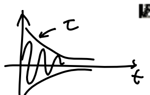
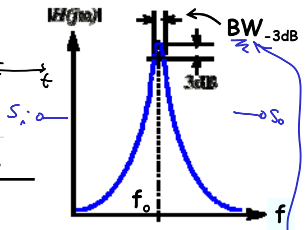
Quality Factor (or *Q*)

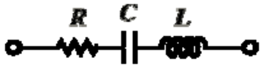
- Measure of the frequency selectivity of a tuned circuit
- **Definition**:



$$Q = \frac{\text{Total Energy Per Cycle}}{\text{Energy Lost Per Cycle}} = \frac{f_o}{BW_{3dB}}$$

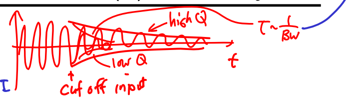


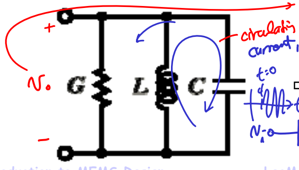
- **Example**: series LCR circuit



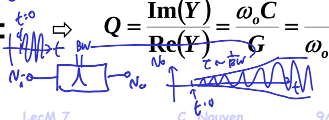
$$\Rightarrow \quad Q = \frac{\mathrm{Im}(Z)}{\mathrm{Re}(Z)} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

- **Example**: parallel LCR circuit



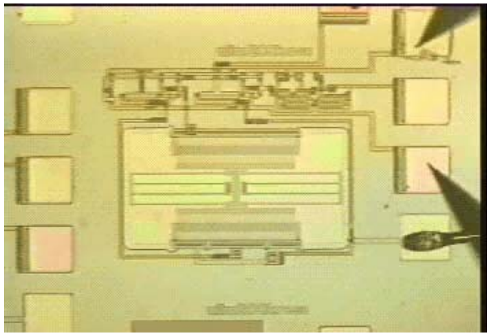


$$\Rightarrow \quad Q = \frac{\mathrm{Im}(Y)}{\mathrm{Re}(Y)} = \frac{\omega_o C}{G} = \frac{1}{\omega_o LG}$$

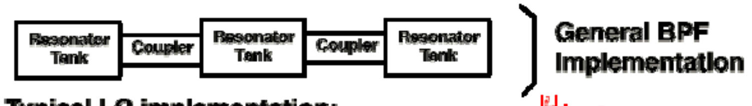


Comb-Drive Resonator in Action

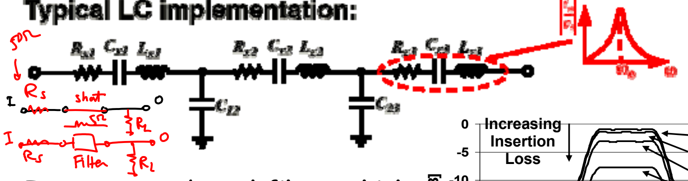
- **Below**: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach



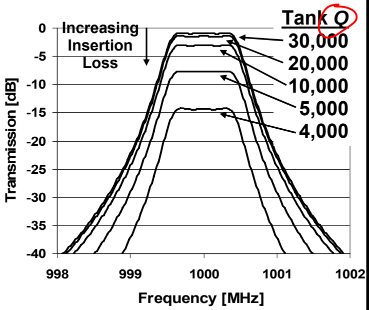
Selective Low-Loss Filters: Need *Q*



Typical LC implementation:

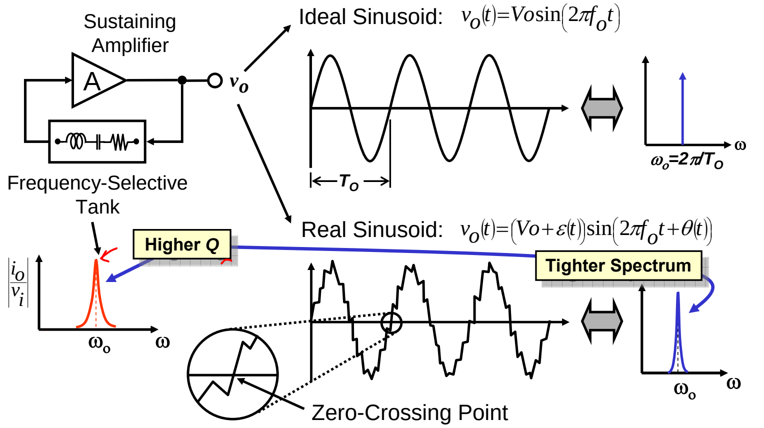


- In resonator-based filters: high tank Q ⇔ low insertion loss
- **At right**: a 0.1% bandwidth, 3-res filter @ 1 GHz (simulated)
  - ↳ heavy insertion loss for resonator Q < 10,000



