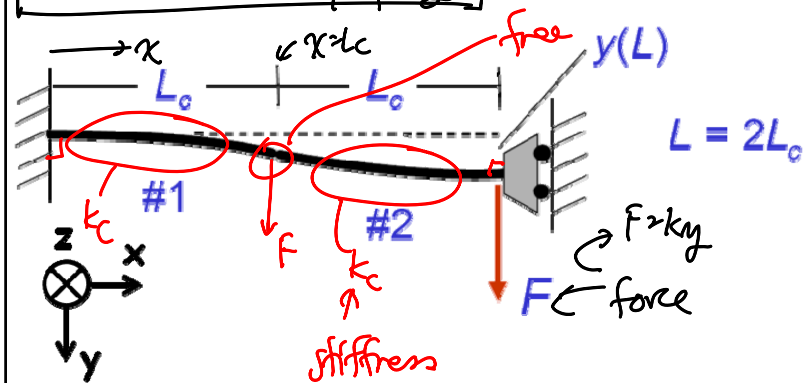
Lecture 16: Beam Combos

- Announcements:
- HW#5 passed out
- Midterm will be Tuesday, Nov. 2
- Class accounts:
  - You can request a new 'named' account by logging into:
- hostname: cory.eecs.berkeley.edu
- username: newacct
- password: newacct
  - You can also login as user 'newacct' in 199 Cory on computers that are under the "newacct" signs
- ------------------------------------
- Reading: Senturia, Chpt. 9
- Lecture Topics:
  - Bending of beams
  - Cantilever beam under small deflections
  - Combining cantilevers in series and parallel
  - Folded suspensions
  - Design implications of residual stress and stress gradients
- ---------------------------------
- Last Time:
- Going through Module 8, looking at folded-flexure suspensions
- Looked at end conditions, i.e., boundary conditions
- Started considering beam combos

Series Combination of Springs



Series: total displacement equals the sum of the displacement of each beam

$$y(L) = \frac{F}{k} = 2y(L_c) = 2\left(\frac{F}{k_c}\right) = F\left(\frac{1}{k_c} + \frac{1}{k_c}\right)$$

total stiffness (k); compliance ($\frac{1}{k_c}$)

∴ compliances add!

$$\boxed{\frac{1}{k} = \frac{1}{k_c} + \frac{1}{k_c}} \rightarrow \boxed{k = k_c \| k_c}$$

For springs (k_c) in series

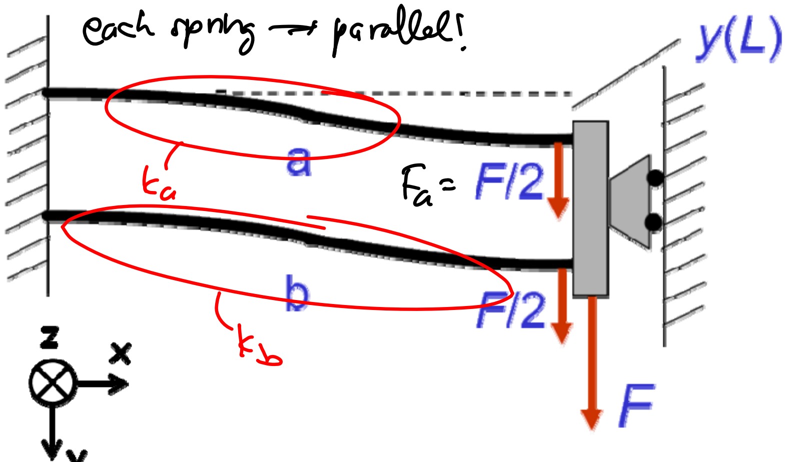
"parallel" operator

$$A \| B = \frac{1}{\frac{1}{A} + \frac{1}{B}}$$

operator

Parallel Combination of Springs

→ if y(L) = total displacement = displacement of each spring → parallel!

