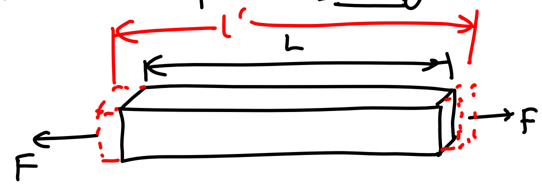
↓

w/ this, can easily assume the stress is uniform on each face (not @ just the point of the vector)



Differential volume element

Strain (other part of elasticity)



Strain: $\left\{ \begin{array}{l} \text{Fractional Change} \\ \text{in length} \end{array} \right\} = \varepsilon = \dfrac{L'-L}{L} = \dfrac{\Delta L}{L}$ [unitless]

Sometimes use "microstrain" where

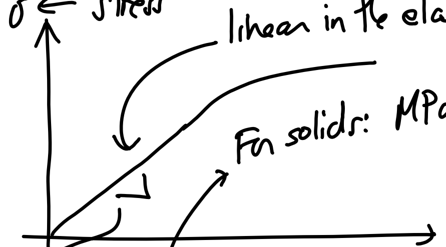
$1\mu\varepsilon = \dfrac{\Delta L}{L}$ of 1 part in 10^6

↑ just $\times 10^{-6}$

In the elastic regime

"small" stresses at "low" temperature

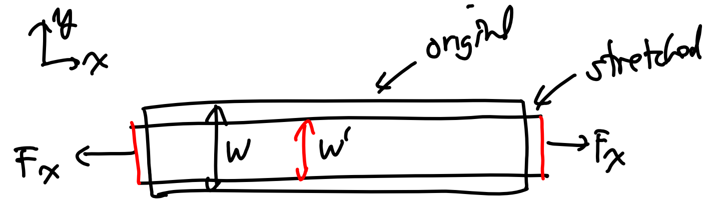
strain is proportional to stress:



slope = E = Young's modulus of elasticity

$\sigma = \varepsilon E \rightarrow \boxed{\varepsilon = \dfrac{\sigma}{E} \text{ [unitless]}}$ ✓

The Poisson Ratio



Apply normal stress } → uniaxial strain
→ but also contraction in directions transverse to the uniaxial strain

⇒ contraction creates a (−) strain:

$$\varepsilon_y = \frac{W'-W}{W} = \frac{\Delta W}{W} = -\nu \varepsilon_x$$

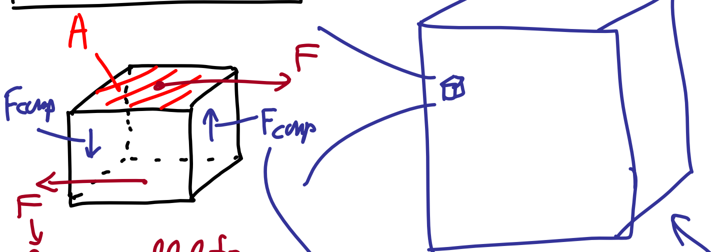
ν = Poisson ratio [unitless]

↳ typical values: 0 → 0.5

→ inorganic solids: 0.2 – 0.3

→ elastomers (e.g., rubber): ~0.5

Shear Stress & Strain



force parallel to the surface = shear stress

Note: Assume compensating forces applied to prevent a net torque (i.e., to prevent rotation → which doesn't happen when the element is embedded in this)

Shear Stress = $\left\{ \begin{array}{l} \text{Force per unit area} \\ \text{parallel to the surface} \end{array} \right\}$

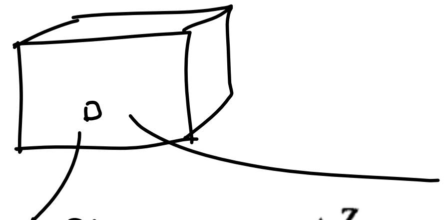
$$= \tau = \frac{F}{A} \quad [\mathrm{Pa}]$$