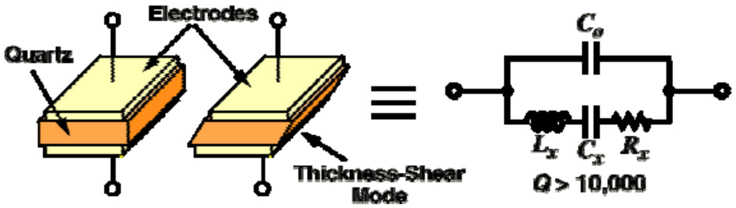
Oscillator: Need for High *Q*

- **Main Function**: provide a stable output frequency
- **Difficulty**: superposed noise degrades frequency stability

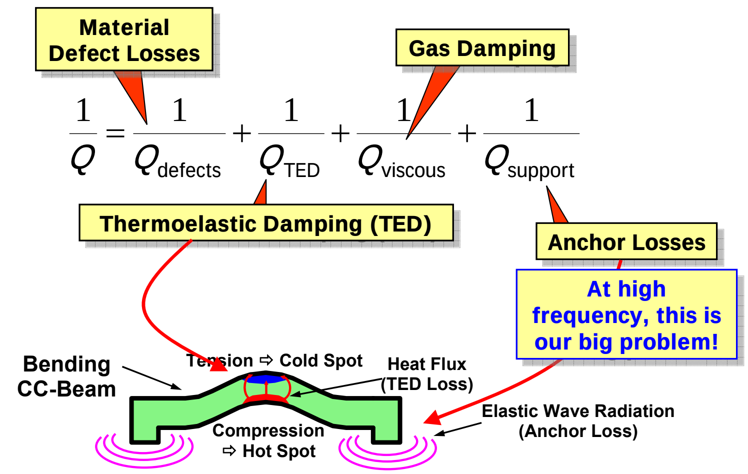


Attaining High Q

- Problem: IC's cannot achieve Q's in the thousands
  - transistors ⇨ consume too much power to get Q
  - on-chip spiral inductors ⇨ Q's no higher than ~10
  - off-chip inductors ⇨ Q's in the range of 100's
- Observation: vibrating mechanical resonances ⇨ Q > 1,000
- Example: quartz crystal resonators (e.g., in wristwatches)
  - extremely high Q's ~ 10,000 or higher (Q ~ 10^6 possible)
  - mechanically vibrates at a distinct frequency in a thickness-shear mode



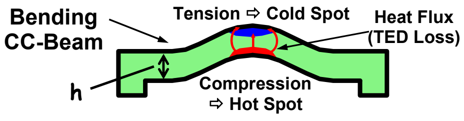
Energy Dissipation and Resonator Q



$$\frac{1}{Q} = \frac{1}{Q_{\text{defects}}} + \frac{1}{Q_{\text{TED}}} + \frac{1}{Q_{\text{viscous}}} + \frac{1}{Q_{\text{support}}}$$

Thermoelastic Damping (TED)

- Occurs when heat moves from compressed parts to tensioned parts → heat flux = energy loss



$$\varsigma = \Gamma(T)\Omega(f) = \frac{1}{2Q}$$

$$\Gamma(T) = \frac{\alpha^2 TE}{4\rho C_p}$$

$$\Omega(f_o) = 2\left[\frac{f_{TED}\, f}{f_{TED}^2 + f^2}\right]$$

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

ζ = thermoelastic damping factor
α = thermal expansion coefficient
T = beam temperature
E = elastic modulus
ρ = material density
C_p = heat capacity at const. pressure
K = thermal conductivity
f = beam frequency
h = beam thickness
f_{TED} = characteristic TED frequency

TED Characteristic Frequency

$$f_{TED} = \frac{\pi K}{2\rho C_p h^2}$$

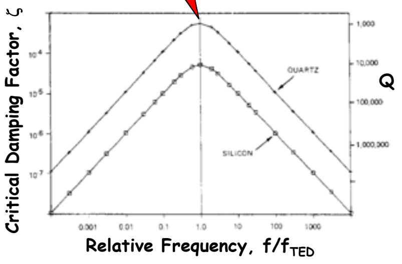
ρ = material density
C_p = heat capacity at const. pressure
K = thermal conductivity
h = beam thickness
f_{TED} = characteristic TED frequency