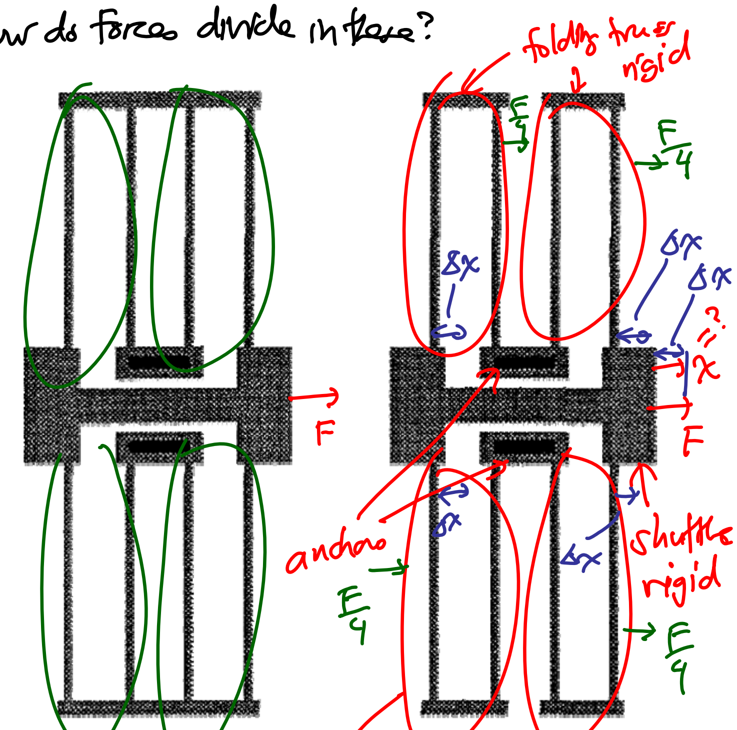


$$y(L) = \frac{F}{k} = \frac{F_a}{k_a} = \frac{F_b}{k_b} = \left(\frac{F}{2}\right)\left(\frac{1}{k_a}\right)$$

$$\boxed{k = 2k_a}$$

For springs in parallel stiffnesses add directly.

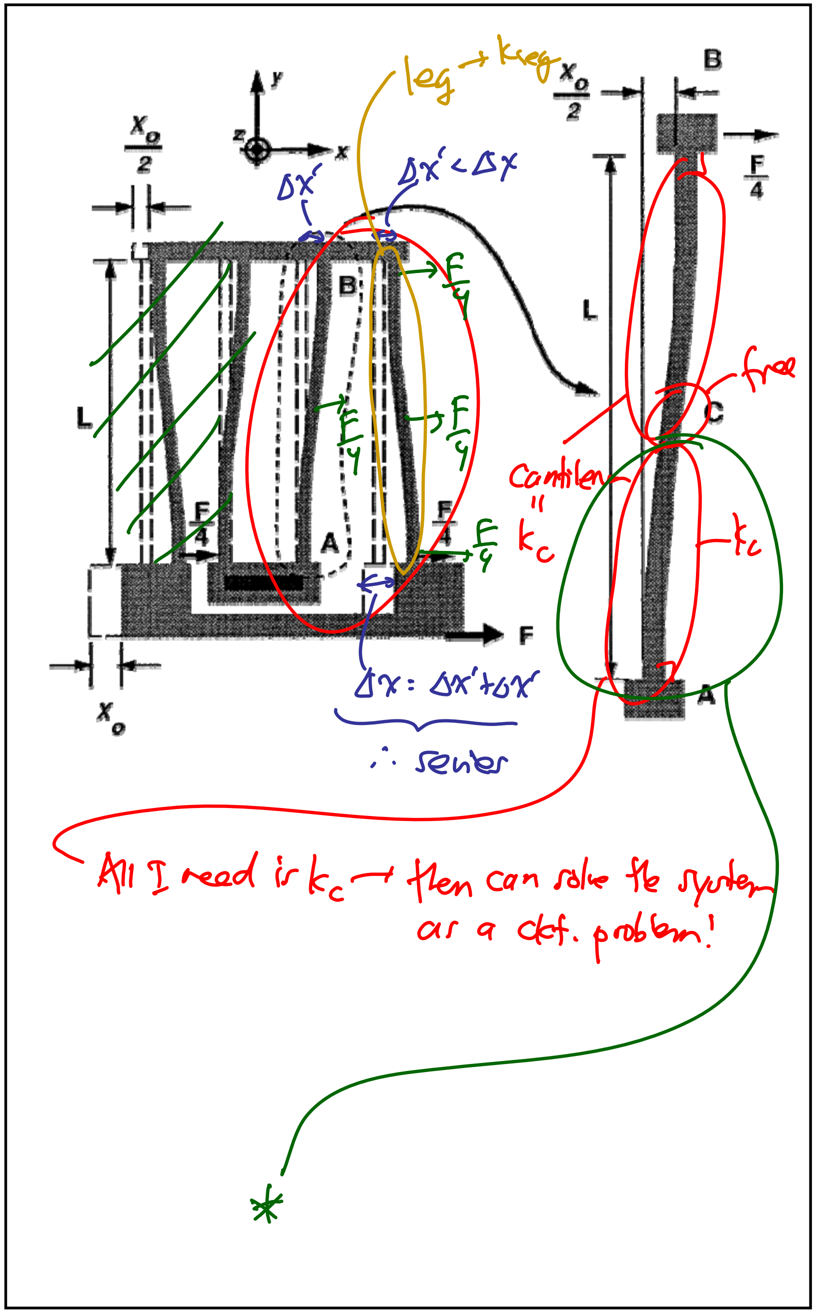
How do forces divide in these?

