Lecture 13: Mechanics of Materials II

- **Announcements**:
- HW#4 passed out and online Nov. 2: 7 Oct. 28: 2
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- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
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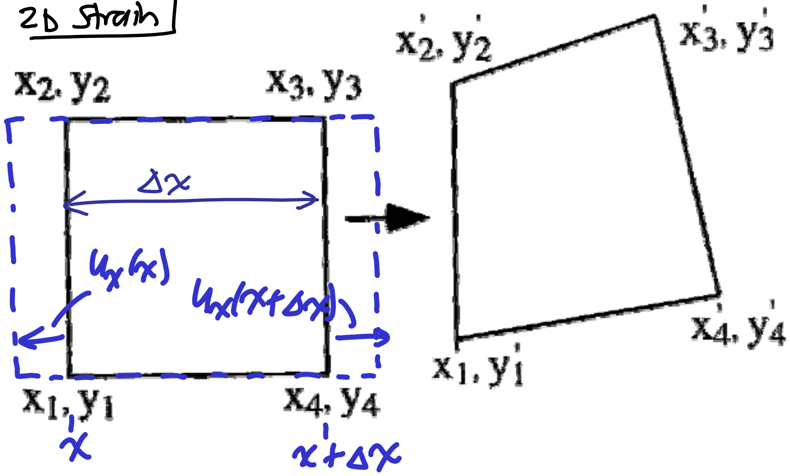
- **Last Time**:

2D Strain



In general, motion consists of:

① Rigid body displacement (motion of the center of mass)

② Rigid body rotation (rotation about the center of mass)

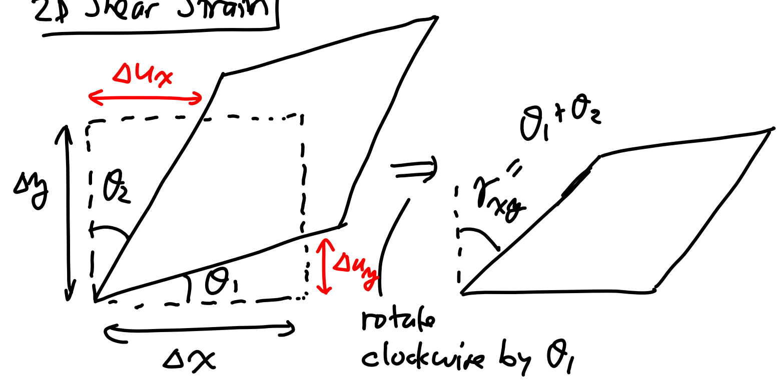
③ Deformation relative to displacement & rotation.

→ handle by displacement vector → do it an axis at a time

For axial strain in the x-direction:

$$\varepsilon_x = \frac{u_x(x+\Delta x) - u_x(x)}{\Delta x} = \frac{\partial u_x}{\partial x}$$

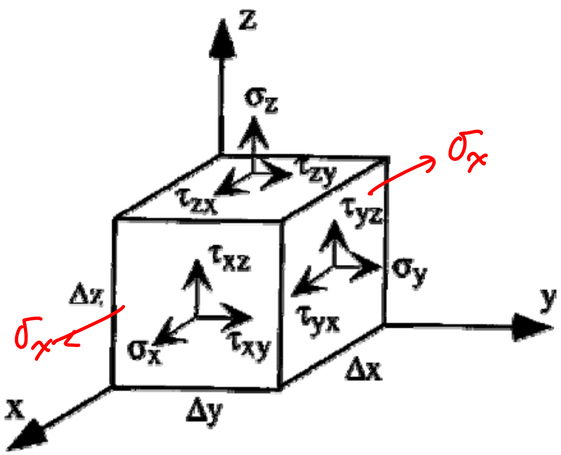
2D Shear Strain



⇒ For shear strain, must remove any rigid body rotation that accompanies the deformation:

$$\gamma_{xy} = \theta_1 + \theta_2 \approx \left(\frac{\Delta u_x}{\Delta y} + \frac{\Delta u_y}{\Delta x}\right) = \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right)$$

Volume Change for Uniaxial Stress



Given an x-directed uniaxial stress, σ_x:

before → after application of σ_x

$\Delta x \rightarrow \Delta x(1+\epsilon_x)$

$\Delta y \rightarrow \Delta y(1-\nu\epsilon_x)$

$\Delta z \rightarrow \Delta z(1-\nu\epsilon_x)$

↓ The resulting change in volume ΔV

$$\Delta V = \Delta x \Delta y \Delta z (1+\epsilon_x)(1-\nu\epsilon_x)^2 - \Delta x \Delta y \Delta z$$

$$= \Delta x \Delta y \Delta z\left[(1+\epsilon_x)(1-\nu\epsilon_x)^2 - 1\right]$$

[Assume small strains]

$\left[(1+mx)^n \approx 1 + nmx\right]$

$$= \Delta x \Delta y \Delta z\left[\cancel{1} + \epsilon_x - 2\nu\epsilon_x - \cancel{2\nu\epsilon_x^2} - \cancel{1}\right]$$

neg!

