Lecture 14: Beam Bending

- Announcements:
- HW#4 is now due on Tuesday, Oct. 19
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- Reading: Senturia, Chpt. 8
- Lecture Topics:
  - Stress, strain, etc., for isotropic materials
  - Thin films: thermal stress, residual stress, and stress gradients
  - Internal dissipation
  - MEMS material properties and performance metrics
- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients
- ---------------------------------
- Last Time:
- Going through material properties in Module 7
- Continue with this now
- New Topic: Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients

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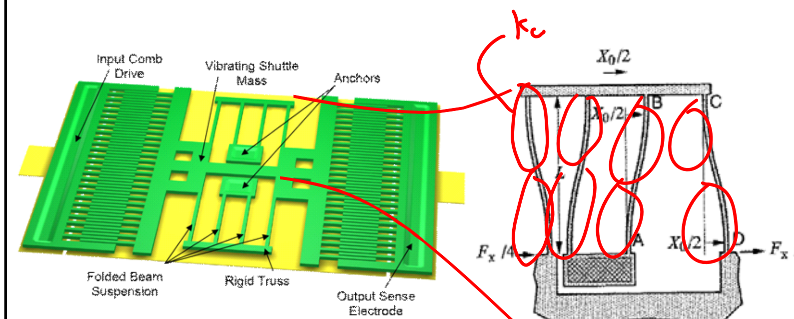
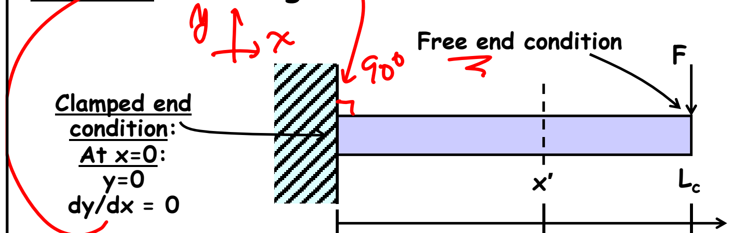
- Springs and suspensions very common in MEMS
- Coils are popular in the macro-world; but not easy to make in the micro-world
- Beams: simpler to fabricate and analyze; become "stronger" on the micro-scale → use beams for MEMS