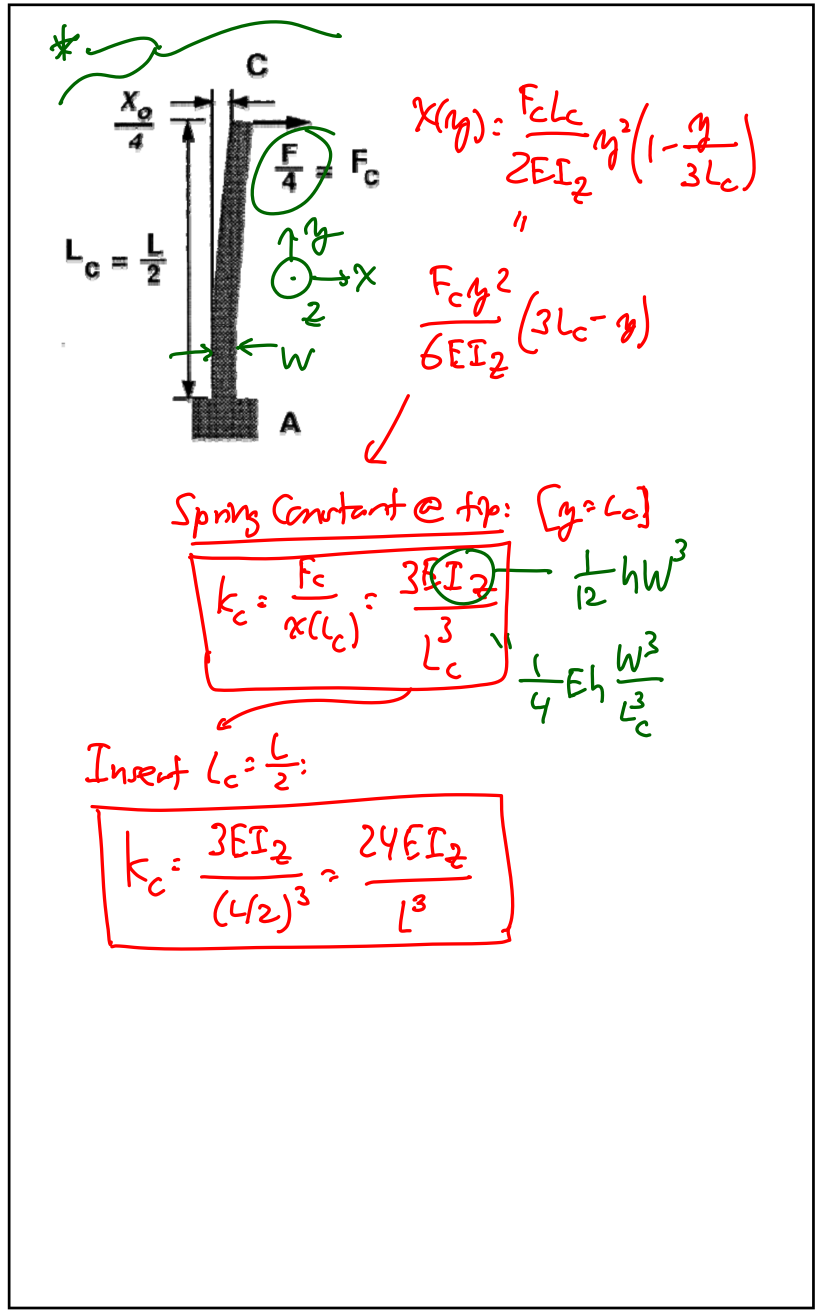


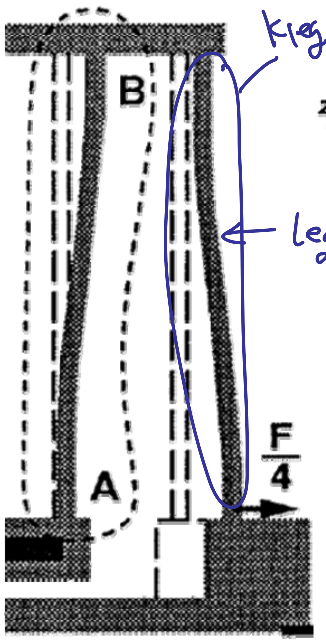
(a) Inner fold, continuous truss (b) Inner fold, discontinuous truss

4 of these; identical

Series or Parallel?


leg → kseg
$\frac{X_o}{2}$
B
A
C
L
$\frac{F}{4}$
F
X_o
$\Delta x'$
$\Delta x' < \Delta x$
$\Delta x = \Delta x' + \Delta x'$
∴ series
free
cantilever
k_c
All I need is k_c → then can solve the system as a def. problem!


C
$\frac{X_o}{4}$
$\frac{F}{4} = F_c$
$L_c = \frac{L}{2}$
W
A
$x(y) = \frac{F_c L_c}{2EI_z} y^2 \left(1 - \frac{y}{3L_c}\right) = \frac{F_c y^2}{6EI_z}(3L_c - y)$
