EE C245 – ME C218
Introduction to MEMS Design
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Lecture Module 9: Energy Methods

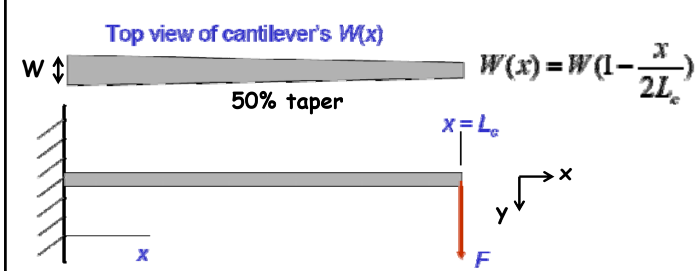
Lecture Outline

- Reading: Senturia, Chpt. 10
- Lecture Topics:
  - Energy Methods
    - Virtual Work
    - Energy Formulations
    - Tapered Beam Example

Energy Methods

More General Geometries

- Euler-Bernoulli beam theory works well for simple geometries
- But how can we handle more complicated ones?
- Example: tapered cantilever beam
- Objective: Find an expression for displacement as a function of location x under a point load F applied at the tip of the free end of a cantilever with tapered width W(x)



Solution: Use Principle of Virtual Work

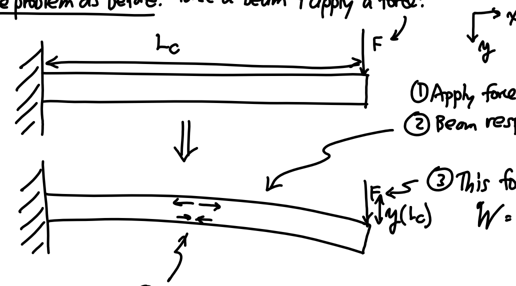
- In an energy-conserving system (i.e., elastic materials), the energy stored in a body due to the quasi-static (i.e., slow) action of surface and body forces is equal to the work done by these forces …
- Implication: if we can formulate **stored energy** as a function of the deformation of a mechanical object, then we can determine how an object responds to a force by determining the shape the object must take in order to **minimize** the **difference U** between the stored energy and the work done by the forces:

$$U = \text{Stored Energy} - \text{Work Done}$$

- Key idea: we don't have to reach U = 0 to produce a very useful, approximate *analytical* result for load-deflection

More Visual Description …

Same problem as before: Take a beam & apply a force:



① Apply force.
② Beam responds by bending.
③ This force has done work: $W = F \cdot y(L_c)$
④ Strain generated → This means the beam has received an influx of stored energy → magnitude determined by its deformed shape
⑤ Then:
$U = \text{Stored Energy} - \text{Work Done} \rightarrow 0$
(When we choose the right shape! (This is how we get the beam's response to F!)