- Governed by
  - Resonator dimensions
  - Material properties

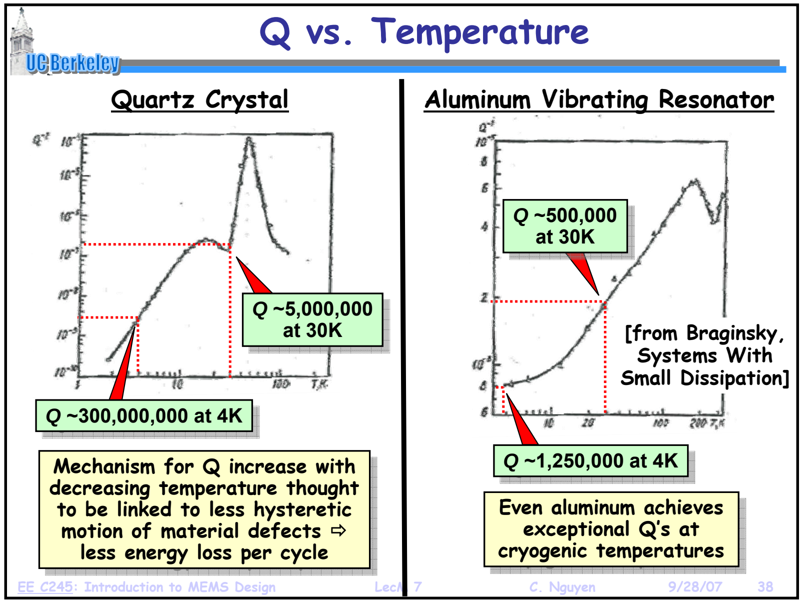
TABLE 1. MATERIAL PROPERTIES

Property	Silicon	Quartz	Units
Thermal expansion	2.60	13.70	ppm/°K
Elastic modulus	1.70	0.78	10^{12} dyne/cm^2
Material density	2.33	2.60	g/cm^3
Heat capacity	0.70	0.75	J/g/°K
Thermal conductivity	1.50	0.10	10^7 dyne/°K/s
Peak damping @ 300°k	1.06	11.34	10^{-4}

Peak where Q is minimized



[from Roszhart, Hilton Head 1990]



Q vs. Temperature

Quartz Crystal

Q ~5,000,000 at 30K

Q ~300,000,000 at 4K

Mechanism for Q increase with decreasing temperature thought to be linked to less hysteretic motion of material defects ⇨ less energy loss per cycle

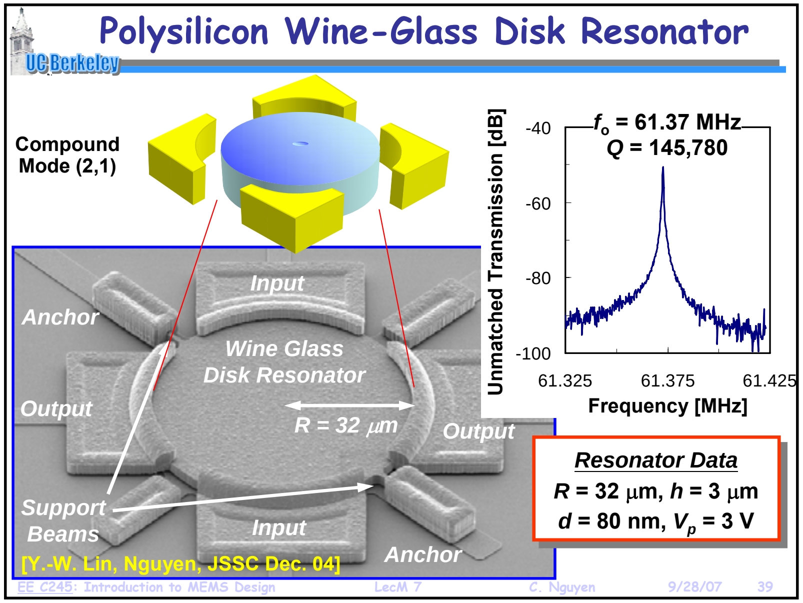
Aluminum Vibrating Resonator

Q ~500,000 at 30K

Q ~1,250,000 at 4K

[from Braginsky, Systems With Small Dissipation]