Spring Constant @ tip: $[y = L_c]$
$k_c = \frac{F_c}{x(L_c)} = \frac{3EI_z}{L_c^3} = \frac{1}{4}Eh\frac{W^3}{L_c^3}$
$I_z = \frac{1}{12}hW^3$
Insert $L_c = \frac{L}{2}$:
$k_c = \frac{3EI_z}{(L/2)^3} = \frac{24EI_z}{L^3}$



leg → Displacement of the two legs add ∴ they are in series

$$x = \frac{F_{pair}}{k_{pair}} = \frac{F_{pair}}{(k_{leg} \| k_{leg})}$$

$$= \left(\frac{F}{4}\right)\left(\frac{1}{k_{leg}} + \frac{1}{k_{leg}}\right)$$

stiffness of pair

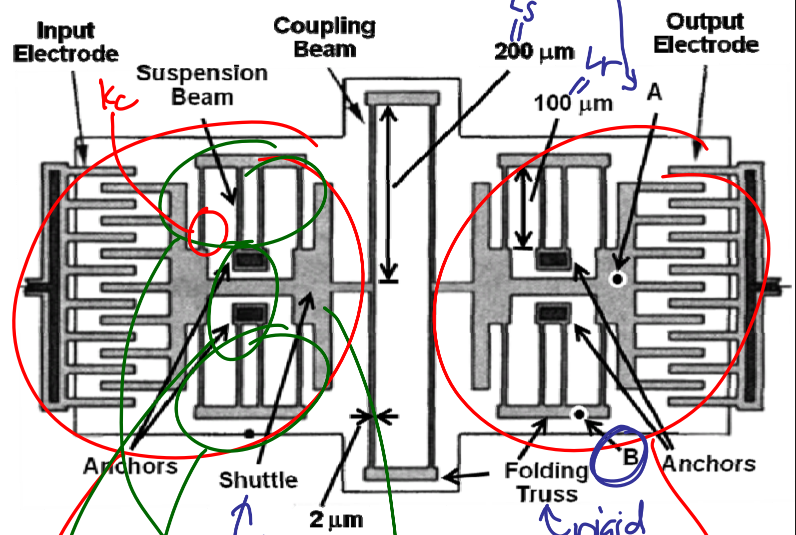
From before: $k_{leg} = k_c \| k_c = \frac{k_c}{2}$

