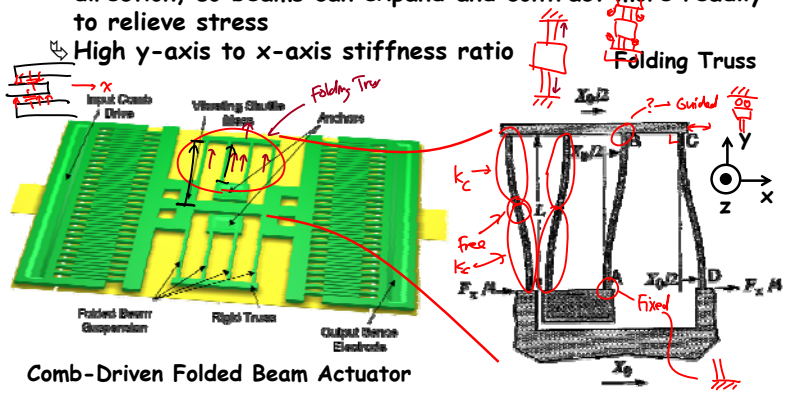
Folded-Beam Suspension

- Use of folded-beam suspension brings many benefits
  - Stress relief: folding truss is free to move in y-direction, so beams can expand and contract more readily to relieve stress
  - High y-axis to x-axis stiffness ratio



Comb-Driven Folded Beam Actuator

Beam End Conditions

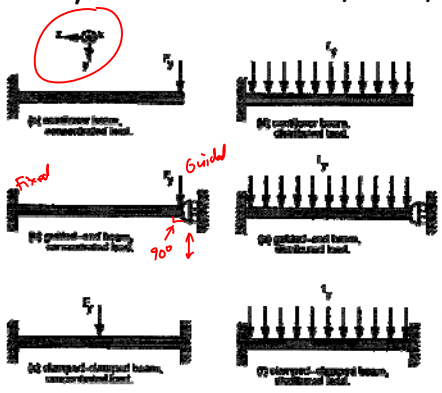
TABLE 4.1
Types of commonly used support conditions for beams and frames

Type of support	Displacement boundary conditions	Force boundary conditions
FREE	None	All, as specified
PINNED	$u = 0$, $w = 0$	Moment is specified
ROLLER (vertical)	$u = 0$	Transverse force and moment are specified
ROLLER (horizontal)	$w = 0$	Horizontal force and bending moment are specified
FIXED or CLAMPED	$u = 0$, $w = 0$, $dw/dx = 0$	None specified

[From Reddy, Finite Element Method]

Common Loading & Boundary Conditions

- Displacement equations derived for various beams with concentrated load F or distributed load f
- Gary Fedder Ph.D. Thesis, EECS, UC Berkeley, 1994



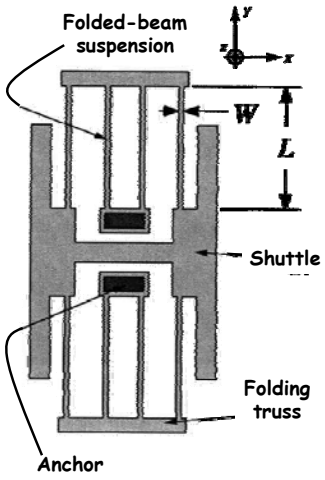
cantilever	guided-end	fixed-fixed
$x = \frac{F_x L}{Ehw}$	$x = \frac{F_x L}{Ehw}$	$x = \frac{F_x L}{4Ehw}$
$y = 4\frac{F_y}{Eh}\frac{L^3}{w^3}$	$y = \frac{F_y}{Eh}\frac{L^3}{w^3}$	$y = \frac{1}{16}\frac{F_y}{Eh}\frac{L^3}{w^3}$
$z = 4\frac{F_z}{Ew}\frac{L^3}{h^3}$	$z = \frac{F_z}{Ew}\frac{L^3}{h^3}$	$z = \frac{1}{16}\frac{F_z}{Ew}\frac{L^3}{h^3}$

(a) Concentrated load.

cantilever	guided-end	fixed-fixed
$x = \frac{f_x L}{E}$	$x = \frac{f_x L}{E}$	$x = \frac{f_x L}{4E}$
$y = \frac{3}{2}\frac{f_y}{Eh}\frac{L^4}{w^3}$	$y = \frac{1}{2}\frac{f_y}{Eh}\frac{L^4}{w^3}$	$y = \frac{1}{32}\frac{f_y}{Eh}\frac{L^4}{w^3}$
$z = \frac{3}{2}\frac{f_z}{Ew}\frac{L^4}{h^3}$	$z = \frac{1}{2}\frac{f_z}{Ew}\frac{L^4}{h^3}$	$z = \frac{1}{32}\frac{f_z}{Ew}\frac{L^4}{h^3}$

(b) Distributed load.

Folded-Beam Stiffness Ratios



- In the x-direction:

$$k_x = \frac{24EI_z}{L^3}$$

- In the z-direction:
  - Same flexure and boundary conditions

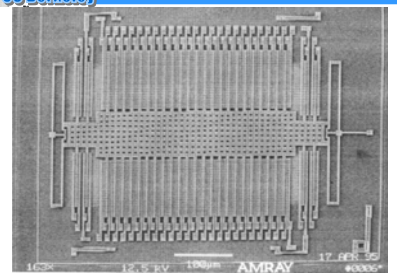
$$k_z = \frac{24EI_x}{L^3}$$

- In the y-direction:

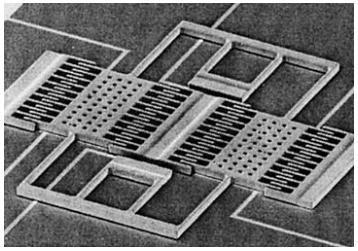
[See Senturia, §9.2] $k_y = \frac{8EWh}{L}$

- Thus: $\frac{k_y}{k_x} = 4\left(\frac{L}{W}\right)^2$ Much stiffer in y-direction!

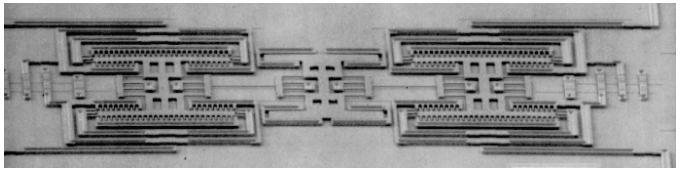
Folded-Beam Suspensions Permeate MEMS



Accelerometer [ADXL-05, Analog Devices]



Gyroscope [Draper Labs.]



Micromechanical Filter [K. Wang, Univ. of Michigan]

Folded-Beam Suspensions Permeate MEMS

- Below: Micro-Oven Controlled Folded-Beam Resonator