Even aluminum achieves exceptional Q's at cryogenic temperatures



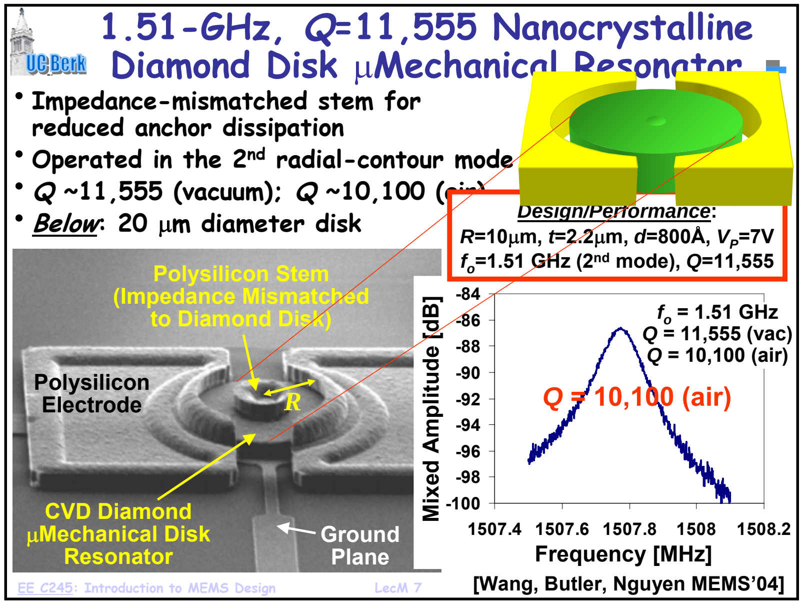
Polysilicon Wine-Glass Disk Resonator

Compound Mode (2,1)

f_o = 61.37 MHz
Q = 145,780

Resonator Data
R = 32 μm, h = 3 μm
d = 80 nm, V_p = 3 V

[Y.-W. Lin, Nguyen, JSSC Dec. 04]



1.51-GHz, Q=11,555 Nanocrystalline Diamond Disk μMechanical Resonator

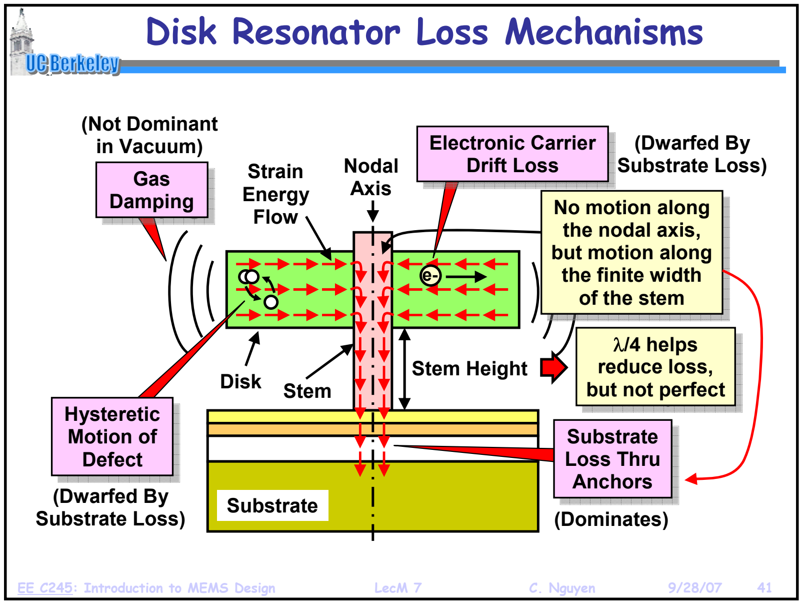
- Impedance-mismatched stem for reduced anchor dissipation
- Operated in the 2nd radial-contour mode
- Q ~11,555 (vacuum); Q ~10,100 (air)
- *Below*: 20 μm diameter disk

Design/Performance:
R=10μm, t=2.2μm, d=800Å, V_P=7V
f_o=1.51 GHz (2nd mode), Q=11,555

f_o = 1.51 GHz
Q = 11,555 (vac)
Q = 10,100 (air)

Q = 10,100 (air)

[Wang, Butler, Nguyen MEMS'04]



Disk Resonator Loss Mechanisms

(Not Dominant in Vacuum)
Gas Damping

Electronic Carrier Drift Loss
(Dwarfed By Substrate Loss)

No motion along the nodal axis, but motion along the finite width of the stem

λ/4 helps reduce loss, but not perfect

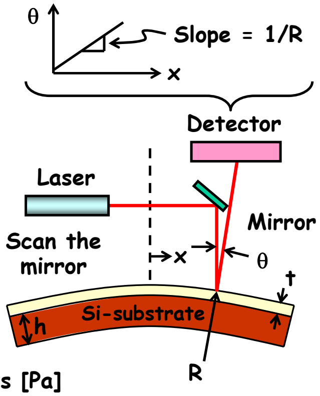
Hysteretic Motion of Defect
(Dwarfed By Substrate Loss)