ϵ_x = small

$$\boxed{\delta V \cong \Delta x \Delta y \Delta z (1-2\nu)\epsilon_x}$$

For $\nu = 0.5$ (rubber) → no ΔV!

$\nu < 0.5$ → finite ΔV

Isotropic Elasticity in 3D

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \qquad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

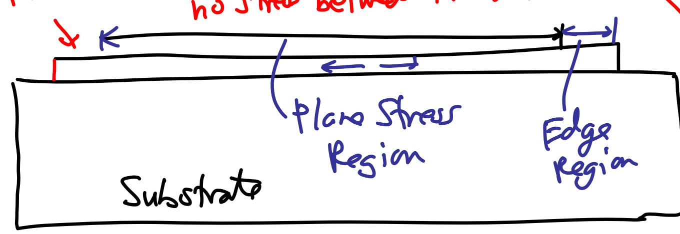
$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_z + \sigma_x)\right] \qquad \cdots$$

$\vdots$

Ex.

Important Case: Plane Stress

Thin Film → deposited @ high temp. @ which there is no stress between the substrate & film



take T↓ → to room temperature → get stress!

Plane Stress Region

Get two components: $\sigma_z = 0$

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + 0)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \nu(\sigma_x + 0)\right]$$

[Symmetry in the xy-plane] ⇒ $\sigma_x = \sigma_y = \sigma$

∴ the in-plane strain components are: $\epsilon_x = \epsilon_y = \epsilon$

$$\epsilon_x = \frac{1}{E}(\sigma - \nu\sigma) = \frac{\sigma}{\left[\frac{E}{(1-\nu)}\right]} = \frac{\sigma}{E'}$$

where

$E' \triangleq$ Biaxial Modulus $= \dfrac{E}{1-\nu}$

Edge Region

Film shrinks w/ T↓ faster than substrate

Linear Thermal Expansion

⇒ As T↑ → most solids expand → V↑

Definition. Linear Thermal Expansion Coefficient

$$\left.\begin{array}{l}\text{Linear Thermal}\\ \text{Expansion Coeff.}\end{array}\right\} \triangleq \alpha_T = \frac{d\epsilon_x}{dT} \quad [\text{kelvin}^{-1}]$$

[μstrain/K]

Remarks:

① α_T values range: 10^{-6} to 10^{-7}

② Can capture the → using μstrain/K

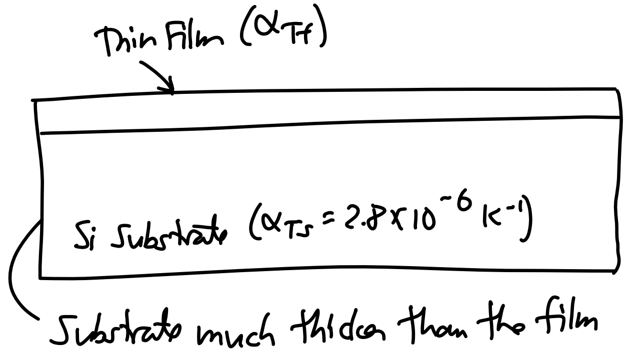
③ In 3D, get volume thermal expansion coeff.

$$\frac{\Delta V}{V} = 3\alpha_T \Delta T$$

④ For moderate ΔT's → $\alpha_T \approx$ constant

→ In actuality, $\alpha_T = f(T)$

Thin Film Thermal Stress



Assume:

1. Film deposited stress free @ T_d
2. Whole thing cooled to room temp., T_r

What is the thermal mismatch strain?

⇒ substrate much thicker than film → substrate dictates the amount of contraction for both

Thermal Strain in Substrate: (in one in-plane dim)

$$\epsilon_s = -\alpha_{Ts}\Delta T, \text{ where } \Delta T = T_d - T_r$$

If the film were not attached to the substrate:

$$\epsilon_{f,free} = -\alpha_{Tf}\Delta T$$

But since the film is attached to substrate, actual strain in film is

$$\epsilon_{f,attached} = -\alpha_{Ts}\Delta T$$

Thus:

$$\boxed{\text{Thermal Mismatch Strain} = \epsilon_{f,mismatch} = (\alpha_{Tf} - \alpha_{Ts})\Delta T}$$

Note that this is biaxial strain:

$$\sigma_{f,mismatch} = \left(\frac{E}{1-\nu}\right)\epsilon_{f,mismatch}$$

Ex. Thin film is polyimide → $\alpha_{Tf} = 70\times10^{-6}K^{-1}$

$E' = 4\ GPa$

deposited @ 250°C, then cooled to RT = 25°C → $\Delta T = 225K$

$$\epsilon_{f,mismatch} = (70-2.8)\mu(225) = 1.5\times10^{-2}$$

$$\sigma_{f,mismatch} = (4G)(1.5\times10^{-2}) = \underline{60.5\ MPa}$$

(+) ∴ tensile

[(-) would be compressive]