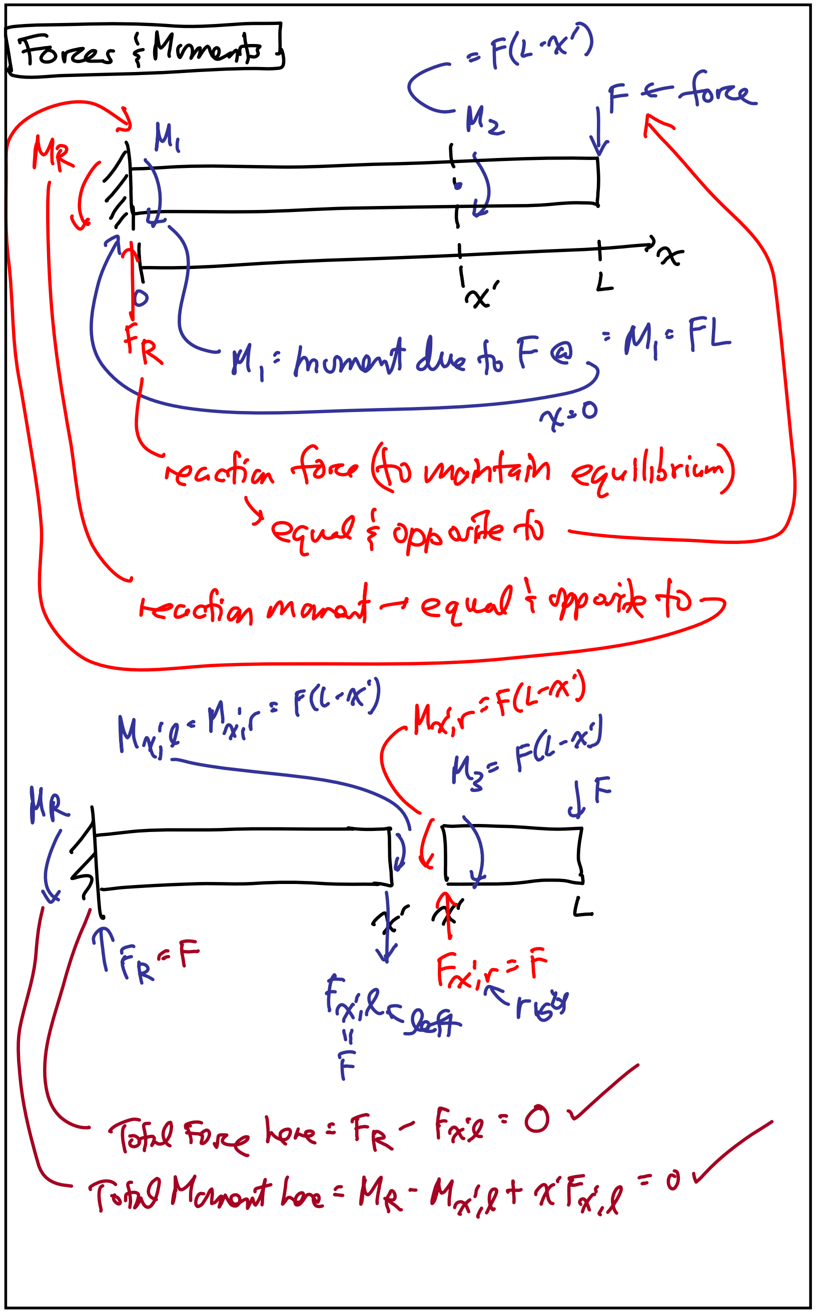


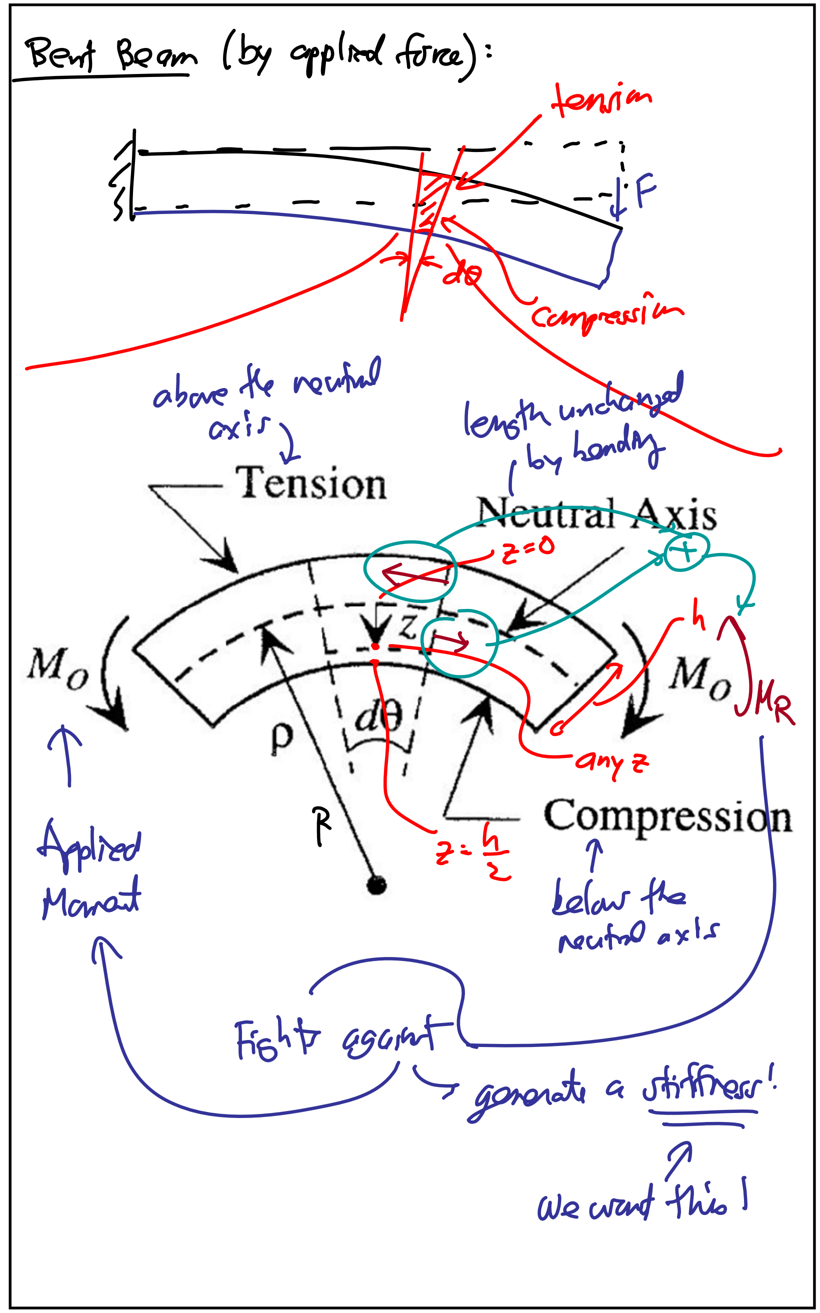
Comb-Driven Folded Beam Actuator

Problem: Bending a Cantilever Beam



- Objective: Find relation between tip deflection $y(x=L_c)$ and applied load F
- Assumptions:
  1. Tip deflection is small compared with beam length
  2. Plane sections (normal to beam's axis) remain plane and normal during bending, i.e., "pure bending"
  3. Shear stresses are negligible


Forces & Moments
M_R
M_1
M_2
$= F(L-x')$
F ← force
x
0
x'
L
F_R
M_1 = moment due to F @ $x=0$ = M_1 = FL
reaction force (to maintain equilibrium) → equal & opposite to
reaction moment → equal & opposite to
$M_{x',l} = M_{x',r} = F(L-x')$
$M_{x',r} = F(L-x')$
$M_3 = F(L-x')$
M_R
F
$F_R = F$
x'
L
$F_{x',l}$ ← left = F
$F_{x',r} = F$ ← right
Total Force here = $F_R - F_{x',l} = 0$ ✓
Total Moment here = $M_R - M_{x',l} + x'F_{x',l} = 0$ ✓


Bent Beam (by applied force):
tension
F
$d\theta$
compression
above the neutral axis
Tension