Substrate Loss Thru Anchors
(Dominates)

MEMS Material Property Test Structures

Stress Measurement Via Wafer Curvature

- Compressively stressed film → bends a wafer into a convex shape
- Tensile stressed film → bends a wafer into a concave shape
- Can optically measure the deflection of the wafer before and after the film is deposited
- Determine the radius of curvature R, then apply:

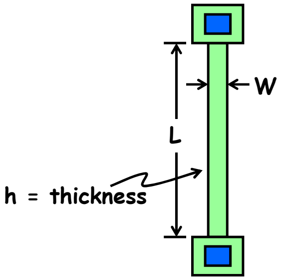


$$\sigma = \frac{E'h^2}{6Rt}$$

σ = film stress [Pa]
E' = E/(1-ν) = biaxial elastic modulus [Pa]
h = substrate thickness [m]
t = film thickness
R = substrate radius of curvature [m]

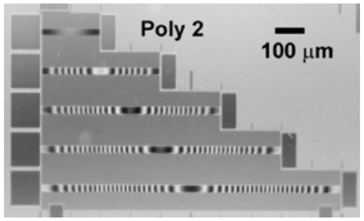
MEMS Stress Test Structure

- Simple Approach: use a clamped-clamped beam
  - Compressive stress causes buckling
  - Arrays with increasing length are used to determine the critical buckling load, where



$$\sigma_{critical} = -\frac{\pi^2}{3}\frac{Eh^2}{L^2}$$

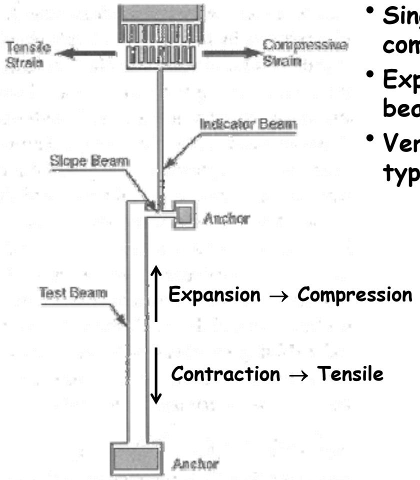
E = Young's modulus [Pa]
I = (1/12)Wh^3 = moment of inertia
L, W, h indicated in the figure

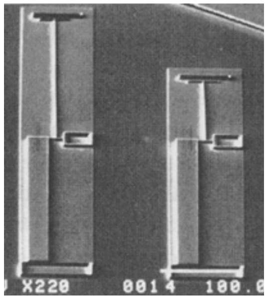


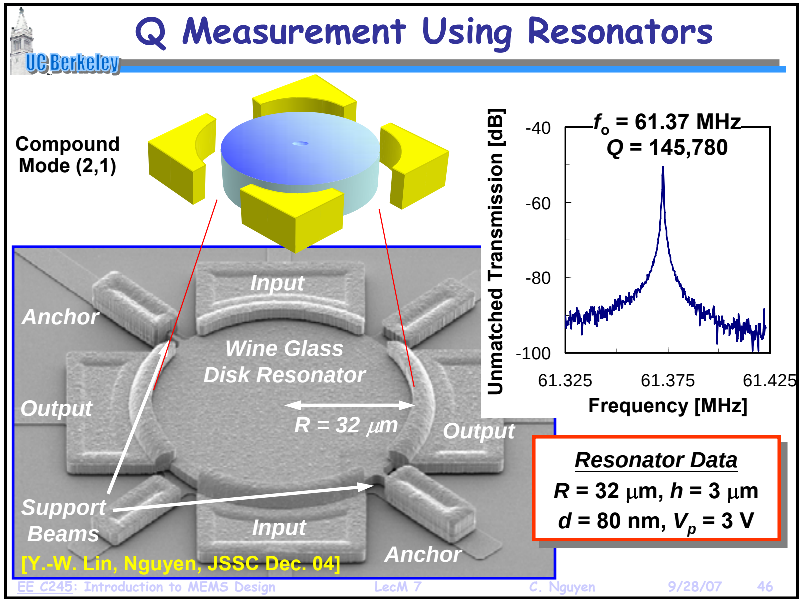
  - Limitation: Only compressive stress is measurable

More Effective Stress Diagnostic

- Single structure measures both compressive and tensile stress
- Expansion or contraction of test beam → deflection of pointer
- Vernier movement indicates type and magnitude of stress







