Lecture 20: Equivalent Circuits

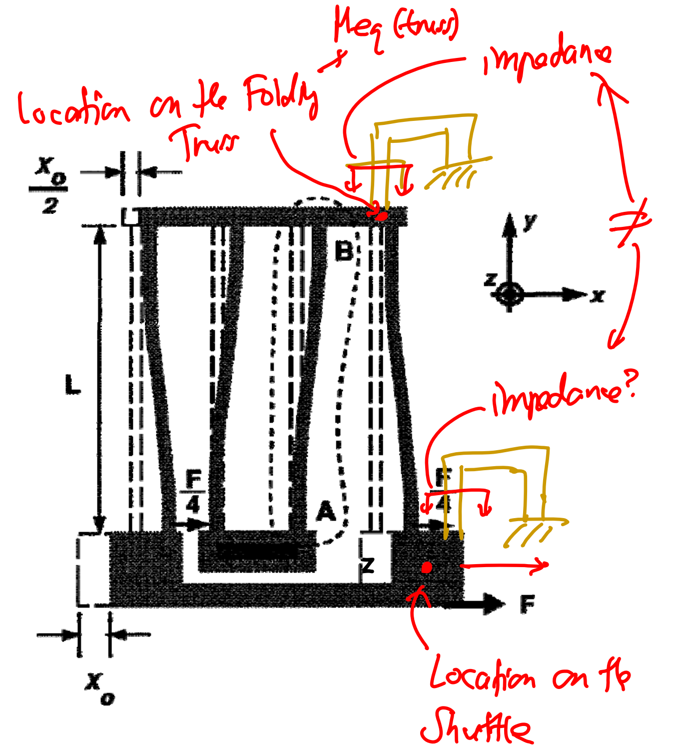
- Announcements:
- HW#6 will be posted online today
- Project handed out and described today
- Looking for handouts? Box outside my office.
- -----------------------------------
- Reading: Senturia, Chpt. 5
- Lecture Topics:
  - Lumped Mechanical Equivalent Circuits
  - Electromechanical Analogies
- Reading: Senturia, Chpt. 5, Chpt. 6
- Lecture Topics:
  - Energy Conserving Transducers
  - Parallel-Plate Capacitive Transducers
- -----------------------------------
- Last Time:
- Derived the following for the resonance frequency of a folded beam resonator:

$$\omega_0 = \left[\frac{k_c}{M_{eq}}\right]^{\frac{1}{2}}$$

where $M_{eq} = M_s + \frac{1}{4}M_t + \frac{12}{35}M_b$

(Resonance Freq. of a Folded-Beam Suspended Shuttle)

Dynamic Mass !!!



Meq (shuttle)

Maximum Kinetic Energy

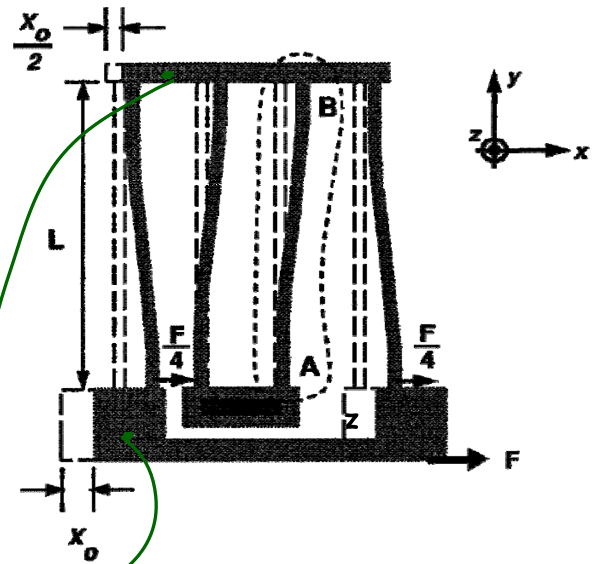
Equivalent Mass:

$$\text{Equiv. Mass: } M_{eq_x} = \frac{KE_{max}}{\frac{1}{2}V_x^2} = \frac{\cancel{\frac{1}{2}}\rho A\int_0^l V_x^2(x)\,dx}{\cancel{\frac{1}{2}}V_x^2}$$