Thus:

$$x = \left(\frac{F}{4}\right)\left(\frac{2}{k_c} + \frac{2}{k_c}\right) = \frac{F}{k_c} = \frac{F}{k_{tot}}$$

$$k_{tot} = k_c = \frac{24EI_z}{L^3}$$

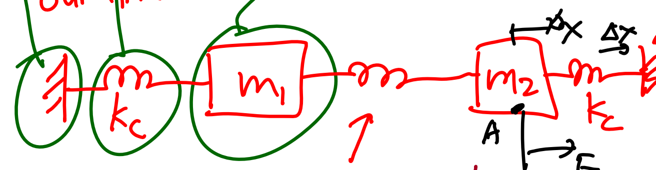
Find the stiffness @ point A:



→ apply F@A → get x ⇒ F = kx

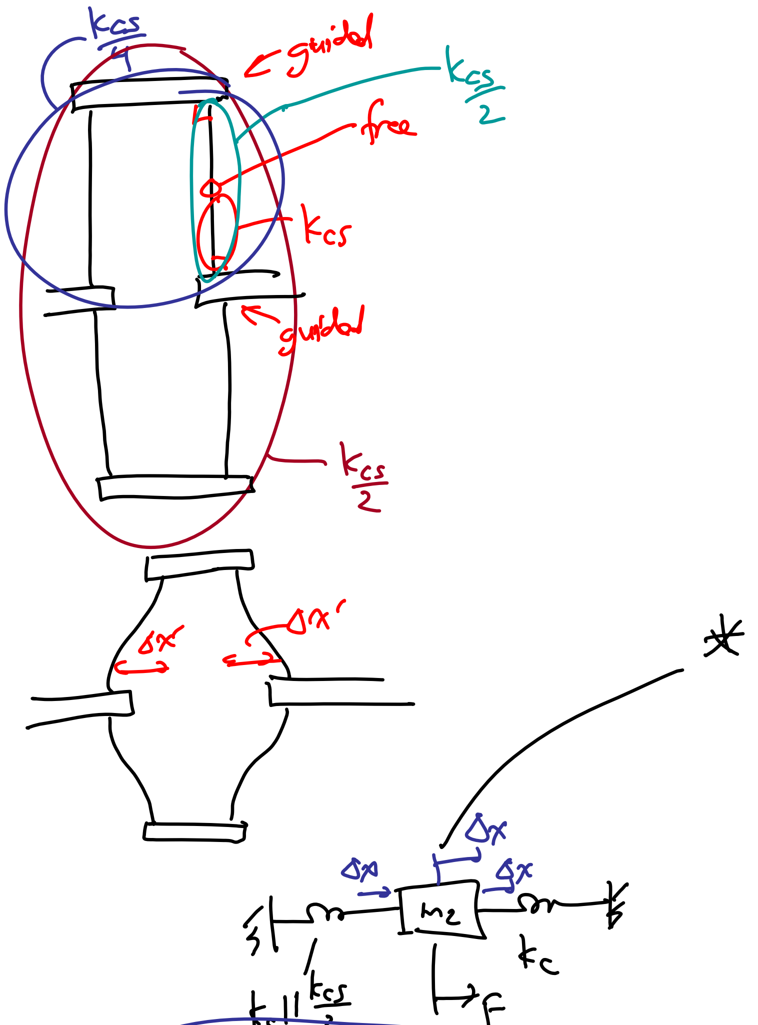
↑ want this!

our friend!



k_s = ? ⇒ $\frac{k_{cs}}{2}$

Δx $k_c \| \frac{k_{cs}}{2}$



$$k_{xA} = k_c + k_c \| \frac{k_{cs}}{2}$$

where $k_c = \frac{24EI_z}{L_r^3}$ and $k_{cs} = \frac{24EI_z}{L_s^3}$