Q Measurement Using Resonators

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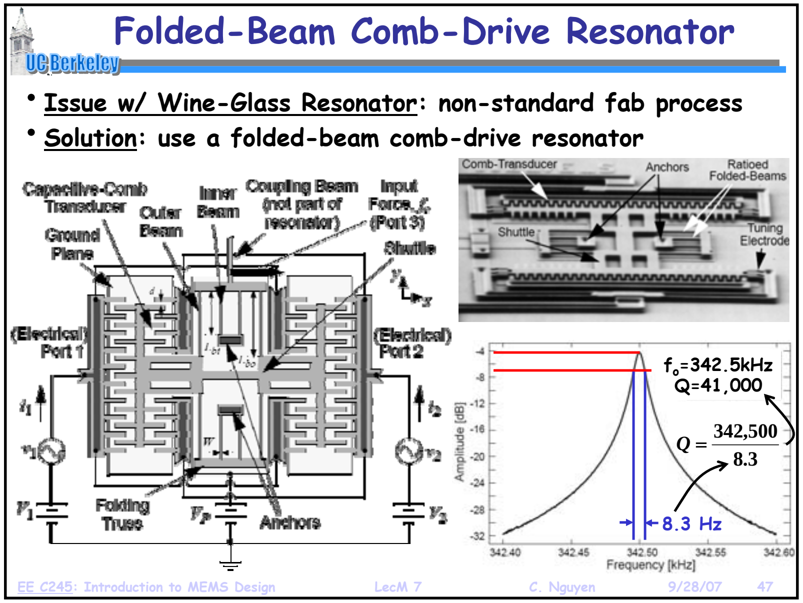
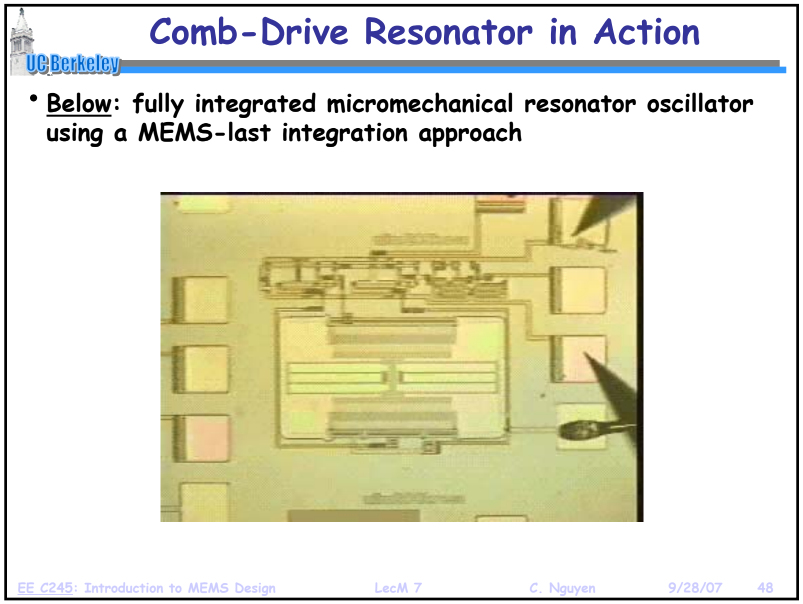


Folded-Beam Comb-Drive Resonator

- **Issue w/ Wine-Glass Resonator**: non-standard fab process
- **Solution**: use a folded-beam comb-drive resonator