Dynamic

x-direction

velocity in x direction @ the location in question



Location on the Shuttle

$$M_{eq(shuttle)} = \frac{KE_{max}}{\frac{1}{2}V_{shuttle}^2} = \frac{\cancel{\omega_o^2 x_o^2 \left(\frac{1}{2}\right)}\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]}{\cancel{\frac{1}{2}\omega_o^2 x_o^2}}$$

$$\therefore M_{eq(shuttle)} = M_s + \frac{1}{4}M_t + \frac{12}{35}M_b$$

Location on the Truss

$$M_{eq(truss)} = \frac{KE_{max}}{\frac{1}{2}V_{truss}^2} = \frac{\cancel{\omega_o^2 x_o^2 \left(\frac{1}{2}\right)}\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]}{\cancel{\frac{1}{2}}\left(\frac{1}{4}\right)\cancel{\omega_o^2 x_o^2}}$$

$$\therefore M_{eq(truss)} = 4\left[M_s + \frac{1}{4}M_t + \frac{12}{35}M_b\right]$$

Once mass is known → can get:

Stiffness

y ≡ location of interest

$$\omega_o = \sqrt{\frac{K_{eq}(y)}{M_{eq}(y)}} \rightarrow \boxed{K_{eq}(y) = \omega_o^2 M_{eq}(y)}$$

if mass @ location y is large, so will be the stiffness

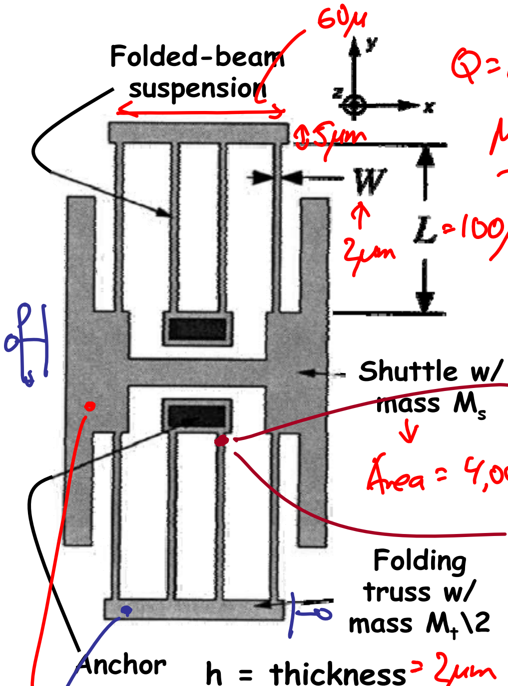
Damping → follows from knowledge of Q

$$Q = \frac{\omega_o M_{eq}(y)}{C_{eq}(y)} \rightarrow \boxed{C_{eq}(y) = \frac{\omega_o M_{eq}(y)}{Q} = \frac{\sqrt{K_{eq}(y) M_{eq}(y)}}{Q}}$$

damping

$$\left[m\ddot{x} + c\dot{x} + kx = F\right]$$

sometimes will call it b



Q = 100k

Material: poly Si

E = 150 GPa

ρ = 2300 kg/m³

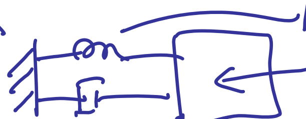
Area = 4,000 μm²

h = thickness = 2μm

$K_{eq(shuttle)} = 4.8$ N/m

$M_{eq(shuttle)} = 2.16 \times 10^{-11}$ kg

$C_{eq(shuttle)} = 1.02 \times 10^{-10}$ kg/s

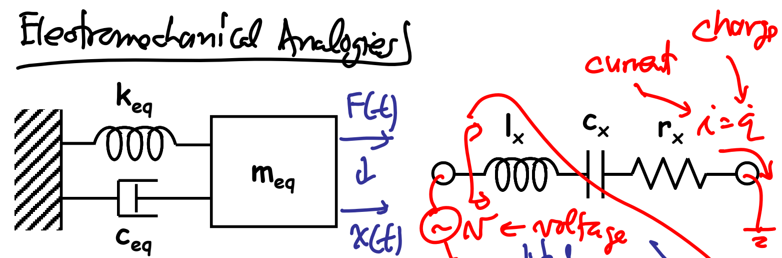


$K_{eq(truss)} = 19.2$ N/m

$M_{eq(truss)} = 8.64 \times 10^{-4}$ kg

$C_{eq(truss)} = 4.08 \times 10^{-10}$ kg/s

Electromechanical Analogies



$F(t) = F\cos(\omega t) \to x(t) = X\cos(\omega t)$

lower case variable → signal

upper case → dc

Equation of Motion:

$$m_{eq}\ddot{x} + c_{eq}\dot{x} + k_{eq}x = F(t)$$

⇒ use phasor concepts:

$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

$v(t) = V\cos(\omega t) \to i(t) = I\cos(\omega t)$

Impedance looking in:

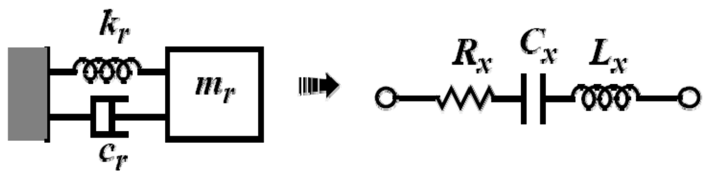
$$\frac{v}{i} = j\omega l_x + \frac{1}{j\omega C_x} + r_x$$

$$v = j\omega l_x i + \frac{(1/C_x)}{j\omega} i + r_x i$$

⇒ by analogy:

$$\boxed{F \to v \quad m_{eq} \to l_x \quad c_{eq} \to r_x \quad \dot{x} \to i \quad k_{eq} \to \frac{1}{C_x}}$$

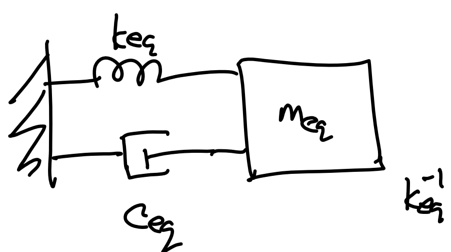
Parameter Relationships in the Current Analogy

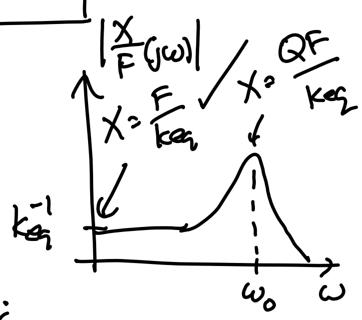


- Mechanical-to-electrical correspondence in the current analogy:

Mechanical Variable	Electrical Variable
Damping, c	Resistance, R
Stiffness^{-1}, k^{-1}	Capacitance, C
Mass, m	Inductance, L
Force, f	Voltage, V
Velocity, v	Current, I

~~Bandpass~~ Lowpass Biquad Transfer Function





$$F = j\omega m_{eq}\dot{x} + \frac{k_{eq}}{j\omega}\dot{x} + c_{eq}\dot{x}$$

$\Rightarrow$ convert to full phasor form:

$$F = (j\omega)j\omega X m_{eq} + \frac{k_{eq}}{\cancel{j\omega}}(\cancel{j\omega}X) + c_{eq}(j\omega X)$$

$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\omega^2\frac{m_{eq}}{k_{eq}} + 1 + j\frac{c_{eq}\omega}{k_{eq}}\right]^{-1} \quad *$$

$$\left[\frac{k_{eq}}{m_{eq}} = \omega_o^2,\quad Q = \frac{m_{eq}\omega_o}{c_{eq}} = \frac{k_{eq}}{\omega_o c_{eq}} \rightarrow \frac{k_{eq}}{c_{eq}} = Q\omega_o\right]$$

$*$

$$\frac{X}{F}(j\omega) = \frac{1}{k_{eq}}\left[-\left(\frac{\omega^2}{\omega_o^2}\right) + 1 + j\frac{\omega}{Q\omega_o}\right]^{-1}$$

$$\boxed{\frac{X}{F}(j\omega) = \frac{k_{eq}^{-1}}{1 - \left(\frac{\omega}{\omega_o}\right)^2 + j\frac{\omega}{Q\omega_o}}}$$

- Go through pages 11-22 of Module 11
- Then, start into Module 12