length unchanged by bending
Neutral Axis
z=0
M_o
M_o
M_R
z
h
ρ
$d\theta$
any z
R
$z = \frac{h}{2}$
Compression
Applied Moment
below the neutral axis
Fights against
→ generate a stiffness!
We want this!

Beam Segment in Bending

⇒ consider the segment bounded by the dashed lines defined by $d\theta$

At $z=0$: neutral axis → segment length $= dx = R\,d\theta$ (1)

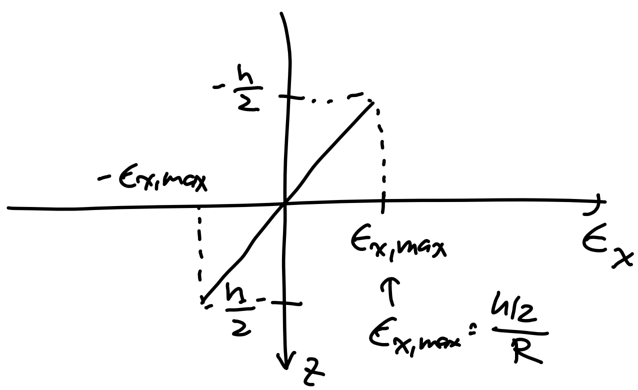
At any z: segment length $= dL = (R-z)\,d\theta$ (2)

Combine (1)&(2): $dL = dx - z\,d\theta = dx - \frac{z}{R}dx$

Thus, the axial strain @ z:

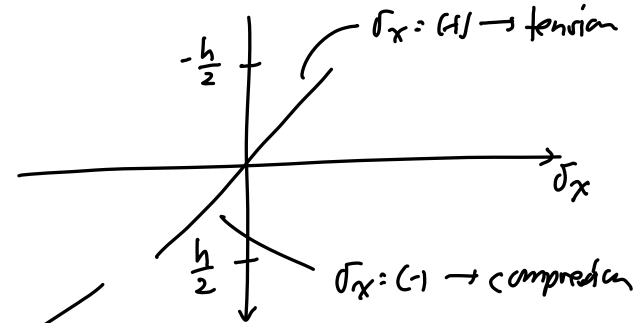
$$\epsilon_x = \frac{dL-dx}{dx} = -\frac{z}{R} \Rightarrow \boxed{\epsilon_x = -\frac{z}{R}}$$

Thus, the strain varies linearly along the beam thickness:



Of course, there is a corresponding axial stress:

$$\sigma_x = \epsilon_x E = \boxed{-\frac{zE}{R} = \sigma_x}$$



This gradient in stress → generates a bending moment! in response to the original applied moment!

Get Bending Moment