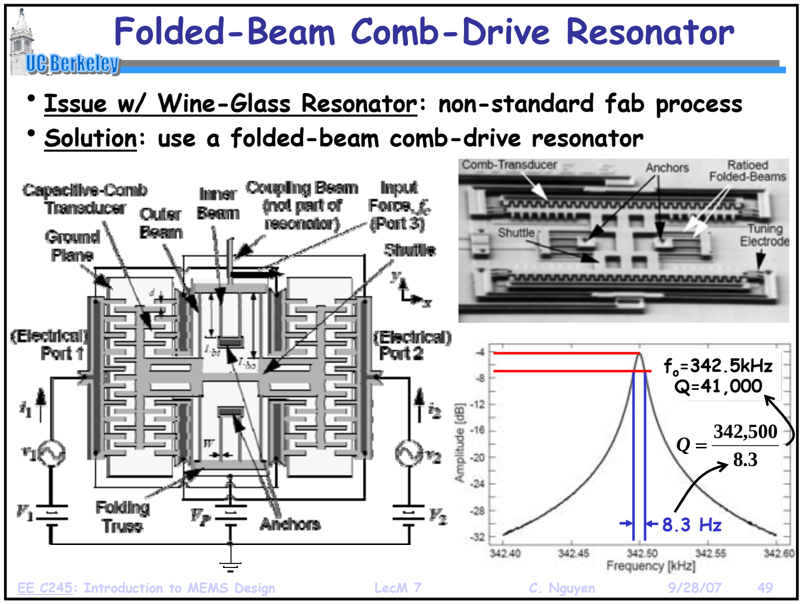
f_o=342.5kHz
Q=41,000

$$Q = \frac{342{,}500}{8.3}$$



Comb-Drive Resonator in Action

- **Below**: fully integrated micromechanical resonator oscillator using a MEMS-last integration approach



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f_o=342.5kHz
Q=41,000

$$Q = \frac{342{,}500}{8.3}$$

Measurement of Young's Modulus

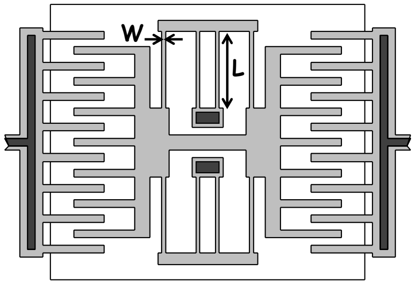
- Use micromechanical resonators
  - ✓ Resonance frequency depends on E
  - ✓ For a folded-beam resonator:

Young's modulus

$$\text{Resonance Frequency} = f_o = \left[\frac{4Eh(W/L)^3}{M_{eq}}\right]^{1/2}$$

Equivalent mass

h = thickness



- Extract E from measured frequency f_o
- Measure f_o for several resonators with varying dimensions
- Use multiple data points to remove uncertainty in some parameters

Anisotropic Materials

Elastic Constants in Crystalline Materials

- Get different elastic constants in different crystallographic directions → 81 of them in all
  - ✓ Cubic symmetries make 60 of these terms zero, leaving 21 of them remaining that need be accounted for
- Thus, describe stress-strain relations using a 6x6 matrix

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{bmatrix} = \underbrace{\begin{bmatrix}C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16}\\ C_{12} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26}\\ C_{13} & C_{23} & C_{33} & C_{34} & C_{35} & C_{36}\\ C_{14} & C_{24} & C_{34} & C_{44} & C_{45} & C_{46}\\ C_{15} & C_{25} & C_{35} & C_{45} & C_{55} & C_{56}\\ C_{16} & C_{26} & C_{36} & C_{46} & C_{56} & C_{66}\end{bmatrix}}_{\text{Stiffness Coefficients}} \begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{bmatrix}$$