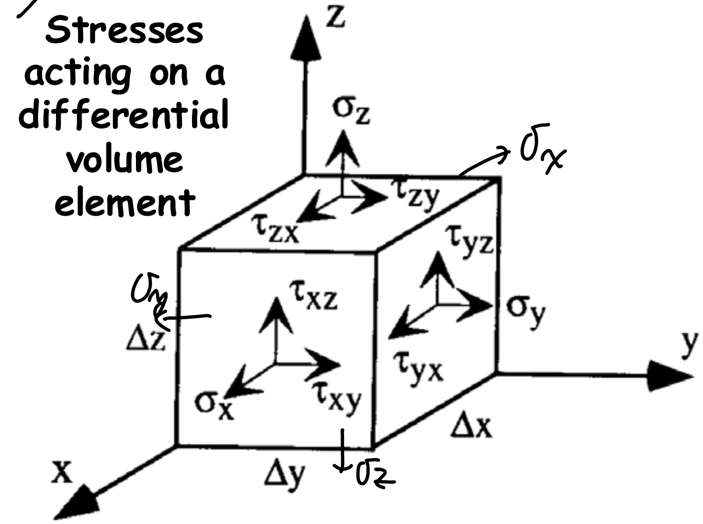
generates a Shear Strain:

$$\text{Shear Strain} = \theta = \frac{\tau}{G} \leftarrow G \triangleq \text{shear modulus}$$

$$G = \frac{E}{2(1+\nu)}$$

2D & 3D Considerations





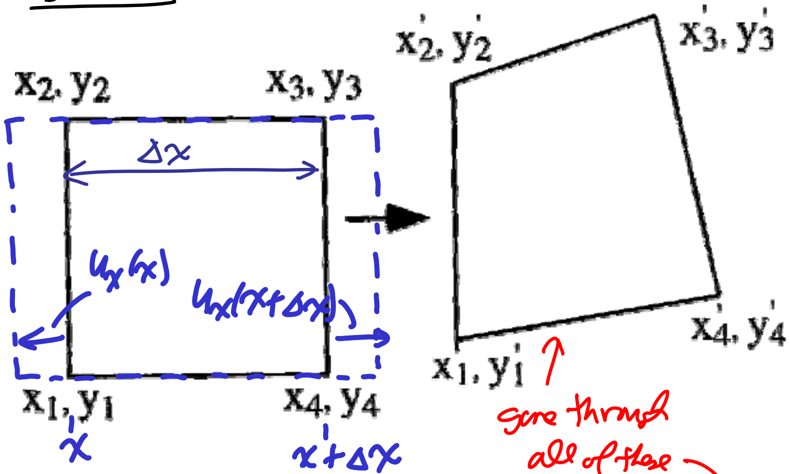
For a Stable Element:

⇒ Every σ must have an equal σ in the opposite direction on the other side of the element

⇒ No net torque:

$$\tau_{xy} = \tau_{yx} \quad \tau_{xz} = \tau_{zx} \quad \tau_{yz} = \tau_{zy}$$

2D Strain



gone through all of these

In general, motion consists of:

① Rigid body displacement (motion of the center of mass)

② Rigid body rotation (rotation about the center of mass)

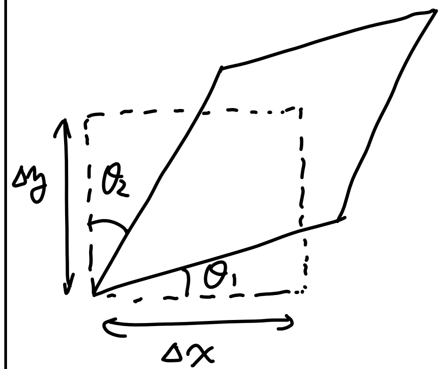
③ Deformation relative to displacement & rotation.

→ handle by displacement vector → do it one axis at a time

For axial strain in the x-direction:

$$\varepsilon_x = \frac{u_x(x+\Delta x) - u_x(x)}{\Delta x} = \frac{\partial u_x}{\partial x}$$

2D Shear Strain



→ For shear strain, γ

$$\gamma_{xy} = \theta_1 + \theta_2 \approx \left(\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x}\right) = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$