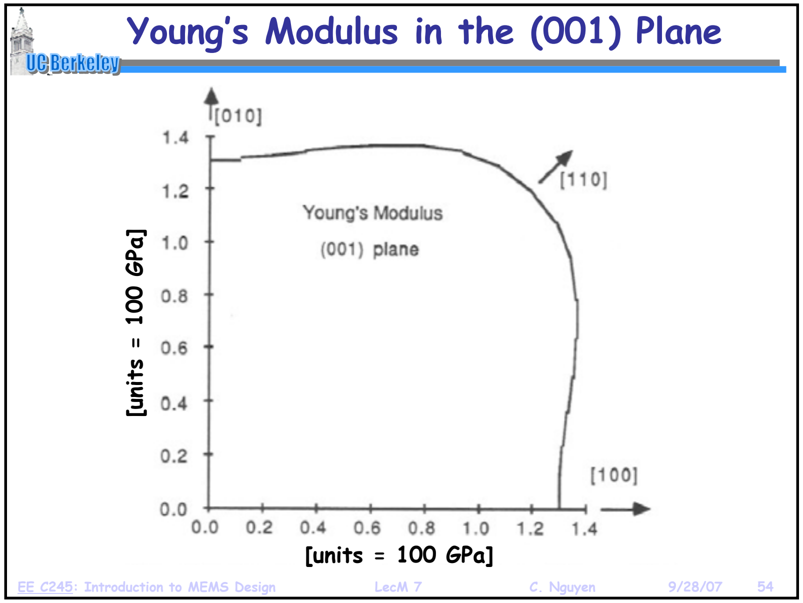
Stresses Stiffness Coefficients Strains

Stiffness Coefficients of Silicon

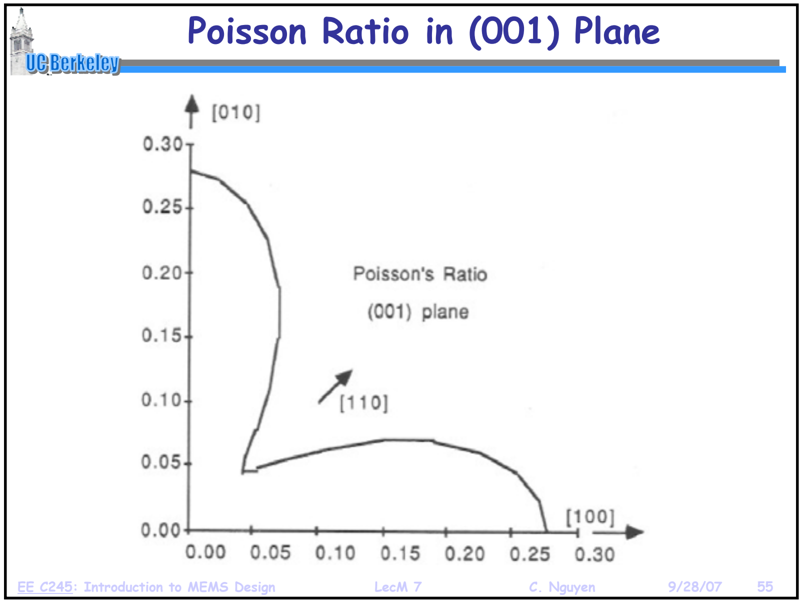
- Due to symmetry, only a few of the 21 coefficients are non-zero
- With cubic symmetry, silicon has only 3 independent components, and its stiffness matrix can be written as:

$$\begin{bmatrix}\sigma_x\\ \sigma_y\\ \sigma_z\\ \tau_{yz}\\ \tau_{zx}\\ \tau_{xy}\end{bmatrix} = \begin{bmatrix}C_{11} & C_{12} & C_{13} & 0 & 0 & 0\\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0\\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & C_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & C_{55} & 0\\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ \gamma_{yz}\\ \gamma_{zx}\\ \gamma_{xy}\end{bmatrix}$$

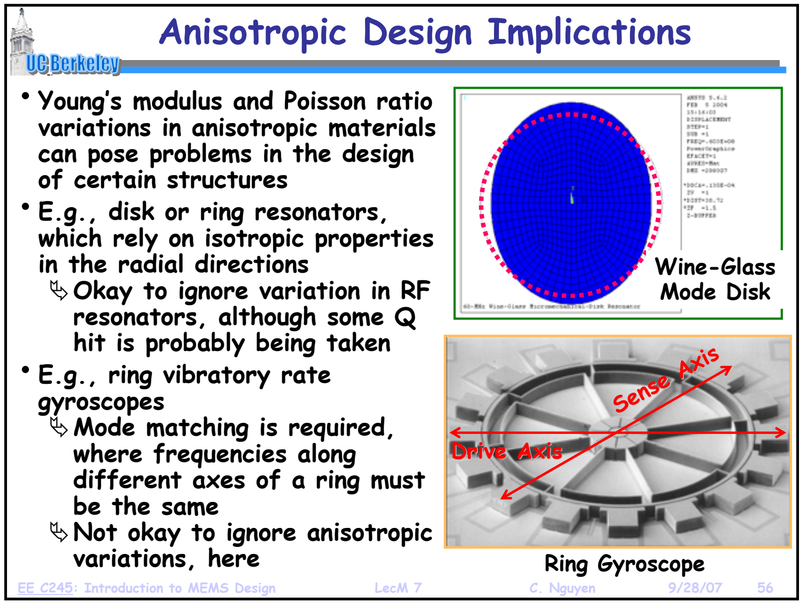
$$\text{where} \begin{cases} C_{11} = 165.7\ \text{GPa}\\ C_{12} = 63.9\ \text{GPa}\\ C_{44} = 79.6\ \text{GPa}\end{cases}$$



Young's Modulus in the (001) Plane



Poisson Ratio in (001) Plane



Anisotropic Design Implications

- Young's modulus and Poisson ratio variations in anisotropic materials can pose problems in the design of certain structures
- E.g., disk or ring resonators, which rely on isotropic properties in the radial directions
  - Okay to ignore variation in RF resonators, although some Q hit is probably being taken
- E.g., ring vibratory rate gyroscopes
  - Mode matching is required, where frequencies along different axes of a ring must be the same
  - Not okay to ignore anisotropic variations, here

Wine-Glass Mode Disk

Ring Gyroscope